RELIABILITY, SCHEDULING MARKETS, AND ELECTRICITY PRICING

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RELIABILITY, SCHEDULING MARKETS, AND ELECTRICITY PRICING

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Advance scheduling of electricity markets could create a conflict between reliability requirements and market operations. Unlike the case for the real-time market alone, an independent system operator charged with preserving system reliability may face a new computational challenge in balancing the requirements of markets and system security. A bid-based, security-constrained, economic-dispatch formulation identifies the criteria for scheduling, interprets the market-clearing prices, and points to a simple approximation that accommodates market choices and is consistent with traditional dispatch procedures.

INTRODUCTION

An independent system operator (ISO) for an electricity market will have responsibility for ensuring the reliability of the system. To guarantee reliability, the ISO must schedule enough generating capacity to provide sufficient operating reserves of spinning and fast-start generation in excess of the energy producing capacity needed to meet load. In the context of day-ahead scheduling markets and a two-settlement system, a question arises as to the treatment of reliability constraints and the impact on locational prices and other components of the market as implemented by an ISO. The purpose of this paper is to outline a general framework for determination of prices when the ISO is coordinating market bids and committing additional generation to meet anticipated reliability constraints.

Special attention to the scheduling market formulation is important because it imposes a new obligation on the ISO. System operators have been implementing security-constrained, economic dispatch for many years. Adapting the operating protocols to a competitive electricity

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market has required little change in the formulation of the dispatch problem, with all of the innovation coming in rules for access eligibility, pricing and settlements. Since these latter elements would not change the form of the dispatch criteria, movement to a competitive market has been greatly simplified, with the complexity of operating the system left to familiar methods in the hands of the system operator. However, in principle, this continuity in the formulation of the dispatch problem could be disrupted in part by the demands of a voluntary scheduling market coupled with the reliability responsibilities of the ISO.[2]

The disconnect would arise in the case of the voluntary bids or advance schedules of market participants being materially different from the ISO's forecast of real-time load that could develop and would need an advance commitment of capacity to ensure reliability. In the world of vertical integration and monopoly, there could have been no divergence because there was only one decision-maker. In the real-time dispatch, there is only one system load, and any previous forecast is by then irrelevant. But in a scheduling market, there can be more than one forecast. A formulation of this scheduling market identifies the principal difference from the traditional formulation of the dispatch problem. An analysis of the generic model identifies the pricing and other implications for the market, and then relates the results to a practical implementation in terms of the traditional dispatch protocols and models. The scheduling market innovation can be accommodated without major difficulty, and the resulting pricing and settlements can be interpreted in terms of the traditional formulation for economic dispatch. Any additional capacity commitments could be treated as a form of ancillary service with payments included as part of an uplift charge.

ENERGY PRICING AND ANCILLARY SERVICES

The costs associated with delivery of electric energy include many services. The direct fuel cost of generation is only one component. Others include reactive power support, spinning reserve, regulation and so on. In analyses of energy pricing, there is no uniformity in the treatment of these other, ancillary services.[3] The typical approach formulates an explicit model approximating the full electricity system, computing both a dispatch solution and associated prices for the explicit variables. Everything that is not explicit is treated as an ancillary service, for which the assumption must be that the services will be provided and charged for in some way other than through the explicit prices in the model. Given the complexity of the real electric system, such approximations or simplifications are found in every model, and there is always a boundary between the explicit variables modeled and the implicit variables that will be treated as separate ancillary services. Payment for the ancillary services may be through an average cost uplift applied to all loads. However, the explicit model is usually silent on the connection between the pricing of the explicit and implicit components.

Development of a full description of the interactions with ancillary services is beyond the scope of the present discussion. In some cases, such as for losses and reactive power, the connection is well understood, in principle, and could be interpreted as already included in the

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[2]   This problem was called to our attention by Robert Pike.

[3]   E Hirst and B. Kirby, "Creating Competitive Markets for Ancillary Services," ORNL/CON-448, Oak Ridge National Laboratory, Oak Ridge, TN, October 1997.

explicit model.[4] In other cases, such as for black-start capability, the connection to market prices would not be straightforward. In the present discussion, formulation of the model makes explicit a treatment of spinning or fast-response operating reserves and reliability commitments that have in other contexts been treated implicitly as ancillary services. Hence, part of the analysis of the model consists of moving between alternative formulations of the problem to identify the effects of different treatments of these services. In the end, application of the pricing model might take a simpler form, with some of the payments included as part of ancillary services. The purpose here is to identify the connections to alternative formulations of the scheduling and dispatch problem.

A second thread in the analysis is found in the underlying formulation of the problem. Procedures for achieving an efficient and reliable economic dispatch have received extensive attention and the problem is familiar to electric system operators. One approach formulates the dispatch problem under uncertainty.   From this perspective, the ISO faces a probability distribution for load, generation, and the availability of the transmission system. The task is to find the expected least-cost or welfare-maximizing dispatch subject to the various operating constraints on the system. Under certain conditions, this solution gives rise to a set of prices for various products that would be consistent with a competitive market equilibrium.[5]

Formulation of the dispatch problem to take explicit account of uncertainty would impose substantial informational and computational requirements on the ISO and the market participants. Importantly, with this approach market participants and the ISO would be assumed to know the probability distribution across the set of all possible events. Given the practical limitations of the full treatment of uncertainty, therefore, a common second-best approach to the dispatch problem in practice has been to select a subset of the possible events and treat these contingencies as constraints. This formulation of a security-constrained, economic-dispatch problem balances load and generation at least cost in the normal or "no-event" case, subject to the constraint that the resulting dispatch would be feasible in the event of any of the contingencies. This approach also produces prices and allows contracts, such as transmission congestion contracts, that would be consistent with a market equilibrium under the same security conditions.[6]   This contingency-constrained framework defines the scheduling and dispatch problem examined below.

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[4]   William W. Hogan, "Markets in Real Electric Networks Require Reactive Prices," Energy Journal, Vol.14, No.3, 1993.

[5]   Michael C. Caramanis, Roger E. Bohn, aand Fred C. Schweppe, "System Security Control and Optimal Electricity Pricing," Electrical Power and Energy Systems, Vol. 9, No. 4, October 1987, pp. 217-224. Fred C. Schweppe, Michael. C. Caramanis, Richard. D. Tabors, and Roger.E. Bohn, Spot Pricing of Electricity, Kluwer Academic Publishers, Norwell, MA, 1988. E. Grant Read, Glenn R. Drayton-Bright and Brendan J. Ring: "An Integrated Energy/Reserve Market for New Zealand", in G. Zaccours and A. LaPointe (eds) Deregulation of Electric Utilities, Kluwer, Boston (to appear).   Brendan Ring and Grant Read, "Pricing for Reserve Capacity in a Competitive Electricity Market," University of Canterbury, New Zealand, June 1994.   R. John Kaye, Felix F. Wu, Pravin Varaiya, "Pricing for System Security," IEEE Transactions on Power Systems, Vol. 10, No. 2, May 1995, pp. 575-583. Laurence D. Kirsch and Rajesh Rajaraman, "Profiting from Operating Reserves," The Electricity Journal, March 1998, pp. 40-49.

[6]   Scott M. Harvey, William W. Hogan and Susan L. Pope, "Transmission Capacity Reservations and Transmission Congestion Contracts," Harvard University, June 6, 1996, (revised March 8, 1997). (http://ksgwww.harvard.edu/people/whogan).

SCHEDULING MARKET

The lead time required to commit and start generating units means that the system operator must make decisions in advance of real-time operation. Although the advance notice required for particular generating units varies from a few minutes to days or longer, a common decision in the design of new competitive electricity markets has been to settle on a day-ahead schedule and commitment. Before the day-ahead decision, market participants make their own commitment decisions.

The day-ahead scheduling opportunity should provide other benefits for market participants in the form of greater certainty about market transactions. In a two-settlement system, the day-ahead schedule would define a set of prices and contracts. The settlement for the day-ahead commitments would be completed at the day-ahead prices and quantities. Then in real time, any deviations from the contracts would be settled at the real-time prices. Hence, there would be two sets of dispatch quantities and associated prices, the first for the day-ahead bids and the second based on real-time bids.[7]

We assume the ISO has a responsibility to ensure that enough generation capacity is scheduled a day ahead to meet reliability requirements. In a market structure with day-ahead bidding and scheduling, the question arises as to the economic criterion to apply in order to meet the load and the reliability requirements. If the day-ahead schedule and the actual dispatch were the same, there would be no difficulty. The operator would commit enough total capacity to allow for generation to meet load. The network reliability requirements could be represented as a system of contingency constraints on spinning reserve and net loads. Subject to these constraints, the ISO would choose the dispatch or schedule that maximized benefits minus costs, in the usual way.

When the ISO makes a judgment that the day-ahead bids do not capture the full load that is likely to appear in real time, the ISO may modify the formulation in order to meet the reliability requirements. In this event, the bids would apply to the day-ahead schedule, but the ISO would have a different forecast of load. The ISO must decide what commitments to make in the face of the different forecasts. The extreme case of "letting the market decide," by treating the day-ahead bids as the best forecast of the actual load, would appear to discard the concerns for reliability. By definition, ignoring the expected discrepancy between the bids and the forecast would require the ISO to accept what it sees as unreliable operating conditions. This is unlikely to be the choice.

The complementary approach of ignoring the bids and scheduling the system for both energy and capacity to meet the forecast rather than the bid-in load would compromise the market. The ISO would be augmenting the load bids of market participants, buying power and spreading the risk. A major objection would be that the ISO would be choosing a least-cost solution even for the benefit of load that chose not to participate in the day-ahead market. The

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[7]   The same principle could apply to a three settlement system, with day-ahead, hour-ahead, and real-time dispatch and prices. Here deviations from the day-ahead contracts would be settled at the hour-ahead prices, and deviations from the hour-ahead contracts would be settled at the real-time prices. The essential requirement for consistency is to have a separate settlement for each clearing market based on the prices for that market. The deviations at period "n" from the contracts at period "n-1" would be settled at the period "n" prices.

cost of energy for the day-ahead bidders would be correspondingly higher, and those who had not bid-in would have less incentive to participate in the day-ahead market or provide meaningful bids. The gaming opportunities could create perverse incentives that would exacerbate the difference between the bids and the ISO load forecast. Rather than forcing greater reliance on the ISO to overcome perverse incentives, a better goal would be to provide market incentives to support meaningful participation in the bidding and reduce the need for independent action by the ISO.

In order to provide the incentive for a meaningful day-ahead market, an alternative approach would distinguish between the energy bids, with their associated forward contracts for the purchase and sale of energy, and the commitment of generating capacity. The reliability needs focus on the commitment of capacity, but the economic incentives would be dominated by the energy bids. Therefore, given a policy decision that imposes the reliability responsibility on the ISO, an approach is to allow the ISO to commit additional units whenever necessary to meet the reliability standards for any ISO forecast of load that is different from the day-ahead schedules of the market participants.[8]

Given that the ISO is committing units to meet a forecast load, rather than just the bid load, we would change the formulation of the economic dispatch model. The "extra" units committed and available provide a reliability benefit for everyone. The model here approaches this problem by choosing the economic dispatch according to the day-ahead bids, subject to the constraint that the resulting dispatch commitment for spinning reserve and network limits would be feasible for the forecast conditions. In effect, the forecast load defines another set of contingency conditions. Prices and quantities determined in the advance schedule apply as contracts. Prior energy and transmission congestion contracts would be settled at the prices for the scheduling settlements. The real time balancing market prices would be used to settle deviations from the scheduling contracts. Any revenue deficiencies, where market-clearing prices do not cover the full costs of capacity commitments, would be included in an uplift charge that would apply to all load in real time. The basic structure of this market is illustrated in the accompanying figure.

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[8] Pennsylvania-New Jersey-Maryland (PJM) Interconnection, "Amended and Restated Schedule 1: Multi-Settlement System and FTR Auctions," Operating Agreement of the PJM Interconnection, Section 1.10.9, December 31, 1997. New York Power Pool, "Supplemental Filing to the Comprehensive Proposal to Restructure the New York Wholesale Electricity Market", Vol. III, New York State ISO Tariff, Section 4.12 (Original Sheet No. 11), December 19, 1997.

A Structure for Spot Market Dispatch & Settlements

Scheduling Transactions   *Settlements*
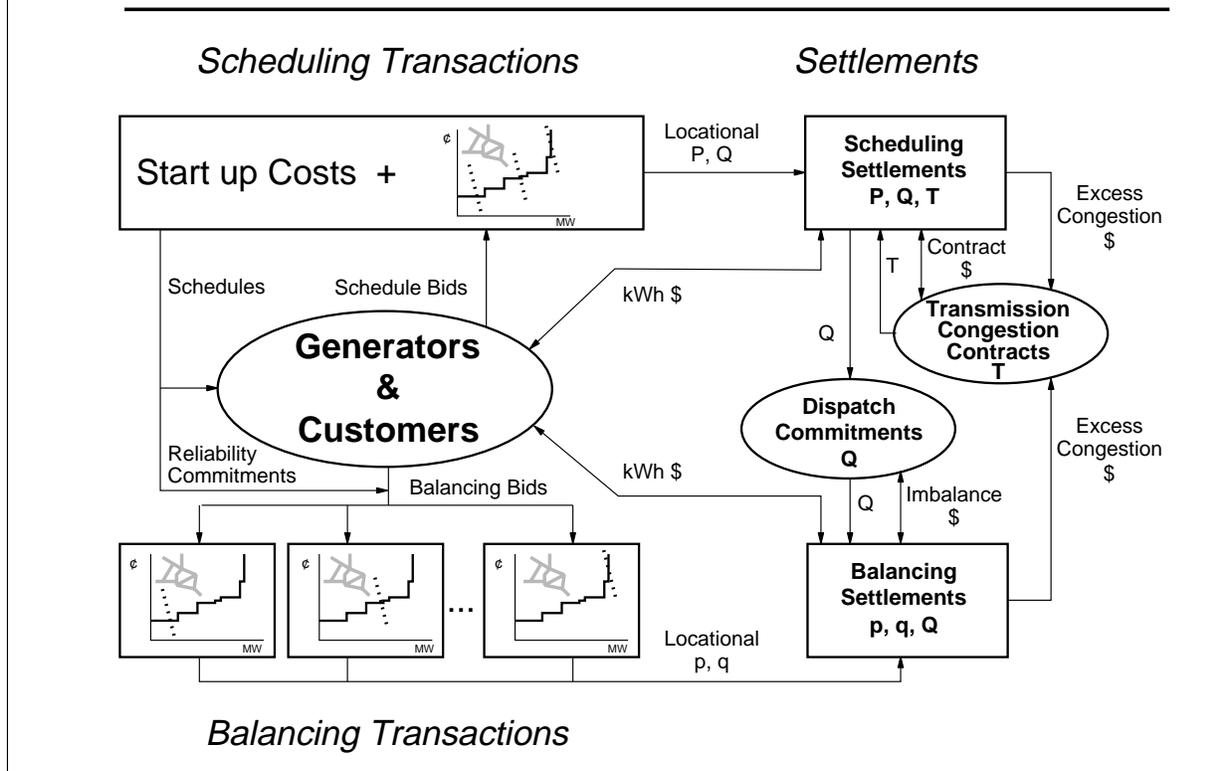
If the day-ahead bids were substantially below the ISO forecast of demand, then the ISO would commit extra units, choosing the units with low commitment cost, but not necessarily low energy costs. The day-ahead dispatch would exploit all the committed units to find the lowest energy costs, with the resulting and presumably low energy prices locked in through the contracts. In real time, the presumably higher load would produce an increase in total energy costs. Although sufficient units would be available to meet the higher load, the real-time load that had not participated in the day-ahead schedule would face these higher costs and the associated prices. All load that had been scheduled in the day-ahead market would be protected from higher real-time prices. Hence, there would be a natural market incentive to participate in the day-ahead schedule, although such participation would not be required.

SECURITY CONSTRAINED ECONOMIC DISPATCH

The following develops a formulation of the underlying security-constrained economic-dispatch formulation for which we will identify the associated market models and pricing interpretation when the commitment of units for reliability may create different transmission constraints for the forecast load. The ISO is scheduling the bids while including enough units to meet the reliability requirements for its exogenous forecast of load, D, which may be different than the bid-in load. The bid-in values are indexed by market participant. All the variables are vectors across locations in the grid.

Let:

d_j bid in load (for participant j),

D   the ISO forecast load,

g_j bid-in generation,

s_j spin for the bid-in schedule,

G_j the generation needed to meet the forecast load,

S_j the spin that would be used with forecast load, D,

y   the net load, $y = \Sigma(d_j - g_j)$,

Y   the net load for the forecast, $Y = D - \Sigma G_j$,

K(y,s)   the usual transmission network, balancing and spinning reserve constraints,

K(Y,S)   the transmission network, balancing and spinning reserve constraints for the forecast,

κ_j generation capacity,

R_j generation ramping limit on the exercise of spinning reserve,

$B_j(d_j)$ the bid-in load benefit function,

$C_j(g_j, \kappa_j, R_j)$ the bid-in generation cost function.

For simplicity, we focus on the loads and assume that all generation is subject to dispatch or available to provide spinning reserves. Bilateral contracts and must-run generation could be accommodated, just as could fixed loads. Generation without associated operating reserves would be treated as though all the capacity was in use, and so on. The ISO forecast of load is denoted by D and assumed to be determined independent of the bids.

For further notational simplicity, the network and electrical constraints are aggregated into the function K(.,.). These constraints include the usual load balancing conditions and all the possible network transmission limits in any of the possible contingencies. In principle, there could be many such contingencies and constraints. This large model presents a familiar computational challenge for the dispatchers, but this challenge is not of central concern here. The principal simplification in the present formulation is the use of network design such that the constraints can be characterized in terms of the ex ante net loads, y, and availability of spinning reserve, s. For example, the loss of a line may require redispatch to use some of the spinning

reserve to meet the load. We suppress explicit representation of the redispatch variables and assume that the resulting constraint on the net loads is differentiable. Dealing with the more general case with explicit representation of all the underlying constraints and variables would not affect the results here.

Another feature of the electrical model is to limit attention to contingencies that appear in the form of the loss of a transmission line. As usual, this allows us to work with the formulation in terms of net loads, rather than explicitly separating load and generation. This framework is not very restrictive. For instance, a contingency for loss of a generator can be modeled by connecting the generator to the grid through a zero-impedance radial line. The loss of the generator is then the same as the loss of this line. A similar approach would apply to transformers and other facilities. These standard modeling practices support the formulation of the network constraints in terms of the net loads and the magnitude of spinning reserves.

It is clear that the load, generation and spin variables are additive across participants. This is not so obvious, however, for the capacity and ramping constraint variables.[9] Hence, to begin, we distinguish these variables for each bidder and derive the result of common prices that allow later simplifications of the model formulation. With these assumptions, we emphasize a formulation of the ISO's day-ahead scheduling problem that includes the spinning reserve and various capacity constraints, as in:

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[9] For example, given the two constraints $S_1 \leq R_1$ and $S_2 \leq R_2$, the feasible set is not the same as for $S_1 + S_2 \leq R_1 + R_2$. However, an equilibrium solution will support either formulation.

$$Max \quad \sum_j [B_j(d_j) - C_j(g_j, \kappa_j, R_j)]$$

$$d_j, g_j, s_j, G_j, \kappa_j, R_j, S_j, s, G, S, \kappa, R \geq 0$$

$$y, Y$$

subject to:

λ_1	$y = \sum_j (d_j - g_j)$,
λ_2	$Y = D - G$,
λ_3	$\sum_j s_j = s$,
λ_4	$\sum_j G_j = G$,
λ_5	$\sum_j S_j = S$,
λ_6	$\sum_j \kappa_j = \kappa$,
λ_7	$\sum_j R_j = R$,
θ_{1j}	$g_j + s_j \leq \kappa_j$,
γ_{1j}	$s_j \leq R_j$
θ_{2j}	$G_j + S_j \leq \kappa_j$,
γ_{2j}	$S_j \leq R_j$,
μ_1	$K(y,s) \leq 0$,
μ_2	$K(Y,S) \leq 0$.

The associated multiplier variables for the ISO problem appear to the left of each constraint, and will be utilized below in the market equilibrium and pricing analysis. For convenience below, this formulation includes the aggregation variables s, S, κ, and R for the spin, capacity, and ramping limits, respectively.

Hence the day-ahead schedule maximizes the benefits minus the costs for the bid-in load, generation and capacity, subject to the constraints that the capacity must be sufficient to meet the contingency requirements for the day-ahead schedule and the contingency requirements for the forecast load. This formulation adopts the usual static model for one period. In practice, there may be a dynamic model that covers multiple periods, in which case we interpret this formulation as applicable after the inter-period decisions have been made.

This formulation requires two representations of the network contingency constraints, one for the bid-in conditions and one for the ISO forecast. This is the conceptual change in the formulation of the ISO problem. Traditional software and procedures would have no difficulty incorporating a few capacity limits on individual plants. But the full description of a parallel network, with all the associated contingencies, would require a major overhaul of the tools and practices. There is nothing difficult, in principle, and the analysis below illustrates the pricing implications of this formulation. The general model is followed later by an approximate implementation within the limits of the currently available tools.

Note that the ISO forecast of the load and associated generation and spin requirements interacts with the bid-in schedules only through the generation capacity variables, spin and the ramping constraints.[10] There is no attempt to choose G to minimize the actual or expected dispatch cost in real time. The only burden is to make sure that the real-time dispatch would be feasible given the reliability requirements. The way for market participants to hedge their energy cost in the real dispatch would be by bidding into the day-ahead market and getting a dispatch schedule.[11]

This is the ISO's security constrained, bid-based, economic dispatch problem for the day-ahead scheduling market. Associated with the solution to this problem will be a commitment schedule, dispatch, and a set of prices. The pricing rules depend upon the underlying formulation of the market institutions and the definition of the competitive equilibrium.

COMPETITIVE MARKET EQUILIBRIUM

Under the usual conditions, a solution for the ISO central dispatch problem is equivalent to a competitive market equilibrium.[12] The formulation of the market problem is based on the conventional partial equilibrium framework that stands behind the typical supply and demand curve analysis.[13] The market consists of the supply and demand of electric energy and transmission service plus an aggregate or numeraire "good" that represents the rest of the economy. The transmission operator (TO) provides transmission service at non-discriminatory market prices. Each customer with its own benefit maximizing problem (CB_j) is assumed to have an initial endowment w_j of the numeraire good. In addition, each customer has an ownership of π_j of the profits, π, of the electricity transmission provider. By definition, a competitive market equilibrium is a consistent set of prices, quantities and profit allocations which balances the market and includes optimal solutions for the transmission operator and each customer.

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[10] The generation ramping constraints appear as simple upper bounds. A more general formulation could be accommodated to distinguish different ramping constraints at different load levels or periods of response, but this is not important here. See Brendan Ring and Grant Read, "Pricing for Reserve Capacity in a Competitive Electricity Market," University of Canterbury, New Zealand, June 1994.

[11] Market participants could also enter into bilateral hedging contracts, but the hedge provider itself would be unhedged unless it scheduled energy or transmission purchases in the day-ahead market.

[12] For the DC-Load representation of the electrical constraints, the feasible set is convex. Hence, concavity of the benefits-minus-cost function would suffice for this formulation. In the full AC representation of the electrical constraints, the feasible set is not necessarily globally convex. Hence, in principle, there could be solutions to an ISO economic dispatch problem that would not have a corresponding competitive market equilibrium; there would be no set of prices that would support the solution and meet the no arbitrage condition. However, such circumstances should be rare, with the implication that the implied prices would not cover the full costs and there would be an added contribution to uplift. Similarly, start-up and minimum load costs could introduce non-convexities that would lead to market prices that would not cover costs without a supplemental payment through uplift. For notational simplicity, the present discussion does not address these details, which are separate from the issue of scheduling markets and reliability commitments. We consider only the competitive case. In the presence of market power, the opportunities for gaming should be examined, but this is beyond the scope of the present discussion.

[13] The partial equilibrium assumptions are that electricity is a small part of the overall economy with consequent small wealth effects, and prices of other goods and services are approximately unaffected by changes in the electricity market. See Mas-Colell, A., M.D. Whinston, and J.R. Green, Microeconomic Theory, Oxford

In formulating the competitive equilibrium, both the transmission operator and the customers are described as price takers. Of course, the transmission service provider is a monopoly and would not be expected to follow the competitive assumption in the absence of regulatory oversight. However, the conventional competitive market definition provides the standard for the service that should be required of the system operator.[14]

There is no unique decomposition of the ISO problem. Here we select a convenient version that defines trading in terms of load, generation, spin, capacity, and ramping limits. Market participants buy and sell these items at their various locations, trading through the transmission operator. Transmission service is represented as a combined trade of load and generation, with the transmission price being the difference in energy prices at the locations.

The explicit treatment of spin as a service the transmission operator purchases in the market, separate from transmission operator recognition of capacity or ramping limits, is the least obvious choice. The ISO problem does include the constraints on spin, but the decomposition here assigns these constraints to the market participants through (CB_j). This choice preserves the natural condition that the price of generation and load, from the perspective of the price-taking transmission operator, is the same at each location. The price of "x" is represented by P_x. With these conventions, the (TO) problem becomes:[15]

University Press, 1995, pp. 311-343. Importantly, we adopt here a relaxed set of assumptions that do not include convexity of the set of feasible net loads.

[14] It is the standard formulation to include (CB_j) and (TO) as part of the definition of competitive market equilibrium. Failure to follow this well established convention leads to confusion when the term "market equilibrium" is applied excluding the producing sector in (TO), as in Wu, F., P. Varaiya, P. Spiller, and S. Orren, "Folk Theorems on Transmission Access," Journal of Regulatory Economics, Vol. 10, No. 1, 1996, pp. 5-24.

[15] For notational simplicity, we treat price vectors as row vectors and quantity vectors as column vectors. This permits the use of $P_{dj}d_j$ in place of the transpose notation of $P_{dj}^t d_j$.

$$\text{Max} \qquad \sum_j P_{dj} d_j - \sum_j P_{gj} g_j - \sum_j P_{sj} s_j - \sum_j P_{\kappa j} \kappa_j - \sum_j P_{Rj} R_j$$

$$d_j, g_j, s_j, G_j, \kappa_j, R_j, S_j, s, G, S \geq 0$$

$$y, Y$$

subject to:

$$\lambda_1 \qquad y = \sum_j (d_j - g_j),$$

$$\lambda_2 \qquad Y = D - G,$$

$$\lambda_3 \qquad \sum_j s_j = s,$$

$$\lambda_4 \qquad \sum_j G_j = G,$$

$$\lambda_5 \qquad \sum_j S_j = S,$$

$$\theta_{2j} \qquad G_j + S_j \leq \kappa_j,$$

$$\gamma_{2j} \qquad S_j \leq R_j,$$

$$\mu_1 \qquad K(y,s) \leq 0,$$

$$\mu_2 \qquad K(Y,S) \leq 0.$$

The corresponding equilibrium conditions for non-zero variables in the dispatch solution for this problem include:

$$P_{dj} = \lambda_1 \quad,$$

$$P_{gj} = \lambda_1 \quad,$$

$$P_{sj} = \lambda_3 = -\mu_1 \nabla K_s \quad,$$

$$\lambda_2 = \lambda_4 \quad,$$

$$\lambda_4 = \theta_{2j} \quad,$$

$$\lambda_5 = -\mu_2 \nabla K_S \quad,$$

$$\lambda_5 = \theta_{2j} + \gamma_{2j} \quad,$$

$$P_{\kappa j} = \theta_{2j} \quad,$$

$$P_{Rj} = \gamma_{2j} \quad,$$

$$\lambda_1 = \mu_1 \nabla K_y \quad,$$

$$\lambda_2 = \mu_2 \nabla K_Y \quad.$$

The apparent equality of prices across participants, as well as between generation and load, justifies a restatement of the (TO) problem for equilibrium solutions in a more transparent version:

$$\textit{Max} \qquad P_y y - P_s s - P_\kappa \kappa - P_R R$$

$$d_j, g_j, G_j, s_j, \kappa_j, R_j, S_j, G, s, S, \kappa, R \geq 0$$

$$y, Y$$

subject to:

$$\lambda_1 \qquad y = \Sigma_j (d_j - g_j),$$

$$\lambda_2 \qquad Y = D - G,$$

$$\lambda_3 \qquad \Sigma_j s_j = s,$$

$$\lambda_4 \qquad \Sigma_j G_j = G,$$

$$\lambda_5 \qquad \Sigma_j S_j = S,$$

$$\lambda_6 \qquad \Sigma_j \kappa_j = \kappa,$$

$$\lambda_7 \qquad \Sigma_j R_j = R,$$

$$\theta_{2j} \qquad G_j + S_j \leq \kappa_j,$$

$$\gamma_{2j} \qquad S_j \leq R_j,$$

$$\mu_1 \qquad K(y, s) \leq 0,$$

$$\mu_2 \qquad K(Y, S) \leq 0.$$

The corresponding equilibrium conditions for this version of the problem include:

$$P_y = \lambda_1 \quad,$$
$$P_s = \lambda_3 = -\mu_1 \nabla K_s \quad,$$
$$\lambda_2 = \lambda_4 \quad,$$
$$\lambda_4 = \theta_{2j} \quad,$$
$$\lambda_5 = -\mu_2 \nabla K_S \quad,$$
$$\lambda_5 = \theta_{2j} + \gamma_{2j} \quad,$$
$$\lambda_6 = \theta_{2j} \quad,$$
$$P_\kappa = \lambda_6 \quad,$$
$$\lambda_6 = \gamma_{2j} \quad,$$
$$P_R = \lambda_7 \quad,$$
$$\lambda_1 = \mu_1 \nabla K_y \quad,$$
$$\lambda_2 = \mu_2 \nabla K_Y \quad.$$

Hence, trade is in terms of the individual participant capacity and ramping variables, in order to make certain that we maintain an accurate representation of the constraints. But we take advantage of the knowledge that the equilibrium conditions must have common prices for these

variables to simplify to a single vector of capacity and ramping constraints for the reliability component.

The surplus or trading profit for the transmission operator is $\pi = P_y y - P_s s - P_\kappa \kappa - P_R R$. Under the assumption of the partial equilibrium model, the profit is divided among the market participants according to some rule such that $\pi = \Sigma \pi_j$. Further, the individual market participants are endowed with wealth, w_j, in terms of the numeraire good. The market participants choose the amount of consumption, c_j, of the numeraire good along with electricity goods and services to maximize the net benefits and costs of consumption, electricity load, capacity, spin, ramping limits and an income constraint.

The individual (CB$_j$) problems would be formulated as:

$$Max \qquad c_j + B_j(d_j) - C_j(g_j, \kappa_j, R_j)$$
$$c_j, d_j, g_j, s_j, \kappa_j, R_j \geq 0$$

subject to:

$$\varphi_j \qquad c_j + P_y(d_j - g_j) - P_s s_j - P_\kappa \kappa_j - P_R R_j \leq w_j + \pi_j,$$
$$\theta_{1j} \qquad g_j + s_j \leq \kappa_j,$$
$$\gamma_{1j} \qquad s_j \leq R_j .$$

The related equilibrium conditions for the (CB$_j$) problem include:

$$P_y = \nabla B_j = \nabla C_{jg} + \theta_{1j} \quad ,$$
$$P_s = \theta_{1j} + \gamma_{1j} \quad ,$$
$$P_\kappa = \nabla C_{j\kappa} - \theta_{1j} \quad ,$$
$$P_R = \nabla C_{jR} - \gamma_{1j} \quad .$$

Assuming the existence of an equilibrium, we can return to the (ISO) problem of security constrained economic dispatch and interpret the dual solution in terms of the equilibrium conditions that follow from this market formulation. These become:

$$P_y = \lambda_1 = \nabla B_j = \nabla C_{jg} + \theta_{1j} \quad ,$$
$$P_s = \lambda_3 = -\mu_1 \nabla K_s = \theta_{1j} + \gamma_{1j} \quad ,$$
$$\lambda_2 = \lambda_4 = \theta_{2j} \quad ,$$
$$\lambda_5 = -\mu_2 \nabla K_S = \theta_{2j} + \gamma_{2j} \quad ,$$
$$P_\kappa = \lambda_6 = \theta_{2j} = \nabla C_{j\kappa} - \theta_{1j} \quad ,$$
$$P_R = \lambda_7 = \gamma_{2j} = \nabla C_{jR} - \gamma_{1j} \quad ,$$
$$\lambda_1 = \mu_1 \nabla K_y \quad ,$$
$$\lambda_2 = \mu_2 \nabla K_Y \quad .$$

The prices and revenues for this version of the ISO problem would have essentially the same properties as for the simpler case that suppresses the explicit spin and capacity variables. Payments for load, generation, spin, and transmission congestion rents in the scheduling market would be determined as for the usual economic dispatch problem, given the commitment of capacity. These payments would net to zero in the linear DC-load model case, meaning the net payments to the ISO would equal the congestion rents for transmission.

The reliability constraints for the ISO load forecast could induce positive opportunity cost prices, P_K and P_R, for capacity or ramping capability, respectively, to meet the forecast load. These payments would be made to all suppliers of such capabilities. The revenues would not be part of the market clearing prices for scheduled energy. Payments for these additional units of capacity could be not be included in the locational prices for energy. Presumably these net "reliability" payments would be treated as an ancillary service and covered as part of an uplift payment.

Note that although there would be common prices for all participants for generation, spin, capacity and the ramping constraints, the decomposition of the prices into marginal costs and opportunity costs could be different for each participant.

Locational prices for the load would be determined in the usual way as the dual variables for the constraints defining the net bid-in loads, $y = d-g$. The revenues for generation would come from energy sales, g, at the locational prices and spinning reserve provisions at the (locational) prices of spin, and the separate payment for generating and ramping capacity.

Every supplier of operating reserves would be paid the market-clearing price of spin, P_s, at its location. If the ramping constraints are not binding, then the locational price of spin is just the opportunity cost of foregone generation, equal to the price of energy less the marginal energy cost of generation. If the ramping constraint is binding, the price of spin could be greater, reflecting the opportunity cost of the ramping limit. If $G > g$, then the price of spin for scheduled load might be small or zero, depending on the locational constraints and the pattern of load.

AN EXAMPLE SCHEDULE WITH RELIABILITY COMMITMENTS

An example illustrates the calculations. Here the supply and demand conditions appear in the figure below summarizing the supply and demand curves. The cost functions include a marginal cost of capacity as well as a marginal cost of energy. The network includes the canonical three-bus structure needed to include the effects of loop flow. The fourth node introduces the single network contingency in the possible loss of the line between nodes 2 and 4. Excess capacity is needed for spin at nodes 1 and 2. In addition, there is a ramping constraint, which limits the spin to no more than 400 MW at node 1 and no more than 100 MW at node 2. For simplicity, it is assumed that the maximum ramping limits at nodes 1 and 2 are fixed. More general conditions for ramping supply would not change the basic relationship among the prices for the scheduled energy given the reliability constraints. With these ramping constraints, we know that there can be no more than 500 MW of generation at node 4, and there may be less if there is a generation capacity constraint or a binding transmission constraint. The potential transmission constraint is 600 MW on the line between nodes 1 and 3, with the same limit in both the pre and post contingency conditions.

Supply and Demand Conditions

(WITH IDENTICAL LINKS, CONSTRAINT ON LINE 1-3)
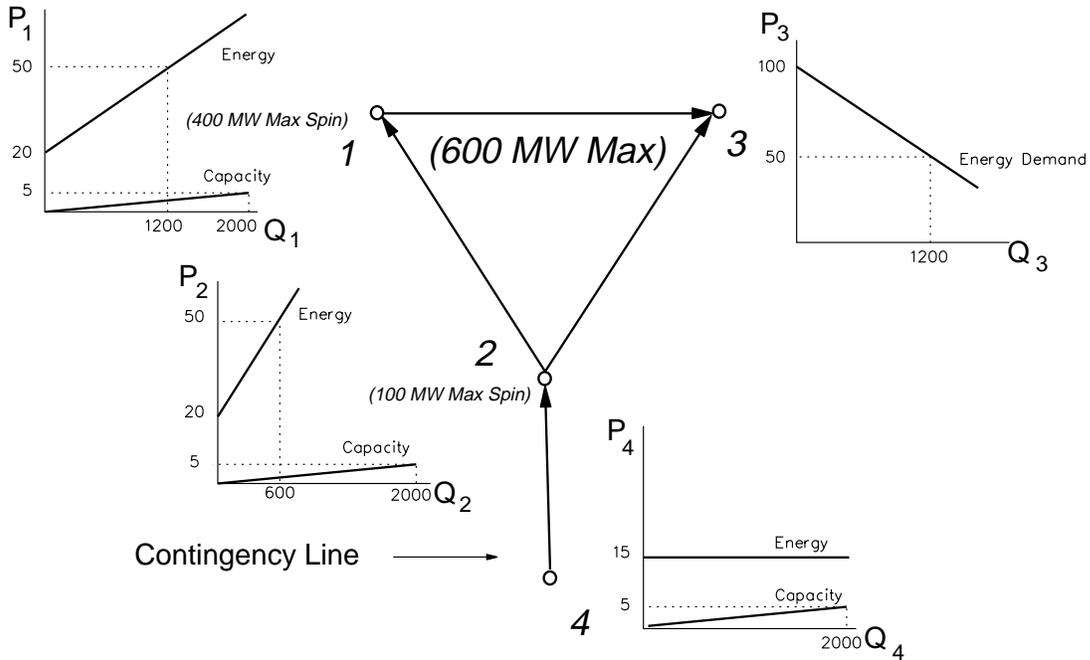
The numbers shown in the figure are provided to calibrate the supply and demand conditions, for which an equilibrium solution must be determined. Hence, at a price of $50/MWh, the load is 1200 MW. At a price of $100, the load would be zero, implying a demand slope of 24=1200/50.

The first example applies this system under the condition that the ISO has no forecast schedule of load that differs from the bid-in schedule. The transmission constraint would be binding in the event of the contingency, so it limits the pre-contingency dispatch. In addition, the full spin is utilized and the ramping constraints are binding, yielding substantial spin prices in this example.

Schedules With No Forecast Load Reliability Constraints

Pre-contingency Bid-In Flows and Prices
(WITH IDENTICAL LINKS, CONSTRAINT ON LINE 1–3)

Post-contingency Bid-In Flows
(WITH IDENTICAL LINKS, CONSTRAINT ON LINE 1–3)
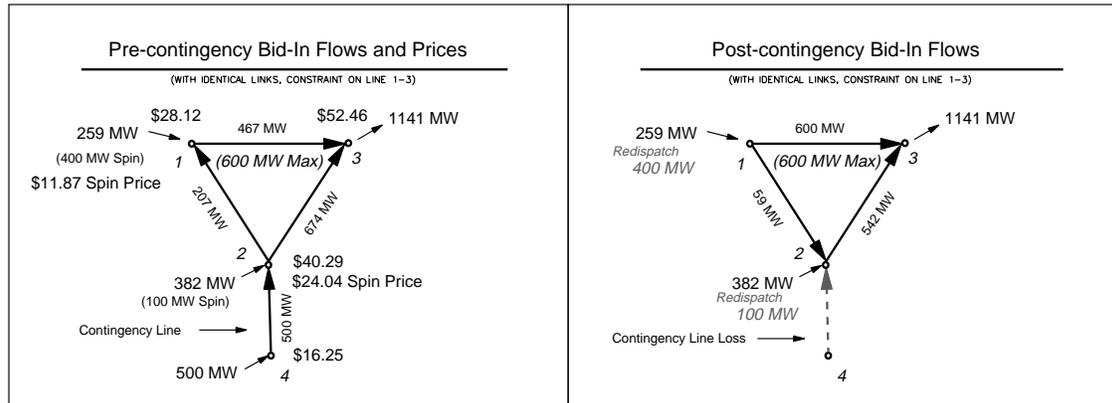
The large operating reserve prices help to make the analytical point, but in a real system the spin prices are more likely to be small or zero.

Note that the energy prices at nodes 1 and 3 are the same as the marginal energy plus marginal capacity costs, and the price of spin at these nodes includes both the marginal cost of capacity and the opportunity cost of the ramping constraint. The payments at market-clearing prices include:

Payments at Market Equilibrium
With No Added Reliability Constraint
($/Hour)

Load	59854.40
Generation	30798.10
Spin	7152.80
Transmission Congestion	21903.60
Net	0.00

The payments for these components in our examples net to zero in the usual way because of the use of the DC-Load model to represent the transmission network. Here there are two opportunity cost or "congestion" components embedded in the energy prices. One is for transmission, and the other is because of the capacity and ramping constraints on spin.
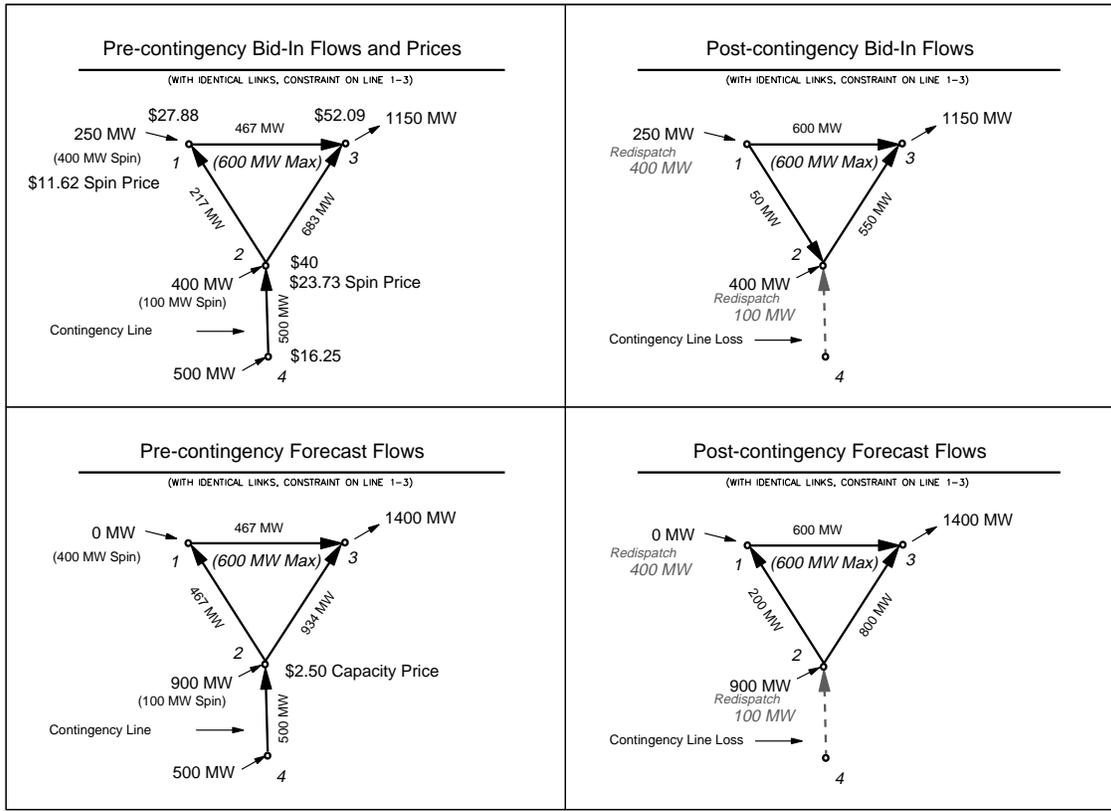
If there were no ramping constraints, then the price of spin would be limited to the capacity opportunity cost. But in the presence of the ramping constraint, we have both a capacity opportunity cost and a ramping opportunity cost. Hence the price of spin is greater than the difference between the price for energy and the marginal energy cost for a given capacity level.

This more traditional example provides a benchmark for comparison. However, the problem of interest here is the new formulation where the ISO has a forecast of net load that may differ from the bid-in schedule. The obligation of the ISO is to schedule enough capacity to meet the generation and spin requirements for the forecast and the bid-in schedule. However, the objective function is to minimize the energy cost for the bid-in schedule plus the total capacity commitment costs. Those who do not bid into the day-ahead schedule are exposed to the energy costs that may appear in real time. However, since the capacity costs will go into the uplift, we want to include them in the optimization.

For simplicity, we have assumed a convex cost function for capacity. The more general case with start-up costs won't affect the basic analysis here except, as usual, to complicate the optimization and leave the potential for some uplift contribution in excess of market-clearing prices.

As shown in the figure, the introduction of the ISO reliability forecast adds a second network with its own contingencies, transmission limits, and spin constraints. Here the ISO forecast is that a load of 1400 MW will appear at node 3, rather than the 1141 MW that would be an equilibrium for the bid-in schedules. This additional load would require a different commitment of capacity in order to meet the contingency and spin constraints.

Schedules With Forecast Load Reliability Constraints

Pre-contingency Bid-In Flows and Prices
(WITH IDENTICAL LINKS, CONSTRAINT ON LINE 1–3)

Post-contingency Bid-In Flows
(WITH IDENTICAL LINKS, CONSTRAINT ON LINE 1–3)

Pre-contingency Forecast Flows
(WITH IDENTICAL LINKS, CONSTRAINT ON LINE 1–3)

Post-contingency Forecast Flows
(WITH IDENTICAL LINKS, CONSTRAINT ON LINE 1–3)

The new solution and a comparison with the previous case illustrate a number of points. First, the commitment of additional capacity alters the equilibrium solution for the bid-in schedules. The prices for spin and energy fall slightly, and bid-in demand increases to 1150 MW.

The generating capacity commitment at node 2 increases from 500 MW to 1000 MW in order to meet the reliability requirements with the ISO forecast. There is a small decrease in generation at node 1 and an increase in generation at node 2, to make up the difference and meet the increased load.

Again, the post-contingency transmission limits constrain the dispatch and the corresponding scheduled flows. Given the capacity commitment, the analysis of the market equilibrium is as before, with slightly adjusted prices and quantities. The revenue components include:

Payments at Market Equilibrium
With ISO Administered Reliability Constraint
($/Hour)

Load	59895.10
Generation	31081.40
Spin	7022.80
Transmission Congestion	21791.00
Net	0.00

Hence, given the unit commitment, the prices and revenues follow the same pattern as before. The unit commitment, however, is driven here by the ISO load forecast of 1400 MW, and there is an opportunity cost for the capacity that is not being covered by the locational prices for energy plus spin. Based only on the energy and spin prices, the generators would not commit this much capacity.

In the present case, the missing piece is the payment for κ at the price P_κ. It turns out the price of ramping for the ISO forecast is zero, because the ramping constraint limits the actual dispatch as well. Likewise, the price of capacity is zero most everywhere, except at node 2. At node 2, however, P_κ is $2.5 per MW, and this applies to the full 1000 MW of capacity committed at node 2. In order to compensate the generators for the marginal commitment of capacity at node 2, we must pay the marginal price of $2.5. And following the competitive market model, this price applies to all capacity committed at the location.

The revenue effect is a deficit of $P_\kappa\kappa$, or $2.2*1000 = $250. This "reliability" payment could be collected as part of the uplift.

PRICING AND TRANSMISSION CONGESTION CONTRACTS

One purpose of transmission congestion contracts (TCC) is to provide the equivalent of perfectly tradeable physical transmission rights. In general a TCC can be defined as a vector of energy inputs and outputs in the network. If P_y is the price of energy in the schedule, and "tcc" is the vector quantity inputs and outputs, then the TCC provides for the payment of $P_y$tcc.[16] A key feature of the analysis of TCCs is the question of feasibility of the vector of net inputs and outputs. In the absence of an explicit treatment of spinning reserve, or if spinning reserve is always freely available, the feasibility condition is equivalent to requiring that the vector (tcc) satisfy the electrical constraints. In the more general case, we would have to account for the spinning reserve

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[16]   The usual application is to apply the definition to congestion payments only, and only for balanced TCCs. In the case of the DC Load model of a network, there is no difference. Here we allow for the more general statement which is convenient for the analysis of equilibrium and revenue adequacy.

which might have an opportunity cost. Hence, associated with the TCC definition would be a parallel spinning congestion contract (SCC) which required payment of $P_s$scc for quantity "scc." This would be the equivalent of an obligation to provide spinning reserve in the amount of scc. In this case, the definition of the feasibility condition would be K(tcc,scc) ≤ 0.

The restatement of the (TO) problem would recognize the existence of the TCC and associated SCC "transmission rights." In effect, we model the (TO) as purchasing these transmission rights at the market clearing prices and selling the transmission services for actual use of the system. Holders of the rights always have the option of using the system in a way that matches their rights, in which case the usage payments equal the payments under the TCC and SCC contracts. Or the users may trade their rights and allow the operator to reconfigure the use of the system. With this interpretation, we expand the (TO) model to include the transmission rights as:

$$Max \qquad P_y(y - tcc) - P_s(s - scc) - P_\kappa \kappa - P_R R$$

$$d_j, g_j, G_j, s_j, \kappa_j, R_j, S_j, G, s, S, \kappa, R \geq 0$$

$$y, Y$$

subject to:

$$\lambda_1 \qquad y = \sum_j (d_j - g_j),$$

$$\lambda_2 \qquad Y = D - G,$$

$$\lambda_3 \qquad \sum_j s_j = s,$$

$$\lambda_4 \qquad \sum_j G_j = G,$$

$$\lambda_5 \qquad \sum_j S_j = S,$$

$$\lambda_6 \qquad \sum_j \kappa_j = \kappa,$$

$$\lambda_7 \qquad \sum_j R_j = R,$$

$$\theta_{2j} \qquad G_j + S_j \leq \kappa_j,$$

$$\gamma_{2j} \qquad S_j \leq R_j,$$

$$\mu_1 \qquad K(y,s) \leq 0,$$

$$\mu_2 \qquad K(Y,S) \leq 0.$$

The corresponding reformulation of the problem (CB$_j$) includes the individual ownership of TCCs and SCCs as part of the income constraint.

$$Max \quad c_j + B_j(d_j) - C_j(g_j, \kappa_j, R_j)$$

$$c_j, d_j, g_j, s_j, \kappa_j, R_j \geq 0$$

subject to:

$$\varphi_j \qquad c_j + P_y(d_j - g_j + tcc_j) - P_s(s_j - scc_j) - P_\kappa \kappa_j - P_R R_j \leq w_j + \pi_j,$$

$$\theta_{1j} \qquad g_j + s_j \leq \kappa_j,$$

$$\gamma_{1j} \qquad s_j \leq R_j \ .$$

Incorporation of the TCCs and SCCs has no impact on the statement of the equilibrium conditions or the interpretation of the prices. Furthermore, we have a form of revenue adeqaucy similar to the result obtained without explicit recognition of the spinning reserve variables or the reliability commitments. In particular, suppose that we have a market equilibrium solution including prices and the quantities y, s, κ, R. Since the vector (tcc,scc) would be part of a feasible solution of the (TO) problem, along with κ and R, we must have,

$$P_y(y - tcc) - P_s(s - scc) - P_\kappa \kappa - P_R R \geq P_y(tcc - tcc) - P_s(scc - scc) - P_\kappa \kappa - P_R R,$$

or

$$P_y(y - tcc) - P_s(s - scc) \geq 0 .$$

Hence,

$$P_y y - P_s s \geq P_y tcc - P_s scc.$$

In other words, we have revenue adequacy.[17] The net payment through the system operator for the load and spin would always be at least as great as the net obligations under the TCCs and SCCs. In the event that spin is free at the margin, then the result reduces to the simpler case of revenue adequacy for TCCs alone. However, the payments for reliability commitments remain as separate from the energy market and may require a separate uplift contribution.

If participant ownership of SCCs does not accompany TCCs, the implict assumption might be that the spinning reserve requirement should be treated as an ancillary service provided through the transmission system operator. In this case, the (likely small) value $P_s scc$ would bound the estimate of the deficit that would be treated as part of ancillary services. Or the equilibrium energy prices could be determined with fixed spin and the usual revenue adequacy result would apply.

IMPLEMENTATION

The introduction of a day-ahead scheduling market and the need for reliability commitments with simultaneous consideration of the day-ahead bids, presents the system operator with a new and more complicated problem. If the bid and forecast load were the same, or if the

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[17] William W. Hogan, "Contract Networks for Electric Power Transmission," Journal of Regulatory Economics, Vol. 4, 1992.

objective were to balance the forecast load at least cost, then the problem would be simple because there would be no separate set of network constraints. However, when there is a significant difference between the bid and forecast loads, and there might be transmission constraints, then it becomes necessary to consider the network limitations that might apply to either set of net loads.

This complication does not arise for the formulation of the real-time dispatch. In this case, only the real-time load matters, and there would be a single set of network constraints. Since the usual security-constrained economic dispatch implementation has been for the equivalent of the real-time application, the software and protocols for this optimization problem are available and well developed. The only new requirement for the competitive market run by the ISO is the conversion to bid-based estimates of the cost and benefit functions, in place of engineering cost estimates. The dispatch would follow the same procedures, and the associated equilibrium prices would be used for settlements that support the competitive interpretation of the transactions.

The case of two possible sets of network limitations in the day-ahead scheduling market imposes a new computational requirement. Although, as shown above, there is nothing difficult in principle, there may be approximations in practice as the system operator adapts traditional dispatch procedures to this new requirement. One such approximation would be to adopt a three-step approach to the dispatch:

1. Formulate the security constrained, bid-based, economic dispatch problem ignoring the ISO forecast and the constraints on reliability commitments.

2. Fixing the commitments and solution to (1), determine the difference in the bid-based load and the forecast load. Using a modified objective function that includes only the commitment costs for incremental capacity, solve for the incremental reliability commitments needed to meet the forecast load.

3. Using the capacity commitments from both (1) and (2), solve for the economic dispatch for the bid-in load to determine the day-ahead schedule and prices.

In terms of the formulation of the (ISO) problem, therefore, the three steps each require solution of a dispatch problem with a single network, which conforms to the standard formulation and protocols. In the first case, the problem would be:

$$\text{Max} \qquad \sum_j [B_j(d_j) - C_j(g_j, \kappa_j, R_j)]$$

$$d_j, g_j, s_j, \kappa_j, R_j, s \geq 0$$
$$y$$

subject to:

$$\lambda_1 \qquad y = \sum_j (d_j - g_j) ,$$

$$\lambda_3 \qquad \sum_j s_j = s ,$$

$$\theta_{1j} \qquad g_j + s_j \leq \kappa_j ,$$

$$\gamma_{1j} \qquad s_j \leq R_j$$

$$\mu_1 \qquad K(y,s) \leq 0 \ .$$

With the solution being d_j', g_j', κ_j', and R_j', the formulation of the second step would be:

$$\text{Min} \qquad \sum_j C_j(g_j', \kappa_j, R_j)]$$

$$G_j, \kappa_j, R_j, S_j, G, S \geq 0$$
$$Y$$

subject to:

$$\lambda_2 \qquad Y = D - G ,$$

$$\lambda_4 \qquad \sum_j G_j = G ,$$

$$\lambda_5 \qquad \sum_j S_j = S ,$$

$$\omega_1 \qquad \kappa_j' \leq \kappa_j ,$$

$$\omega_2 \qquad R_j' \leq R_j ,$$

$$\theta_{2j} \qquad G_j + S_j \leq \kappa_j ,$$

$$\gamma_{2j} \qquad S_j \leq R_j ,$$

$$\mu_2 \qquad K(Y,S) \leq 0 .$$

With the solution for capacity and ramping commitments as κ_j^*, and R_j^*, the formulation of the third step would be the usual economic dispatch problem with given capacity availability:

$$Max \qquad \sum_{j}[B_j(d_j) - C_j(g_j, \kappa_j^*, R_j^*)]$$

$$d_j, g_j, s_j, s \geq 0$$

$$y$$

subject to:

$$\lambda_1 \qquad y = \sum_{j}(d_j - g_j),$$

$$\lambda_3 \qquad \sum_{j} s_j = s,$$

$$\theta_{1j} \qquad g_j + s_j \leq \kappa_j^*,$$

$$\gamma_{1j} \qquad s_j \leq R_j^*,$$

$$\mu_1 \qquad K(y,s) \leq 0.$$

The equilibrium conditions for the price determination for this problem would be the same as above, given the level of the capacity commitments. In the framework of ex post or dispatched-based prices, the prices must satisfy the set of linear constraints defined by the equilibrium conditions. Any deviations form the optimality conditions would be minimized, with the associated payments included as part of any uplift charge.[18]

The resulting dispatch and commitment schedule would then meet the feasibility test for both the bid-in dispatch and the ISO forecast. Given the capacity commitment, the dispatch is also energy least-cost for the bid-in loads. Compared to "letting the market decide" as in the solution to (1), the additional costs for generating capacity commitments would be minimized. The final energy costs for the bid-in loads would be lower than if the ISO ignored its own forecast. The capacity commitment costs could be higher. The only approximation of the full problem is that the complete interaction of the capacity commitments might not be recognized in finding the least-cost solution overall. However, with first step to come close to the best solution, it is reasonable to conjecture that any discrepancy would be small.

CONCLUSION

The simplified formulation of a security-constrained, economic dispatch problem lends itself to a natural adaptation to handle the requirements of day-ahead scheduling and reliability commitments. The resulting formulation of the equilibrium conditions determines the locational prices for energy with a consistent treatment of settlements and transmission congestion contracts. Pricing of spinning reserve could be incorporated explicitly as part of the locational pricing regime. Capacity payments for additional reliability commitments would be treated as an ancillary service and included in an uplift payment.

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[18] William W. Hogan, E. Grant Read, and Brendan J. Ring, "Using Mathematical Programming for Electricity Spot Pricing," Energy Models for Policy and Planning, International Transactions of Operational Research, Vol.3, No. 3/4, 1996.