Cross-product Manipulation in Electricity Markets,

Microstructure Models and Asymmetric Information

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Abstract

Electricity market manipulation enforcement actions have moved from conventional analysis of generator market power in real-time physical markets to material allegations of sustained cross-product price manipulation in forward financial markets. A major challenge is to develop and apply forward market analytical frameworks and models. This task is more difficult than for the real-time market. An adaptation of cross-product manipulation models from cash-settled financial markets provides an existence demonstration under uncertainty and asymmetric information. The implications of this analysis include strong empirical predictions about necessary randomized strategies that are not likely to be observed or sustainable in electricity markets. Absent these randomized strategies and other market imperfections, the means for achieving sustained forward market price manipulation remains unexplained.

Keywords: market manipulation; electricity markets; limits to arbitrage; asymmetric information

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1. Introduction

Electricity market design aims at defining rules and incentives leading to a workably competitive market whose outcome achieves a broad social benefit (Joskow and Schmalensee, 1983). In the United States, the reference framework for electricity market design is given by the model of bid-based, security-constrained economic dispatch with locational marginal prices (Hogan, 2010).[1] Organized electricity markets run by Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs) are built around this model, and have a two-settlement structure with day-ahead and real-time coordinated auctions.[2] The commodity traded in each market is the quantity of power, in MWh, produced and consumed in real-time at a given location on the transmission network. A market auction on the day before actual power dispatch creates a financial obligation to buy or sell power for delivery in real-time. In contrast, the real-time market is a physical market where actual supply and demand of electricity are balanced continuously over the delivery day. In both auctions, the result is market clearing with locational marginal prices that, under competitive market conditions, reflect the short-run marginal cost of serving one incremental megawatt of load at each node (Schweppe et al., 1988). Sales and purchases cleared at the day-ahead price that are not converted into physical positions must be bought or sold back at the real-time price.

Due to the lack of economic energy storage, the existence of capacity and transmission constraints, and the small short-run price elasticity of demand, real-time physical markets are vulnerable to price manipulation. It is well known that power generators may exercise supplier market power by not bidding available capacity into the market ("physical withholding") or by raising offer prices above marginal cost of production ("economic withholding") (Newbery, 1995; Cardell et al., 1997; Borenstein et al., 2002; Joskow and Kahn, 2002; Harvey and Hogan, 2002; Wolak, 2003; Kim and

[1] Security-constrained economic dispatch can be carried out reliably, given the thermal, voltage and stability limits of the transmission network (Stoft, 2002).
[2] ISOs and RTOs serve about two thirds of electricity consumers in the United States (ISO/RTO Council, 2018).

Knittel, 2006; Puller, 2007; Sweeting, 2007; Lo Prete and Hobbs, 2015). Extensive monitoring and mitigation rules are in place to prevent this type of generator manipulation (Helman, 2006). In contrast, a policy concern raised by enforcement actions of the Federal Energy Regulatory Commission (FERC) focuses on price manipulation involving forward electricity markets and related financial positions: market participants may act against their economic interest in the day-ahead market to artificially move prices and benefit related positions in another market (FERC, 2017). For example, several actions brought by the FERC involved uneconomic (i.e., unprofitable) virtual transactions and alleged manipulation of day-ahead prices to benefit related financial transmission rights (FTRs). Cross-product manipulation cases are being litigated or resulted in multi-million dollar settlements where the accused may not admit to the behavior alleged. As a material case in point, an investigation culminating in 2012 alleged electricity market cross-product manipulation by Constellation Energy (138 FERC ¶ 61,168). A settlement resulted in $135 million in civil penalties and $110 million in disgorged profits, but with no agreement on the merits of the claim.[3]

By design, settlements approved by FERC do not disclose details regarding the underlying analysis conducted during the investigation and the market price impact of the manipulation.[4] Further, there is concern that FERC's Anti-Manipulation Rule for electricity markets (18 C.F.R. §1c.2), modeled on Rule 10b-5 of the U.S. Securities and Exchange Commission, is too narrowly focused on the notion of fraud (Pirrong, 2010; Hogan, 2014; Evans, 2015) and has been applied on a case-by-case basis,

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[3] After the FERC decision, the Constellation CEO stated: "We believe Constellation's trading practices in question were lawful portfolio risk management transactions. The company admits to no wrongdoing in this case" (Constellation Energy, 2012). The FERC chairman responded: "In my opinion, clearly that is not the case. … I urge anyone who has any question as to Constellation's actions in this case to read that Stipulation and Consent Agreement" (Wellinghoff, 2012). A reading of the agreement does not provide the necessary data or any theoretical model to evaluate the allegations. However, the agreement does include a *quid pro quo* stipulation granting something of value to Constellation in the form of regulatory approval of a planned merger: "The Effective Date of this Agreement shall be the later of the date on which: (a) the Commission issues an order approving this Agreement without material modification; or (b) the merger pursuant to the Agreement and Plan of Merger among Constellation Energy Group, Inc., Exelon Corporation, and Bolt Acquisition Corporation, dated April 28, 2011, is consummated" (138 FERC ¶ 61,168, p. 44).

[4] "All information and documentation received during an investigation, as well as the existence of an investigation, is treated as non-public information. Disclosure is permitted only at the Commission's direction or authorization" (123 FERC ¶ 61,156, Par. 23).

creating uncertainty as to what types of misconduct constitute manipulative behavior in practice (Scherman et al., 2014). Evans (2015) suggested that FERC adopt the rules of the Commodity Futures Trading Commission (CFTC) as a model to design a new, more flexible regulatory regime that encompasses all forms of potential manipulation in electricity markets.

A proposed framework for the analysis of electricity market manipulation appears in Ledgerwood and Carpenter (2012). The analytical outline breaks the process into several necessary components that would separate prohibited market manipulation from other possible interpretations of market actions. Central to this process would be an explicit model that characterizes the direction and magnitude of market responses, and links the alleged actions to the difference between observed prices and the counterfactual case. An immediate challenge is the lack of such a cross-product price manipulation model for the electricity market.

Absent control over real-time prices, how could a market participant affect day-ahead electricity prices over a sustained period using only financial positions from the forward market? "It is not possible for a market participant that has only a paper electricity position, and no generating assets, to distort prices merely by taking delivery on this position, because electricity must be consumed precisely when it is produced" (Pirrong, 2017). Something more is required than is found in the analysis of real-time market power withholding. Why are other participants unable to restore price convergence by profiting from arbitrage opportunities created by the alleged manipulation? Market monitoring and enforcement activities must distinguish between manipulative and efficient transactions. Yet, the theoretical foundations of day-ahead electricity price manipulation are neither obvious nor well developed. A theory (or theories) of day-ahead price manipulation would explain what market imperfections allow manipulation to be sustained over time, quantify its material effect on prices in a transparent way, and provide empirical implications that may be tested in the data to determine if actions were consistent with manipulation.

The focus is on equilibrium conditions that would support the alleged sustained manipulation. To illustrate, we construct an example of equilibrium manipulation under uncertainty and asymmetric information in the context of a modified Kumar and Seppi (1992) model for cash-settled financial transactions. The equilibrium manipulation strategy consists in incurring losses in one financial market in order to bias a market outcome and benefit related positions in another financial market. Empirical and welfare implications of the equilibrium are compared to those from three benchmark models to examine the impact of manipulation on market liquidity and performance, as well as its distributional effects. Finally, simulated equilibrium outcomes imposing futures position limits illustrate whether theoretical predictions are affected under more restrictive conditions that apply in electricity markets.

The remainder of the paper is organized as follows. Section 2 provides additional background and discusses why equilibrium models are useful and needed for the analysis of day-ahead price manipulation. Section 3 presents the modified Kumar and Seppi (1992) model of cross-product manipulation, adapted to the context of electric power markets. Section 4 derives empirical and welfare implications of the equilibrium and compares them to those from three benchmark equilibrium models. Section 5 describes our simulation results, while Section 6 offers concluding remarks and provides directions for future research. Analytical characterizations of equilibrium outcomes, empirical and welfare implications are presented in the Appendix.

2. Policy context

Organized electricity markets run by RTOs and ISOs are characterized by a two-settlement market structure with day-ahead and real-time coordinated auctions. In the presence of uncertainty, a sequential market structure helps improve the allocation of resources and risks (Anderson, 1984). The day-ahead market plays a pivotal role because it provides system operators with flexibility in planning the commitment of generation resources, as units may have ramping requirements and long lead times for starting up. Further, load-serving entities can manage risk by hedging their exposure to real-time prices through day-ahead purchases. In most organized electricity markets, about 95% of energy transactions are scheduled in the day-ahead market (FERC, 2015). As a result, its performance is critical to ensuring the efficient operation of electric systems, and closely monitored evaluating convergence between day-ahead and real-time prices.

In an efficient commodity market characterized by complete information, risk neutrality, no transaction costs and no market power, forward and spot contracts for delivery at the same time and location should transact at the same price, on average (Weber, 1983). Thus, day-ahead prices for delivery of power at a given hour and location should reflect participant price expectations for the following day, given the information available at the time bids were made in the day-ahead market. In general, locational day-ahead prices will be different from real-time prices on an hourly basis, due to factors like forced generation outages and load forecasting errors. However, day-ahead and expected real-time prices should not diverge systematically over long periods of time (i.e., monthly or annually). Empirical analyses of price differentials in organized electricity markets generally find evidence of a small positive forward premium, defined as the difference between average day-ahead and real-time prices (Pirrong and Jermakyan, 1999; Saravia, 2003; Longstaff and Wang, 2004; Borenstein et al., 2008; Hadsell, 2008; Bowden et al., 2009; Ito and Reguant, 2016).

When day-ahead prices are predictably higher or lower than expected real-time prices, arbitrage opportunities exist. Virtual transactions allow electricity market participants to exploit these arbitrage opportunities (PJM Interconnection, 2015). These are financial positions for the purchase or sale of energy in the day-ahead market that do not correspond to physical load or generation resources and are settled against real-time energy prices. Specifically, decrement bids (or DECs) are financial positions for the purchase of energy in the day-ahead market, which are settled by selling back that energy in the real-time market at the same location. Conversely, increment offers (or INCs) are financial positions for the sale of energy in the day-ahead market, which are settled by buying back that energy in the real-time market at the same location. If day-ahead prices are systematically lower than expected real-time prices, virtual transactions allow market participants to profitably purchase energy in the day-ahead market and sell it back in the real-time market. This tends to increase day-ahead prices and improve price convergence with the real-time market (Saravia, 2003; Hadsell, 2007; Güler et al., 2010; Jha and Wolak, 2015; Li et al., 2015). The available evidence also suggests that virtual bidding positively affects day-ahead unit commitment by bringing it closer to real-time system requirements, and reduces the total cost of serving load (Güler et al., 2010; Jha and Wolak, 2015). Under the assumption of no modeling differences between day-ahead and real-time markets, profitable virtual transactions contribute to better price convergence and tend to improve day-ahead market performance.[5]

In contrast, unprofitable virtual transactions generally diverge prices between day-ahead and real-time markets (Potomac Economics, 2016a), and may create productive, allocative and transactional inefficiencies in the day-ahead market (Kyle and Viswanathan, 2008; Ledgerwood and Carpenter, 2012), with implications for the distribution of profits and losses among market participants. Yet, virtual bids that are unprofitable on a stand-alone basis may be used to move day-ahead prices in a direction that enhances the value of related financial positions. An example of related positions is given by financial transmission rights, which hedge the difference in day-ahead locational marginal prices between two nodes and settle at the day-ahead price (Hogan, 1992; Rosellón and Kristiansen, 2013). The possibility that virtual bidding may be used strategically to enhance the value of FTRs has long been acknowledged (Isemonger, 2006; Celebi et al., 2010), and two electricity markets have monitoring rules to deter and detect this type of gaming *ex post* (PJM Interconnection, 2016; CAISO,

[5] When price differences between day-ahead and real-time markets arise due to inconsistencies in the way in which the transmission network is modeled in the two markets, virtual transactions may be profitable, but not improve day-ahead market performance (Parsons et al., 2015).

2016). However, enforcement actions brought by the FERC against market participants taking unprofitable virtual transactions on a persistent basis to benefit FTR and similar financial positions have drawn new attention to day-ahead price manipulation in recent years (Taylor et al., 2015). FERC investigations often targeted financial market participants and resulted in multi-million dollar settlements (FERC, 2016).

Ledgerwood and Carpenter (2012) present a conceptual framework that describes price-based market manipulation as an intentional act (the "trigger") made to cause a directional price movement (the "nexus") to benefit financially leveraged positions that tie to that price (the "target"). Building on this study, Taylor et al. (2015) introduce a diagnostic framework for the analysis of market manipulation, and discuss how it may assist in the detection, investigation and proof of manipulative behavior in recent enforcement actions. Their framework provides a logical sequence to guide consideration of possible market manipulation, but does not define a theory explaining how forward market manipulation could be sustained over time. Similarly, the economic analysis in Ledgerwood and Pfeifenberger (2013) is instructive to understand the workings of a loss-based price manipulation, but does not represent an economic equilibrium. By repeatedly placing unprofitable virtual bids at a node, a market participant would create a persistent divergence between day-ahead and expected real-time prices. In an efficient market, this should promote competition for arbitrage opportunities, leading to price convergence and making manipulation difficult to sustain.

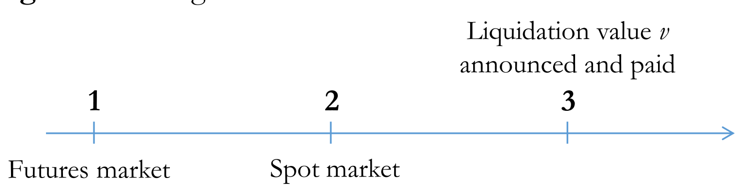
A theory (or theories) of forward market manipulation would help FERC administer principles-based enforcement, as advocated by former Commissioner Tony Clark (Clark and Meidhof, 2014), and serve three main purposes. First, theoretical models could explain what market imperfections (e.g., asymmetric information, imperfect competition, transaction costs, and risk aversion) allow manipulation to exist in equilibrium, rather than as an isolated surprise. This analysis may expose flaws in the current design of electricity markets that create inefficient incentives and unintended consequences, and suggest prospective reforms in market rules if flawed design elements are inconsistent with efficient outcomes. Second, models may be used to determine the material effect of manipulation on prices, which is required for calculating disgorgement (i.e., repayment of unjust profits resulting from the violation of regulations, orders and tariffs in FERC enforcement actions). Finally, theory would provide empirical implications (i.e., model-based predictions) for market performance that may be tested in the data to determine if observed actions implied manipulation. Improved analysis tools would complement existing *ex post* screens, which are not based on economic models of financial trading, and help market monitors and enforcement activities distinguish between manipulative and efficient transactions.

3. Model

An extensive literature in the field of market microstructure has considered strategic behavior in equity markets, building on Kyle's (1985) rational expectations model of batch trading. This model considers a game in quantities between two types of financial market participants characterized by asymmetric information: a single risk neutral informed trader who receives private information about the liquidation value v of a stock, and a number of noise traders who have no information on market fundamentals and trade for hedging or liquidity needs (Black, 1986; Bloomfield et al., 2009). In the single auction setting of Kyle's model, the informed trader and the noise traders submit trade quantities to a risk neutral market maker providing liquidity to the market. The market maker observes the aggregate net order flow and sets an efficient clearing price, equal to the conditional expected value of the asset, given the aggregate order quantity. In line with the rational expectations literature (Grossman and Stiglitz, 1980), the informed trader can infer other participants' information and strategies from market statistics: even though she does not know the actual order flow of the noise traders, she knows the parameters of its distribution, and correctly anticipates the pricing rule used by the market maker. Thus, the informed trader accounts for the fact that her order has an impact on the equilibrium price set by the market maker, and acts strategically to reveal some, but not all, of her private information, in order to preserve some profit margin for her trades. The strategy succeeds because the uninformed trading volume enables the informed trader to hide her trades and make profits at the expense of the noise traders. In equilibrium, prices will not immediately adjust to their full information value, because withholding some private information allows the informed trader to earn positive trading profits.

Several papers have built on Kyle's (1985) seminal work (Kyle, 1989; Jarrow, 1992, 1994; Allen and Gorton, 1992; Allen and Gale, 1992; Kumar and Seppi, 1992; Bagnoli and Lipman, 1996). In particular, Kumar and Seppi (1992) demonstrate that cross-product manipulation could be sustained in sequential financial markets due to asymmetric information. In their setting, a futures market at date 1 is followed by a spot market for the stock at date 2. Futures accounts are closed through cash settlement at the spot market price, rather than physical delivery as in Pirrong (1995). The liquidation value of the stock, v, is exogenous (i.e., cannot be manipulated) and assumed to be normally distributed with mean μ and variance σ_v^2.[6] Figure 1 illustrates the timing of events.

Figure 1. Timing of events



The model presents a game in quantities with noise traders, one informed trader, and one uninformed manipulating trader. All agents are risk neutral. The noise traders are price takers assumed to represent various needs for market hedging. The informed trader receives a signal about the stock's

[6] The assumption of normally distributed random variables allows derivation of closed form solutions in an analytically tractable model. While this implies that prices are not necessarily positive, the assumption does not seem overly restrictive. For example, the significant increase in the supply of renewable resources in California and Texas is contributing to increased frequency of negative electricity prices in their wholesale electricity markets (Bajwa and Cavicchi, 2017).

liquidation value *v*, interpreted as the result of better information or analysis, and then trades to maximize the value of her trades. The uniformed trader has no such information signal, but holds a related financial position and can successfully manipulate the spot settlement price of the stock futures contract to enhance her total profitability. On average, the manipulator expects to lose money in the spot market, but earn higher profits in the futures market by taking long or short positions with equal probability.

This strategy presents an interesting analogy to the use of unprofitable virtual transactions to move day-ahead electricity prices in a direction that benefits FTR positions, absent control over real-time prices. For the adaptation to electricity markets, the liquidation value in Figure 1 would be established by the real-time dispatch, which is assumed to produce a workably competitive market result; the spot market would correspond to the day-ahead electricity market; the futures market would determine the related financial position, such as FTRs. We examine the effects of introducing a second informed trader on the equilibrium of the game presented in Kumar and Seppi (1992). The two informed traders receive a common signal about the stock's liquidation value, and then trade to maximize the value of their respective trades. While Kumar and Seppi (1992) show that profits from manipulation fall to zero as the number of uninformed manipulators grows, we consider a setting with one manipulator, in line with FERC settlements of cross-product manipulation cases.[7] Further, we maintain the original assumption that futures positions are settled through cash settlement (i.e., a payment equal to the spot price minus the futures price). Thus, our work differs from Pirrong (1995), who builds on Kumar and Seppi (1992) to construct a model in which delivery settled contracts are manipulated.

Trading sequence. The manipulator is endowed with some initial wealth $|W|$, where W is assumed to be normally distributed with mean 0 and variance σ_W^2. In period 1, she deposits a portion

[7] Adding a second informed trader or many manipulators to the game diminishes each manipulator's profits. However, equilibrium outcomes, empirical and welfare implications are different in the two cases.

of |W| into a margin account to participate in the futures market. This portion is denoted as $|\Delta| = \gamma |W|$, where $0 \le \gamma \le 1$. Then, the manipulator takes a futures position $\Delta = \gamma W$, which may be positive (i.e., long position) or negative (i.e., short position). The margin assumption is useful because it induces a normal distribution for Δ, and thus makes the model analytically tractable. Noise traders take an aggregate position *e*, which is normally distributed with mean 0 and variance σ_e^2. The market maker observes the aggregate order $y_{1f} = e + \Delta$ and sets the efficient futures price, $F(y_{1f})$, equal to the expected spot price given the net futures order flow, $E(S \mid y_{1f})$.

In period 2, trading takes place in the spot market, which is characterized by asymmetric information. Given the noise traders' futures position, which can be inferred from y_{1f}, the manipulator chooses an optimal spot position *z*. Noise traders submit an aggregate order *u*, which is normally distributed with mean 0 and variance σ_u^2. Given the net futures order flow, y_{1f}, and the liquidation value, *v*, the two informed traders choose optimal spot positions x_1 and x_2, respectively. We assume that each informed trader receives the same private signal about the stock's liquidation value and acts à la Cournot, taking the position of the other informed trader as given. The market maker observes the net spot order flow $y_{2s} = u + z + x_1 + x_2$ and sets the efficient spot price, $S(y_{2s}, y_{1f})$, equal to the expected liquidation value, given the net spot and futures order flows, $E(v \mid y_{2s}, y_{1f})$. The futures contracts are cash-settled at the spot price $S(y_{2s}, y_{1f})$.

In period 3, spot positions are also closed through cash settlement. As noted above, the exogenous liquidation value *v* is assumed normally distributed with mean μ and variance σ_v^2, and cannot be manipulated.

Trader optimization problems. In period 1, the manipulator solves the following problem:

$$Max_{\Delta} E_e \{ Max_z E_{v,u,x_1,x_2} \{ \Delta[S(y_{2s}, y_{1f}) - F(y_{1f})] + z[v - S(y_{2s}, y_{1f})] \mid e \} \} \tag{1}$$

subject to $|\Delta| \leq |W|$ (2)

The trader establishes a futures position Δ, and then takes a position *z* in the spot market to maximize her expected profit, given the noise traders' futures position *e* and subject to the constraint in (2).

In period 2, informed trader 1 solves the problem:

$$Max_{x_1} E_{u,z}\{x_1[v - S(y_{2s}, y_{1f})] \mid v, x_2, y_{1f}\} \tag{3}$$

and informed trader 2 solves the symmetric problem:

$$Max_{x_2} E_{u,z}\{x_2[v - S(y_{2s}, y_{1f})] \mid v, x_1, y_{1f}\} \tag{4}$$

Each trader takes a position *x* in the spot market to maximize her expected profit, given the net futures order flow, y_{1f}, the spot position of the other informed trader, and common information about the liquidation value, *v*.

The noise traders do not solve an optimization problem, because they only transact for non-speculative reasons and do not act strategically.

Equilibrium. Equilibrium models of two-settlement (i.e., forward and spot) markets adopting closed-loop information structures yield subgame perfect equilibria: when making decisions in the first stage, market participants correctly anticipate the reactions of all agents in the second stage (Allaz, 1992; Allaz and Vila, 1993). Equilibrium solutions are obtained using backwards induction: after deriving the spot market equilibrium as a function of forward quantities, the expected optimization problem in the forward market is solved accounting for the spot market optimality conditions.

The equilibrium model presented above is analytically tractable, if random variables are normally distributed, and pricing rules and order strategies are linear. Under these conditions, a unique linear equilibrium exists, and closed form expressions for equilibrium outcomes in the spot and futures markets are obtained using backwards induction. An important feature of strategic trader models from the market microstructure literature is that, at each date, the market maker observes the aggregate net order flow and sets the clearing price to break even. Equilibrium prices are thus equal to the conditional expected value of the asset in the following period, given the aggregate order quantities. We take two steps to find the unique equilibrium of the two-stage game.

First, we solve for the equilibrium quantities and prices in the spot market (period 2), as a function of futures positions. The optimal spot positions are derived from the first order conditions of each player's problem. The spot position of the uninformed manipulator is:

$$z = \frac{\Delta + k(e+\Delta)}{2} \tag{5}$$

where $k = \frac{\gamma^2 \sigma_W^2}{\gamma^2 \sigma_W^2 + \sigma_e^2}$, while the spot position of each informed trader is:

$$x_1 = x_2 = \frac{1}{\sqrt{2}}(v-\mu)\frac{(\sigma_u^2 + \frac{k}{4}\sigma_e^2)^{1/2}}{\sigma_v} \tag{6}$$

The market maker observes the aggregate net futures and spot order flows, y_{1f} and y_{2s}, and sets the efficient equilibrium price *S* that clears the spot market. Market efficiency requires that the spot price be equal to the conditional expected value of the settlement value, given the aggregate orders in the futures and spot markets. Applying the laws of conditional distributions of normally distributed random variables, the equilibrium spot price reduces to:

$$S(y_{2s}, y_{1f}) = E(v \mid y_{2s}, y_{1f}) = \mu + \lambda[y_{2s} - E(y_{2s} \mid y_{1f})] \tag{7}$$

where λ represents the price impact per unit trade (Vayanos and Wang, 2013). After substituting in expressions for the conditional variances and expectations, the spot price in eq. (7) becomes:

$$S(y_{2s}, y_{1f}) = \frac{1}{3}\mu + \frac{2}{3}v + \lambda u + \frac{\lambda}{2}(1-k)\Delta - \frac{\lambda}{2}ke \tag{8}$$

where:

$$\lambda = \frac{\sqrt{2}}{3} \frac{\sigma_v}{(\sigma_u^2 + \frac{k}{4}\sigma_e^2)^{1/2}} \tag{9}$$

Second, we solve for the equilibrium prices and quantities in the futures market (period 1) by substituting the spot equilibrium outcomes in the manipulator's problem, which reduces to:

$$Max_{\Delta}[\frac{\lambda}{4}\Delta^2(k-1)^2 + \frac{\lambda}{4}k^2\sigma_e^2] \tag{10}$$

subject to the constraint in (2). Since the quadratic form in (10) is increasing in Δ, $\gamma = 1$ and the manipulator is indifferent between the two corner solutions. As a result, long and short futures positions are submitted with equal probability (i.e., the uninformed trader randomizes her futures order between long and short positions with equal probability). When the manipulator goes long in the futures market, she expects to buy in the spot market to increase the spot price; similarly, when the manipulator goes short in the futures market, she expects to sell in the spot market to decrease the spot price. The manipulating trader expects to lose, on average, in the spot market, but these losses are offset by higher expected profits in the futures market; as shown below, the strategy is profitable *ex ante*, as long as the futures position Δ is larger than the spot position z (i.e., the trader has sufficient leverage).

4. Empirical and welfare implications

A plausible theory of forward market manipulation would be useful to determine its material effect on prices under transparent and objective standards of analysis. Equilibrium models would also provide empirical implications for market manipulation analysis (i.e., predictions that may be tested in the data to detect manipulation). Finally, models may be used to examine the distributional effects of manipulation. In this section, we present empirical and welfare implications of the equilibrium presented in Section 3, as well as implications from three benchmark models. Our first benchmark is a naïve model of competitive arbitrage that only includes noise traders in the spot market. The second benchmark is Kyle's (1985) model, which adds one strategic trader with private information on the asset liquidation value in the spot market. Our last benchmark is Kumar and Seppi's (1992) model, which builds on Kyle's model by including a futures market with cash settlement and an uninformed manipulating trader in futures and spot markets. A comparison of key equilibrium outcomes for the four models is presented in the Appendix (Table A1). Model predictions for market liquidity and performance, as well as welfare implications, are discussed below and compared in the Appendix (Tables A2 and A3). A numerical illustration of the impacts of manipulation based on a common set of assumed parameter values concludes the section.

4.1 Market liquidity

As noted by Kyle (1985), "market liquidity is a slippery and elusive concept, in part because it encompasses a number of transactional properties of markets." As a result, precise definitions are usually provided in the context of specific models (Hasbrouck, 2007). The dimension of liquidity considered for the present discussion is market depth, defined as "the order flow necessary to induce prices to rise or fall by one dollar" (Kyle, 1985). In the market microstructure literature, market depth is measured by the quantity $1/\lambda$, while its inverse, λ, is a measure of illiquidity and represents the price impact per unit trade (O'Hara, 1995; de Jong and Rindi, 2009; Vayanos and Wang, 2013).

Markets where trades have the least effect on price are generally viewed as liquid. Thus, deep markets are characterized by a value of λ close to zero. In the naïve model of competitive arbitrage, where noise traders are the only market participants, the spot market is infinitely deep (i.e., $\lambda_N = 0$) because the market price does not depend on the size of the aggregate order of the noise traders. As discussed above, an informed trader acts strategically to reveal some of her private information in the spot market; however, not all information will be immediately revealed to preserve some profit margin for her trades. Thus, the introduction of one informed market participant reduces market liquidity. Specifically, in Kyle's (1985) model λ is positive and equal to:

$$\lambda_K = \frac{1}{2}\frac{\sigma_v}{\sigma_u} \tag{11}$$

In contrast, the introduction of one uninformed manipulative trader and a futures market increases spot market liquidity, yielding a lower λ in Kumar and Seppi's (1992) model:

$$\lambda_{KS} = \frac{1}{2}\frac{\sigma_v}{(\sigma_u^2 + \frac{k}{4}\sigma_e^2)^{1/2}} \tag{12}$$

Note that the price impact per unit trade accounts for the variance of the noise futures positions: as a result, λ_{KS} is lower than λ_K (and $1/\lambda_{KS}$ is higher than $1/\lambda_K$). Finally, the introduction of a second informed trader in the modified Kumar and Seppi model further increases spot market liquidity, as can be seen comparing eq. (9) with eq. (12).

4.2 Market performance

4.2.1 Price convergence

Price convergence is an important indicator of market performance. In electricity markets, convergence is typically measured by the difference between average day-ahead and real-time prices at each location on the network on a monthly or annual basis. Over time, price convergence has improved with refinements of electricity market design (Monitoring Analytics, 2016; Potomac Economics, 2016a, 2016b).

We evaluate two dimensions of convergence between spot prices and liquidation values. First, market efficiency implies that the spot price is equal to the expected liquidation value, conditional on the aggregate order flow in the current and past periods. However, when intertemporal trading profits are not arbitraged away, the spot price differs from the unconditional expected liquidation value (i.e., S≠E(v)=μ). In Kyle's (1985) model, the spot premium is equal to $\lambda_K y_{2s}$, where λ_K is defined in eq.

(11). In the context of Kumar and Seppi's (1992) formulation, prices diverge because the spot trades of an uninformed manipulator seeking to profit from her futures positions are confused with those of the informed traders. In this case, the spot premium is $\lambda(y_{2s} - ky_{1f})$, where λ is defined in eq. (9) with two informed traders and eq. (12) with one informed trader, respectively.

Second, we examine convergence between expected spot price and expected liquidation value (i.e., whether E(S)≠E(v)) to investigate whether the theoretical models predict systematic forward premia as a result of cross-product manipulation. When market efficiency is assumed, $E(S) = E(v)$. This result follows from the application of the law of iterated expectations. Thus, theoretical models from the market microstructure literature predict that equilibrium spot prices deviate from expected liquidation values when cross-market manipulation is in the works, and the magnitude of the spot premium depends on the type and number of traders in the spot market. However, spot prices and expected liquidation values do not diverge systematically in the long run, as a result of manipulation. Hence, long-run price convergence does not fully characterize market efficiency.

4.2.2 Variance of the spot price

Even though spot prices may not diverge systematically from expected liquidation values, there may be large but offsetting price spreads. As a result, we also consider the variance of the spot price $Var(S)$ as a measure of performance. In the naïve model of competitive arbitrage, the spot price variance is equal to zero because the price always equals the expected liquidation value. The analytical expressions for the spot price variance in terms of λ differ in Kyle's (1985) and Kumar and Seppi's (1992) models (see Table A2). However, after substituting in the optimal value for λ_K and λ_{KS}, both expressions reduce to $\frac{1}{2}\sigma_v^2$, indicating that the variance of the spot price is unchanged by the introduction of an uninformed manipulator. In contrast, the variance of the spot equilibrium price in the modified Kumar and Seppi model is given by $\frac{2}{3}\sigma_v^2$. Thus, spot price variance increases as more informed traders participate in the spot market, but not as a result of cross-market manipulation.

4.2.3 Variance of the difference between spot price and liquidation value

Our third measure of market performance is the variance of the difference between spot price (S) and liquidation value (v). Note that $Var(v-S)=E(v-S)^2$. A spot market characterized by arbitrage efficiency would minimize this deviation, provided that $E(S)=\mu$. In the naïve model, there is no arbitrage needed to bring convergence between spot prices and liquidation value. As a result, the variance of the price difference is equal to the variance of the liquidation value:

$$Var(v-S)=E(v-\mu)^2=\sigma_v^2 \tag{13}$$

In Kyle's (1985) model, the variance of the difference between spot price and liquidation value decreases by half, relative to the efficient benchmark:

$$Var[v-S(x+u)]=\frac{1}{2}\sigma_v^2 \tag{14}$$

Thus, there exists an arbitrage benefit in the spot market because the informed trader moves the price closer to the asset liquidation value, although she does not fully close the gap in order to preserve some margin for her trades. This arbitrage benefit is unchanged by the introduction of the uninformed manipulator. In the modified Kumar and Seppi (1992) model, arbitrage efficiency is further enhanced by the introduction of a second informed trader, and the variance of the difference between spot price and liquidation value is lower:

$$Var[v-S(x_1+x_2+u+z)]=\frac{1}{3}\sigma_v^2 \tag{15}$$

The variance of the difference between spot price and liquidation decreases as more informed traders participate in the spot market, but not as a result of cross-market manipulation.

4.2.4 Variance of the aggregate spot positions

The fourth metric of market performance considered here is the variance of the aggregate spot positions. In the naïve model of no arbitrage, this equals the variance of the noise trader positions:

$$Var(y_{2s}) = Var(u) = \sigma_u^2 \tag{16}$$

The introduction of one informed trader doubles the variance of spot trading in Kyle's model. Variance increases further as a result of cross-market manipulation in Kumar and Seppi's model, and when a second informed trader participates in the spot market (see Table A2).

4.2.5 Correlation between spot price and manipulator's futures position

Our last measure of market performance is the correlation between futures position of the uninformed manipulating trader and equilibrium spot price. We expect this correlation to be positive: for example, if the manipulator establishes a long futures position, she will expect to buy in the spot market in order to raise the spot price. In Kumar and Seppi's model, correlation is equal to:

$$Corr(\Delta, S) = (\frac{1}{2})^{3/2}(1-k)\frac{\sigma_W}{(\sigma_u^2 + \frac{k}{4}\sigma_e^2)^{1/2}} \tag{17}$$

The introduction of a second informed trader in the spot market slightly decreases the correlation:

$$Corr(\Delta, S) = \frac{1}{2\sqrt{3}}(1-k)\frac{\sigma_W}{(\sigma_u^2 + \frac{k}{4}\sigma_e^2)^{1/2}} \tag{18}$$

4.3 Welfare analysis

By assumption, the real-time market assures productive efficiency. Cross-market manipulation has implications for the distribution of profits and losses among market participants. Kumar and Seppi (1992) present the *ex post* profits associated with the optimization decisions of each player in equilibrium, conditional on the information set available at each market stage. We focus here instead on the *ex ante* (or unconditional) profits of market participants, which represent the expected profits before entering the futures and spot markets. Deviations from the competitive benchmark provide a measure of the allocative effects. We present closed form expressions for the *ex ante* profits associated with equilibrium outcomes in Section 3, as well as Kyle's and Kumar and Seppi's models. Importantly, at each market stage the price is assumed equal to the expected price in the following stage, conditional on the market maker's information. Therefore, at each stage the market maker breaks even in expectation, and market participants engage in a zero-sum game where each one expects to benefit at the expense of others.

Analytical derivations are presented in Table A3, but the discussion here focuses on welfare implications of the modified Kumar and Seppi (1992) model. Informed traders earn positive *ex ante* profits from the spot market:

$$\pi_{I_1,s} = \pi_{I_2,s} = E_{e+\Delta,u,v,x_1,x_2,z}[x(v-S)] = \frac{1}{9\lambda}\sigma_v^2 \tag{19}$$

The uninformed manipulating trader also earns positive *ex ante* profits:

$$\pi_U = E_{e+\Delta,u,v,x_1,x_2,z}[\Delta(S-F)+z(v-S)] = [\frac{\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2 \tag{20}$$

resulting from profits in the futures market:

$$\pi_{U,f} = E_{e,\Delta}\{\Delta[\frac{\lambda}{2}(1-k)\Delta-\frac{\lambda}{2}ke]\} = [\frac{2\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2 \tag{21}$$

and losses in the spot market:

$$\pi_{U,s} = E_{e,\Delta}\{-\frac{\Delta(1+k)+ek}{2}[\frac{\lambda}{2}(1-k)\Delta-\frac{\lambda}{2}ke]\} = -[\frac{\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2 \tag{22}$$

As long as the manipulating trader's futures position Δ is larger than her spot position $z = \frac{\Delta(1+k)+ek}{2}$, cross-market manipulation is profitable *ex ante*.

Finally, noise traders who transact for non-speculative reasons lose money on average, compensating other participants in an informationally efficient market (Bloomfield et al., 2009). The uninformed manipulating trader's profits in the futures market come at the expense of the noise traders, who incur symmetric losses:

$$\pi_{N,f} = E_{e,\Delta}\{e\lambda[\frac{(1-k)\Delta - ke}{2}]\} = -\frac{\lambda}{2}k\sigma_e^2 = -[\frac{2\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2 + \frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2 \tag{23}$$

Ex ante losses of spot noise traders are equal to:

$$\pi_{N,s} = E_{e,\Delta}\{-\lambda\sigma_u^2\} = -\lambda\sigma_u^2 = -[\frac{\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2 + \frac{k}{4}\sigma_e^2)^{1/2}}]\sigma_u^2 \tag{24}$$

4.4 Numerical illustration

We compare the equilibrium models in Sections 4.1-4.3 based on a common set of parameters and realizations for the liquidation value, spot and futures positions. These values are not intended to be representative of specific markets, but to illustrate the impacts of manipulation on market liquidity, performance and welfare. Table 1 presents the empirical implications of our model and the three benchmarks. As noted above, introduction of two informed traders, one uninformed trader and a futures market decreases spot market liquidity relative to the naïve model of competitive arbitrage, but increases liquidity relative to the other two benchmarks characterized by asymmetric information. Thus, λ decreases in the modified Kumar and Seppi model. Further, the equilibrium spot price differs across models, but no model predicts a systematic divergence between spot price and liquidation value, because $E(S) = E(v)$.

Ex ante profits are presented in Table 2. In the naïve model of the spot market, noise traders break even on average. In Kyle's model, the informed trader earns a profit at the expense of the noise traders. In Kumar and Seppi's model, cross-market manipulation hurts futures noise traders, but benefits spot noise traders and the informed trader (who, respectively, incur lower losses and make larger profits than in Kyle's model). Adding a second informed trader reduces aggregate profits for the informed traders due to increased spot market liquidity. It also benefits noise traders in both futures and spot markets by reducing their losses, and decreases the overall profitability of the cross-market manipulation strategy. On average, the manipulator expects to incur losses in the spot market, but earn higher profits in the futures market at the expense of the noise traders.

5. Simulation results

The equilibrium manipulation strategy described in Section 3 presents an interesting analogy to cross-product manipulation in FTR and day-ahead markets, absent control over real-time prices. Yet, day-ahead electricity markets differ from purely financial markets, because they are subject to physical and reliability power system constraints that are known to affect the next-day dispatch. While the assumption of normally distributed random variables allows derivation of closed form solutions in an analytically tractable model, FTRs are constrained by transmission capacity. Imposing futures position limits is not analytically tractable in the normally distributed models in Section 3. Hence, we use simulation to examine the effects of imposing futures position limits on equilibrium outcomes and model predictions of the modified Kumar and Seppi (1992) model.

5.1 Normal distributions

First, we describe our simulation and show that it allows replication of the analytical results in Section 3 when all random variables are normally distributed.

The equilibrium spot price in eq. (7) may be rewritten in terms of expected liquidation value, spot and futures aggregate positions as follows:

$$S(y_{2s}, y_{1f}) = E(v \mid y_{2s}, y_{1f}) = \mu + \lambda[y_{2s} - E(y_{2s} \mid y_{1f})] = \mu + \lambda y_{2s} - \lambda k y_{1f} = \mu + \lambda y_{2s} + \beta y_{1f} \quad (25)$$

Given the parameter values for $\mu, \sigma_e^2, \sigma_W^2, \sigma_u^2, \sigma_v^2$ in Section 4.4:

1. We select a value of $\gamma \in [0,1]$, the share of wealth deposited into a margin account to participate in the futures market.
2. We draw 50,000 random realizations for the noise trader positions e and u, the liquidation value v, and wealth W, and given γ find the corresponding futures position Δ.
3. We select 20 values of $\lambda \in [0,1]$ and 20 values of $\beta \in [-1,0]$, obtaining 400 possible combinations of λ and β.[8] For each combination, and given the 50,000 random realizations in 2., we derive the optimal spot positions of the uninformed and informed traders, x_1, x_2 and z using eq.(5) and (6), the aggregate futures and spot positions y_{1f} and y_{2s}, and the spot price S from eq. (25).
4. We now have a range of realizations for y_{1f} and y_{2s}. Each range is divided into thirty equally spaced intervals to create a 30×30 matrix. Each cell in this matrix represents a unique combination of intervals for y_{1f} and y_{2s}, and includes values of spot and futures positions falling within each interval.
5. For each cell, we calculate $|S - E(v \mid y_{1f}, y_{2s})|^2$, where S is obtained from eq.(25) using midpoint values of y_{1f} and y_{2s}, and the estimator of $E(v \mid y_{1f}, y_{2s})$ is the average liquidation value v in that cell. Summing across cells, we obtain the error associated with a given combination of λ and β. The optimal combination is the one that minimizes the sum of squared differences between spot price and estimator of $E(v \mid y_{1f}, y_{2s})$, on average, over 50,000 simulation runs.

[8] Values of estimated price impact vary in the empirical literature, but simulation studies have used values of λ in the range [0,5] (Collin-Dufresne and Fos, 2016).

We repeat steps 2.-5. for multiple values of γ, each one yielding a different optimal combination λ and β, and verify that γ=1 maximizes profits, in line with the analytical results. Table 3 reports simulation results for the optimal value of λ and γ=1, and compares them with the analytical results.

5.2 Bounded normal distributions

As discussed, FTRs may correspond to futures contracts in the analysis. In actual electricity markets, FTRs are constrained by the capacity of the transmission system, and the total amount of contracts that may be awarded must achieve simultaneous feasibility (Rosellón and Kristiansen, 2013). Accounting for transmission capacity constraints and other network characteristics, and regardless of physical generation and load, system operators determine the maximum amount of FTRs that would be dispatchable on the transmission path from source to sink; the set of FTRs awarded is simultaneously feasible only if it does not exceed network capacity. In the context of the model presented in Section 3, futures contracts would be subject to position limits. We modify our simulation accordingly, and examine the effects of imposing futures position limits on equilibrium outcomes and model predictions.

Aggregate futures positions are assumed to have a mixed type distribution (Kumar, 2004), given by a normal distribution with point mass at the boundaries *–a* and *a*. We refer to this as a bounded normal distribution in the interval [*-a, a*]. Formally, let X be a continuous random variable with bounded normal distribution $BN(0,\sigma^2,a)$. Its cumulative distribution function is given by:

$$F(x)=\Phi(-\frac{a}{\sigma})I_{\{x\geq -a\}}+\int_{-a}^{x}\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{x^2}{2\sigma^2})I_{\{x\in(-a,a)\}}+(1-\Phi(-\frac{a}{\sigma}))I_{\{x\geq a\}} \tag{26}$$

where $\Phi(x)$ is the cumulative distribution function for the standard normal distribution, and I_A represents the indicator function of a subset A of X. A bounded normal distribution differs from a truncated normal distribution because the probability that the random variable equals the boundary values is nonzero. Since there exists a positive probability that futures positions will be equal to their limits, a bounded normal distribution seems more appropriate in our context.

Our simulation proceeds as follows. As in Section 5.1, the spot position u, futures positions e and Δ, and liquidation value v are drawn from a normal distribution. If the aggregate futures position y_{1f} (in absolute value) exceeds the bound a, we award e or Δ in full, and set the other equal to the difference between a and the awarded position; each futures position has equal probability of being scaled down. As a result, e and Δ are no longer independent. In contrast, if the aggregate futures position (in absolute value) is lower than or equal to the bound, we make no adjustment, and e and Δ are independent. We denote the set of futures positions after the potential adjustment as $(\tilde{\Delta}, \tilde{e})$, their sum as $\tilde{y}_{1f}$, and obtain 50,000 realizations within the interval $[-a, a]$.

Next, we derive the optimal spot positions for the informed and uninformed traders. Recall that the uninformed trader solves the following problem in the spot market:

$$Max_z E_{v,u,x_1,x_2}\{\tilde{\Delta}[S(y_{2s}, \tilde{y}_{1f}) - F(\tilde{y}_{1f})] + z[v - S(y_{2s}, \tilde{y}_{1f})] \mid \tilde{e}\}$$

while the informed traders solve eq.(3) and (4). By substituting eq.(25) into the spot problems of the uninformed and informed traders, we obtain closed form solutions for the spot positions:

$$x_1 = x_2 = \frac{v - \mu - \lambda E(z \mid \tilde{y}_{1f}) - \beta \tilde{y}_{1f}}{3\lambda} \tag{27}$$

$$z = \frac{\tilde{\Delta}}{2} + \frac{1}{3} E(z \mid \tilde{y}_{1f}) - \frac{\beta}{6\lambda} \tilde{y}_{1f} \tag{28}$$

Taking the expectation of both sides of (28) conditional on $\tilde{y}_{1f}$, we simplify x_1 and x_2 to:

$$x_1 = x_2 = \frac{v - \mu}{3\lambda} - \frac{1}{4} E(\tilde{\Delta} \mid \tilde{y}_{1f}) - \frac{\beta}{4\lambda} \tilde{y}_{1f} \tag{29}$$

and reduce z to:

$$z=\frac{\tilde{\Delta}}{2}+\frac{1}{4}E(\tilde{\Delta}\,|\,\tilde{y}_{1f})-\frac{\beta}{4\lambda}\tilde{y}_{1f} \tag{30}$$

The analytical expressions for the spot positions of the informed and uninformed traders depend on $E(\tilde{\Delta}\,|\,\tilde{y}_{1f})$, for which we have no closed form solution when futures positions follow a bounded normal distribution. Given 50,000 simulated futures positions, we obtain an estimator for $E(\tilde{\Delta}\,|\,\tilde{y}_{1f})$ non-parametrically using the OLP method by Ortobelli Lozza et al. (2017). The rest of the simulation proceeds as discussed in the previous section.

In Table 4 we set the bound *a* equal to one or two standard deviations of the joint distribution $\Delta+e\sim N(0,\sigma_{\Delta}^{2}+\sigma_{e}^{2})$, so that aggregate futures positions fall within the bounds about 68% and 95% of the time, respectively. It is useful to read these results in combination with the simulation results for the unbounded case in Table 3. As the interval [*-a, a*] gets smaller, the price impact per unit trade, the variance of the spot price, and the variance of the difference between spot price and liquidation value increase. In contrast, other metrics of market performance do not show increasing or decreasing trends. Imposing futures position limits does not affect the key prediction of the theoretical model: the uninformed manipulating trader randomizes her futures order between long and short positions with equal probability, and loses on average on the spot market while earning higher profits on the futures market. Long-run convergence between spot prices and liquidation values and positive correlation between futures positions of the manipulator and spot prices are also unaffected by futures position limits. Simulation results are robust to alternate choices of the parameter values and distributional assumptions (i.e., uniform distribution for the aggregate futures positions, instead of a bounded normal distribution).[9]

[9] Sensitivity analyses are available from the authors upon request.

6. Conclusions

Due to the lack of economic storage, the existence of capacity and transmission constraints, and the small price elasticity of demand in the short run, physical electricity markets are vulnerable to price manipulation. This may increase the total cost of serving electricity demand, lead to market outcomes that do not reflect underlying fundamentals, and have implications for the distribution of profits and losses among market participants. An extensive literature exists on the exercise of supplier market power in electricity markets, and monitoring and mitigation rules are in place to detect this type of price manipulation in real-time physical markets. In contrast, a policy concern raised by enforcement actions of the Federal Energy Regulatory Commission focuses on price manipulation involving forward electricity markets and related financial positions: market participants may act against their economic interest in the day-ahead market to artificially move prices and gain profits on related financial positions that make up for any direct losses.

Enforcement actions of the FERC in regard to allegations of day-ahead price manipulation in electricity markets raise important issues about electricity market design and energy trading. While some ISOs have established *ex post* screens for cross-product manipulation involving virtual transactions and FTRs, these monitoring rules are not based on economic models of financial trading, and the theoretical foundations and empirical implications of day-ahead price manipulation remain poorly understood. As discussed in the paper, a theory (or theories) of forward market manipulation would help FERC administer principles-based enforcement and serve three main purposes: explain what market imperfections allow day-ahead price manipulation to be sustained over time, quantify its price impact in a transparent way, and provide empirical implications that may be tested in the data to determine if actions were consistent with manipulation.

To illustrate this point, our paper makes three contributions. First, building on the Kumar and Seppi (1992) model, we construct an example of electricity market equilibrium price manipulation under uncertainty where asymmetric information creates limits to arbitrage. A trader without superior information on market fundamentals can successfully manipulate the spot settlement price of a futures contract because her positions are confused with those of two informed traders. On average, the manipulator expects to lose money in the spot market, but earn higher profits in the futures market by randomizing her order between long or short positions with equal probability.

Our second contribution consists in examining the empirical and welfare implications of the equilibrium of the modified Kumar and Seppi (1992) model, and comparing these implications to those from three benchmark models. Cross-product manipulation creates a transitory divergence between spot prices and expected liquidation values. However, spot prices appear unbiased in the long term (i.e., manipulation does not lead to systematic forward premia), suggesting that long-run price convergence does not fully determine market efficiency. Manipulation also increases the variance of the spot aggregate positions, and creates a positive (albeit small) correlation between the spot price and the manipulator's related financial positions. With regard to welfare implications, manipulation determines a redistribution of *ex ante* profits and losses among market participants. With one informed trader in the spot market, manipulation hurts futures noise traders, but benefits spot noise traders and the informed trader. Adding a second informed trader in the spot market reduces the overall profitability of the cross-market manipulation strategy and aggregate profits for the informed traders due to increased market liquidity, while benefiting noise traders in both futures and spot markets.

Third, insights of a model of cross-product manipulation in financial markets may not carry over under conditions that apply in electricity markets, which are characterized by capacity constraints, loop flows and non-convexities. In particular, while the assumption of normally distributed random variables allows derivation of closed form solutions in an analytically tractable model, FTRs are constrained by the capacity of the transmission system. Our simulation results indicate that the main predictions of the theoretical model are not affected by futures position limits, and are robust to alternate parameter values and distributional assumptions.

The adaptation of the Kumar and Seppi analysis illustrates the type of theoretical analysis that should guide market manipulation inquiries. This is an existence demonstration that market manipulation is at least possible in principle. Could cross-market manipulation actions in electricity markets be evaluated and explained in the context of the Kumar and Seppi model? According to this theory, a key element allowing manipulation to exist in equilibrium is given by the randomization of trading strategies, including randomization of related financial positions. This would correspond to randomization of FTR positions in electricity markets. Absent such randomization, the market would soon uncover the arbitrage opportunities and eliminate the profitability for the manipulator.

Although FERC has extensive data collection powers, the information is not generally publicly available. However, on one case, using publicly available data from the Midwest ISO, we analyzed FTR positions taken by Louis Dreyfus Energy Services between November 2009 and February 2010. FERC Enforcement determined that the company placed uneconomic virtual demand bids at a node in the MISO footprint, Velva, to affect the value of its FTRs sinking at that node.[10] Analysis of FTR positions indicates that randomization was not observed empirically in the alleged price manipulation case involving Louis Dreyfus Energy Services. Hence, some other model, not yet defined, would be needed to demonstrate that the observed behavior implied market manipulation.

This suggests that market features other than asymmetric information may limit arbitrage opportunities in sequential electricity markets. Ongoing work by the authors considers alternate market imperfections allowing cross-product manipulation to exist in equilibrium, rather than as an isolated surprise, in the context of multi-stage models that account for features specific to electricity

[10] The case was settled on February 7, 2014, and the company agreed to pay disgorgement of $3,340,000 (plus interest) to MISO, and a civil penalty of $4,072,257. One of the traders, Xu Cheng, also received a civil penalty of $310,000 (146 FERC ¶ 61,072).

systems, like transmission congestion, loop flows and unit commitment decisions (Lo Prete et al., 2018; Guo and Lo Prete, 2018).

Table 1. Numerical illustration of the impacts of manipulation on measures of market liquidity and performance

	Naïve model	**Kyle (1985)**	**Kumar and Seppi (1992)**	**This paper**
	Noise traders	Noise traders, Informed trader	Noise traders, Informed trader, Uninformed manipulator	Noise traders, Two informed traders, Uninformed manipulator
λ	0	0.80	0.78	0.74
$1/\lambda$	∞	1.25	1.28	1.36
E(v)	50	50	50	50
S	50	80	82.54	83.03
E(S)	50	50	50	50
Var(S)	0	32	32	42.67
Var(v-S)	64	32	32	21.33
Var(y_{2s})	25	50	53.75	80
Corr(Δ,S)	-	-	0.14	0.11

Note: we assume σ_e=5; σ_W=2.5; σ_u=5; σ_v=8; μ=50; v=62; e=20; Δ=15; u=30.

Table 2. Numerical illustration of the impacts of manipulation on *ex ante* profits

	Naïve model	**Kyle (1985)**	**Kumar and Seppi (1992)**	**This paper**
	Noise traders	Noise traders, Informed trader	Noise traders, Informed trader, Uninformed manipulator	Noise traders, Two informed traders, Uninformed manipulator
Informed trader 1, spot	-	20	20.49	9.66
Informed trader 2, spot	-	-	-	9.66
Manipulator, futures	-	-	1.95	1.84
Manipulator, spot	-	-	-0.98	-0.92
Noise traders, futures	-	-	-1.95	-1.84
Noise traders, spot	0	-20	-19.52	-18.40

Note: we assume σ_e=5; σ_W=2.5; σ_u=5; σ_v=8; μ=50; v=62; e=20; Δ=15; u=30.

Table 3. Analytical and simulation results assuming normal distributions

	Analytical results	**Simulation results**
λ	0.74	0.70
E(v)	50	49.92 (8.03)
E(S)	50	49.96
Var(S)	42.67	41.33
Var(v-S)	21.33	19.98
Var(y_{2s})	80	93.62
Corr(Δ,S)	0.11	0.09
Pr(Δ>0)	50%	49.9%
	Ex ante profits	Realized profits
Informed trader 1, spot	9.66	10.32 (20.08)
Informed trader 2, spot	9.66	10.32 (20.08)
Manipulator, futures	1.84	1.40 (16.05)
Manipulator, spot	-0.92	-0.55 (8.19)
Noise traders, futures	-1.84	-3.02 (32.35)
Noise traders, spot	-18.40	-17.32 (28.20)

Note: analytical and simulation results assume the following parameters: σ_e=5; σ_W=2.5; σ_u=5; σ_v=8; μ=50. For the simulation results, we present average values (standard deviations in parentheses) from 50,000 runs. Further, we compare ex ante profits (for which we have analytical expressions) to average realized profits over 50,000 simulation runs.

Table 4. Simulation results assuming bounded normal distributions

	Two SD Bound	**One SD Bound**
Lower bound	$-2\sqrt{(\sigma_\Delta^2+\sigma_e^2)}=-11.18$	$-\sqrt{(\sigma_\Delta^2+\sigma_e^2)}=-5.59$
Upper bound	$2\sqrt{(\sigma_\Delta^2+\sigma_e^2)}=11.18$	$\sqrt{(\sigma_\Delta^2+\sigma_e^2)}=5.59$
λ	0.72	0.74
E(v)	50.04 (7.99)	49.96 (7.98)
E(S)	49.99	49.96
Var(S)	41.95	42.54
Var(v-S)	21.06	21.86
Var(y_{2s})	94.92	69.89
Corr(Δ,S)	0.08	0.12
Pr(Δ>0)	49.4%	49.9%
Realized profits		
Informed trader 1, spot	10.13 (19.89)	9.69 (19.32)
Informed trader 2, spot	10.13 (19.89)	9.69 (19.32)
Manipulator, futures	1.36 (16.19)	1.99 (16.20)
Manipulator, spot	-0.24 (8.90)	-1.04 (6.39)
Noise traders, futures	-3.55 (31.81)	-2.34 (27.78)
Noise traders, spot	-18.16 (29.47)	-18.76 (30.25)

Note: the table presents average values (standard deviations in parentheses) from 50,000 simulation runs, assuming σ_e=5; σ_W=2.5; σ_u=5; σ_v=8; μ=50.

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Appendix

Table A1. Key equilibrium outcomes

	Naive Model	Kyle (1985)	Kumar and Seppi (1992)	This paper
y_{1f}	-	-	$e+\Delta$	$e+\Delta$
y_{2s}	u	$x+u$	$x+u+z$	$x+u+z$
x	-	$\frac{v-\mu}{2\lambda_K}=(v-\mu)\frac{\sigma_u}{\sigma_v}$	$\frac{v-\mu}{2\lambda_{KS}}=(v-\mu)\frac{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}{\sigma_v}$	$\frac{2(v-\mu)}{3\lambda}=\sqrt{2}(v-\mu)\frac{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}{\sigma_v}$
x_1, x_2	-	-	-	$\frac{v-\mu}{3\lambda}$
z	-	-	$\frac{\Delta(1+k)+ek}{2}$	$\frac{\Delta(1+k)+ek}{2}$
k	-	-	$\frac{\sigma_W^2}{\sigma_W^2+\sigma_e^2}$	$\frac{\sigma_W^2}{\sigma_W^2+\sigma_e^2}$
S	μ	$S(y_{2s})=E(v\mid y_{2s})=$ $=\mu+\lambda_K[y_{2s}-E(y_{2s})]=$ $=\mu+\lambda_K(x+u)=$ $=\frac{v+\mu}{2}+\lambda_K u$	$S(y_{1f},y_{2s})=E(v\mid y_{1f},y_{2s})=$ $=\mu+\lambda_{KS}[y_{2s}-E(y_{2s}\mid y_{1f})]=$ $=\mu+\lambda_{KS}[(x+u+z)-k(e+\Delta)]=$ $=\frac{v+\mu}{2}+\lambda_{KS}u+\frac{\lambda_{KS}}{2}(1-k)\Delta-\frac{\lambda_{KS}}{2}ke$	$S(y_{1f},y_{2s})=E(v\mid y_{1f},y_{2s})=$ $=\mu+\lambda[y_{2s}-E(y_{2s}\mid y_{1f})]=$ $=\mu+\lambda[(x_1+x_2+u+z)-k(e+\Delta)]=$ $=\frac{1}{3}\mu+\frac{2}{3}v+\lambda u+\frac{\lambda}{2}(1-k)\Delta-\frac{\lambda}{2}ke$
F	-	-	$F(y_{1f})=E(S\mid y_{1f})=$ $=E[E(v\mid y_{1f},y_{2s})\mid y_{1f}]=$ $=E(v\mid y_{1f})=E(v)=\mu$	$F(y_{1f})=E(S\mid y_{1f})=$ $=E[E(v\mid y_{1f},y_{2s})\mid y_{1f}]=$ $=E(v\mid y_{1f})=E(v)=\mu$
λ	0	$\lambda_K=\frac{1}{2}\frac{\sigma_v}{\sigma_u}$	$\lambda_{KS}=\frac{1}{2}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}$	$\lambda=\frac{\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}$

Table A2. Empirical implications of the equilibrium models

Metric	Naive Model	Kyle (1985)	Kumar and Seppi (1992)	This paper
λ	0	λ_K	λ_{KS}	λ
$E(S)$	μ	μ	μ	μ
$Var(S)$	0	$\frac{1}{4}\sigma_v^2+\lambda_K^2\sigma_u^2= =\frac{1}{2}\sigma_v^2$	$\frac{1}{4}\sigma_v^2+\lambda_{KS}^2[\left(\frac{1-k}{2}\right)^2\sigma_W^2+\left(\frac{-k}{2}\right)^2\sigma_e^2+\sigma_u^2]= =\frac{1}{2}\sigma_v^2$	$\frac{4}{9}\sigma_v^2+\lambda^2[(\frac{1-k}{2})^2\sigma_W^2+(\frac{-k}{2})^2\sigma_e^2+\sigma_u^2]= =\frac{2}{3}\sigma_v^2$
$Var(v-S)$	σ_v^2	$\frac{1}{2}\sigma_v^2$	$\frac{1}{4}\sigma_v^2+\lambda_{KS}^2[\left(\frac{1-k}{2}\right)^2\sigma_W^2+\left(\frac{-k}{2}\right)^2\sigma_e^2+\sigma_u^2]= =\frac{1}{2}\sigma_v^2$	$\frac{1}{9}\sigma_v^2+\lambda^2[\left(\frac{1-k}{2}\right)^2\sigma_W^2+\left(\frac{-k}{2}\right)^2\sigma_e^2+\sigma_u^2]= =\frac{1}{3}\sigma_v^2$
$Var(y_{2s})$	σ_u^2	$2\sigma_u^2$	$2\sigma_u^2+\left(\frac{k+k^2}{4}\right)\sigma_e^2+\frac{(1+k)^2}{4}\sigma_W^2$	$3\sigma_u^2+\left(\frac{2k+k^2}{4}\right)\sigma_e^2+\frac{(1+k)^2}{4}\sigma_W^2$
$Corr(\Delta,S)$	-	-	$(\frac{1}{2})^{3/2}(1-k)\frac{\sigma_W}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}$	$\frac{1}{2\sqrt{3}}(1-k)\frac{\sigma_W}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}$

Table A3. Welfare implications of the equilibrium models

Profit	Kyle (1985)	Kumar and Seppi (1992)	This paper
π_I	$\pi_{I,K}=\pi_{I,s,K}=$ $=E_{u,v,x}[x(v-S)]=$ $=E_v\left\{Max_x E_u\left[x(v-S)\mid v\right]\right\}=$ $=\frac{1}{4\lambda_K}\sigma_v^2=\frac{1}{2}\sigma_v\sigma_u$	$\pi_{I,KS}=\pi_{I,s,KS}=$ $=E_{e+\Delta,u,v,x,z}[x(v-S)]=$ $=E_{e+\Delta}\left[E_v\left\{Max_x E_{u,z}\left[x(v-S)\mid v,e+\Delta\right]\right\}\right]=$ $=\frac{1}{4\lambda_{KS}}\sigma_v^2=\frac{1}{2}\sigma_v(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}$	$\pi_{I_1}=\pi_{I_1,s}=E_{e+\Delta,u,v,x_1,x_2,z}[x(v-S)]=$ $=E_{e+\Delta,u,v,x_1,x_2,z}[x(v-S)]=$ $=E_{e+\Delta}\left[E_v\left\{E_{x_2}\{Max_{x_1}E_{u,z}\left[x(v-S)\mid v,e+\Delta\right]\}\right\}\right]=$ $=\frac{1}{9\lambda}\sigma_v^2=\frac{1}{3\sqrt{2}}\sigma_v(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}=\pi_{I_2,s}$
$\pi_{I,f}$	-	-	-
$\pi_{I,s}$	$\pi_{I,s,K}=\frac{1}{4\lambda_K}\sigma_v^2=\frac{1}{2}\sigma_v\sigma_u$	$\pi_{I,s,KS}=\frac{1}{4\lambda_{KS}}\sigma_v^2=\frac{1}{2}\sigma_v(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}$	$\pi_{I_1,s}=\pi_{I_2,s}=\frac{1}{9\lambda}\sigma_v^2=\frac{1}{3\sqrt{2}}\sigma_v(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}$
π_U	-	$\pi_{U,KS}=\pi_{U,s,KS}+\pi_{U,s,KS}=$ $=E_{e+\Delta,u,v,x,z}[\Delta(S-F)+z(v-S)]=$ $=E_{e+\Delta}\{Max_z E_{u,v,x}[\Delta(S-F)+z(v-S)]\mid e,\Delta\}=$ $=\frac{\lambda_{KS}}{4}k\sigma_e^2=[\frac{1}{2}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$	$\pi_U=\pi_{U,s}+\pi_{U,s}=$ $=E_{e+\Delta,u,v,x_1,x_2,z}[\Delta(S-F)+z(v-S)]=$ $=E_{e+\Delta}\{Max_z E_{u,v,x_1,x_2}[\Delta(S-F)+z(v-S)]\mid e,\Delta\}=$ $=\frac{\lambda}{4}k\sigma_e^2=[\frac{\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$
$\pi_{U,f}$	-	$\pi_{U,f,KS}=\frac{\lambda_{KS}}{2}(1-k)\sigma_W^2=$ $=[\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$	$\pi_{U,f}=\frac{\lambda}{2}(1-k)\sigma_W^2=$ $=[\frac{2\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$
$\pi_{U,s}$	-	$\pi_{U,s,KS}=\frac{\lambda_{KS}}{4}(k-1)\sigma_W^2=-\lambda_{KS}\frac{k}{4}\sigma_e^2=$ $=-[\frac{1}{2}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$	$\pi_{U,s}=\frac{\lambda}{4}(k-1)\sigma_W^2=-\lambda\frac{k}{4}\sigma_e^2=$ $=-[\frac{\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$

π_N	$\pi_{N,K} = \pi_{N,s,K} = E_{u,v,x}[u(v-S)] =$ $= -\lambda_K \sigma_u^2 = -\frac{1}{2}\sigma_v \sigma_u$	$\pi_{N,KS} = \pi_{N,f,KS} + \pi_{N,s,KS} =$ $= E_{e+\Delta,u,v,x,z}[e(S-F)+u(v-S)] =$ $= E_{e+\Delta}\left\{E_{u,v,x,z}\left[e(S-F)+u(v-S)\middle\| e,\Delta\right]\right\} =$ $= -\frac{\lambda_{KS}}{2}k\sigma_e^2 - \lambda_{KS}\sigma_u^2 =$ $= -[\frac{1}{2}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}](\frac{1}{2}k\sigma_e^2+\sigma_u^2)$	$\pi_N = \pi_{N,f} + \pi_{N,s} =$ $= E_{e+\Delta,u,v,x_1,x_2,z}[e(S-F)+u(v-S)] =$ $= E_{e+\Delta}\left\{E_{u,v,x_1,x_2,z}\left[e(S-F)+u(v-S)\middle\| e,\Delta\right]\right\} =$ $= -\frac{\lambda}{2}k\sigma_e^2 - \lambda\sigma_u^2 =$ $= -[\frac{1}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}](\frac{1}{\sqrt{2}}k\sigma_e^2+\sqrt{2}\sigma_u^2)$
$\pi_{N,f}$	-	$\pi_{N,f,KS} = -\frac{\lambda_{KS}}{2}k\sigma_e^2 =$ $= -[\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$	$\pi_{N,f} = -\frac{\lambda}{2}k\sigma_e^2 =$ $= -[\frac{2\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\frac{k}{4}\sigma_e^2$
$\pi_{N,s}$	$\pi_{N,s,K} = -\lambda_K\sigma_u^2 = -\frac{1}{2}\sigma_v\sigma_u$	$\pi_{N,s,KS} = -\lambda_{KS}\sigma_u^2 =$ $= -[\frac{1}{2}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\sigma_u^2$	$\pi_{N,s} = -\lambda\sigma_u^2 =$ $= -[\frac{\sqrt{2}}{3}\frac{\sigma_v}{(\sigma_u^2+\frac{k}{4}\sigma_e^2)^{1/2}}]\sigma_u^2$

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