ELECTRICITY MARKET DESIGN: MULTI-INTERVAL PRICING

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A core challenge for all electricity systems is between monopoly provision and market operations. Electricity market design depends on critical choices. There is no escape from the fundamentals.

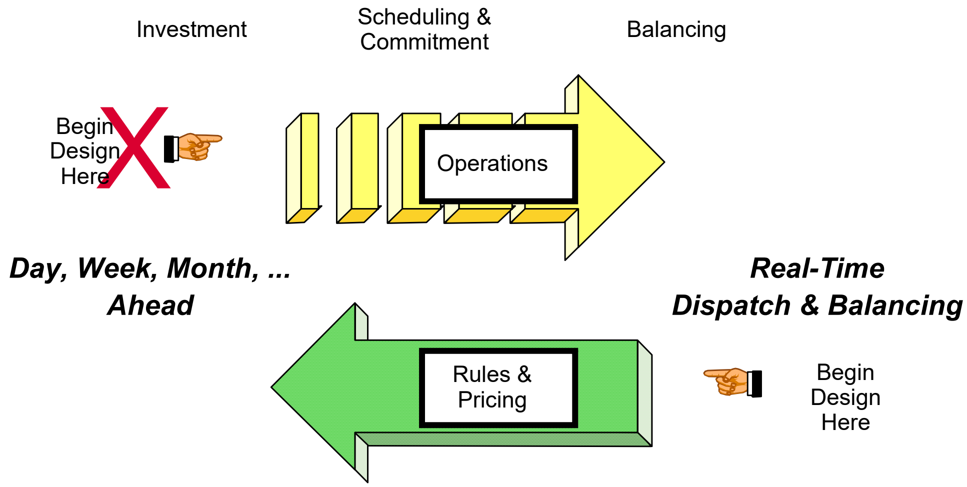
Integrated Monopoly	Competitive Markets
• Mandated	• Voluntary
• Closed Access	• Open Access
• Discrimination	• Non-discrimination
• Central Planning	• Independent Investment
• Few Choices	• Many Choices
• Spending Other People's Money	• Spending Your Own Money
• Average Cost Pricing	• Marginal Cost Pricing

A Key Market Design Objective

Supporting the Solution: Given the prices and settlement payments, individual optimal behavior is consistent with the aggregate optimal solution.

The solution to open access and non-discrimination inherently involves market design. Good design begins with the real-time market and works backward. A common failure mode starts with the forward market, without specifying the rules and prices that would apply in real time.

False Starts for the Electricity Market

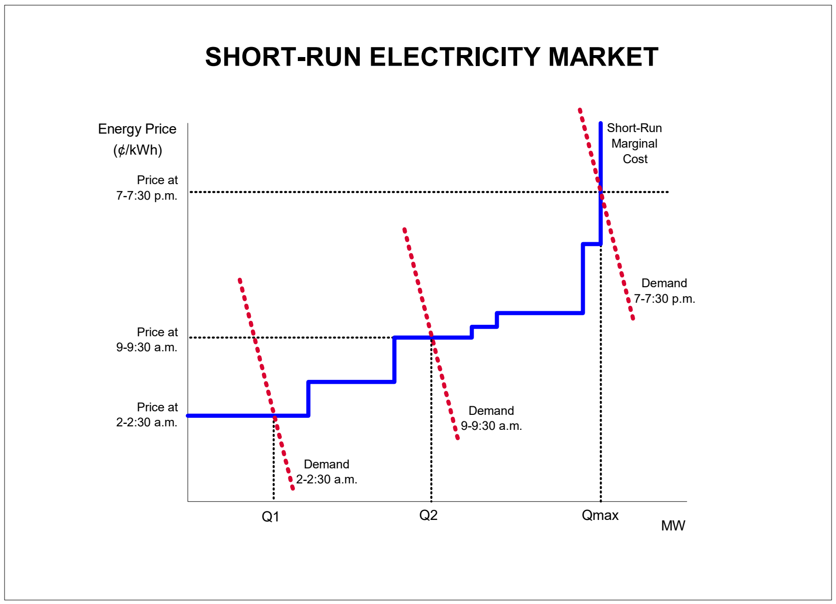


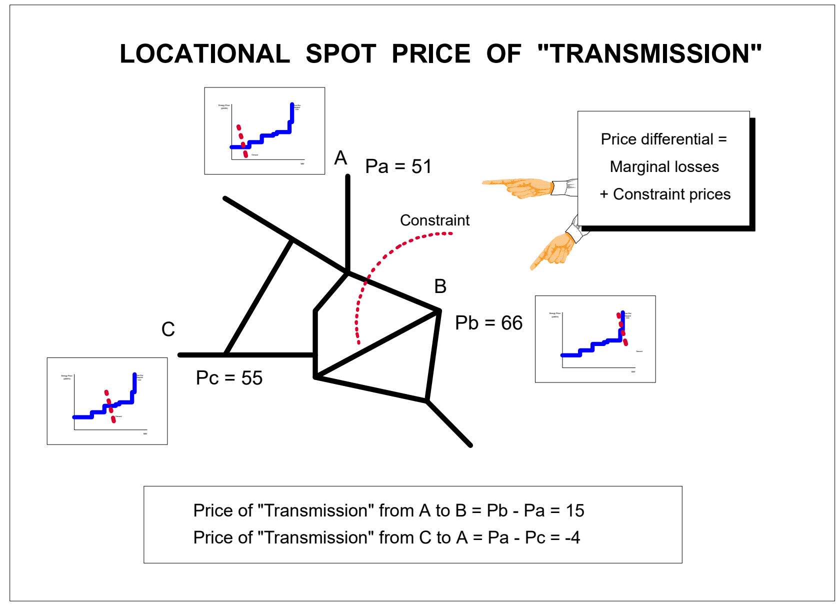
Market expectations determine incentives. Start at the end. Work backward, not forward, in setting market design.

All energy delivery takes place in the real-time market. Market participants will anticipate and make forward decisions based on expectations about real-time prices.

- **Real-Time Prices:** In a market where participants have discretion, the most important prices are those in real-time. "Despite the fact that quantities traded in the balancing markets are generally small, the prevailing balancing prices, or real-time prices, may have a strong impact on prices in the wholesale electricity markets. … No generator would want to sell on the wholesale market at a price lower than the expected real-time price, and no consumer would want to buy on the wholesale market at a price higher than the expected real-time price. As a consequence, any distortions in the real-time prices may filter through to the wholesale electricity prices." (Cervigni & Perekhodtsev, 2013)
- **Day-Ahead Prices:** Commitment decisions made day-ahead will be affected by the design of day-ahead pricing rules, but the energy component of day-ahead prices will be dominated by expectations about real-time prices.
- **Forward Prices:** Forward prices will look ahead to the real-time and day-ahead markets. Although forward prices are developed in advance, the last prices in real-time will drive the system.
- **Getting the Prices Right:** The last should be first. The most important focus should be on the models for real-time prices. Only after everything that can be done has been done, would it make sense to focus on out-of-market payments and forward market rules.

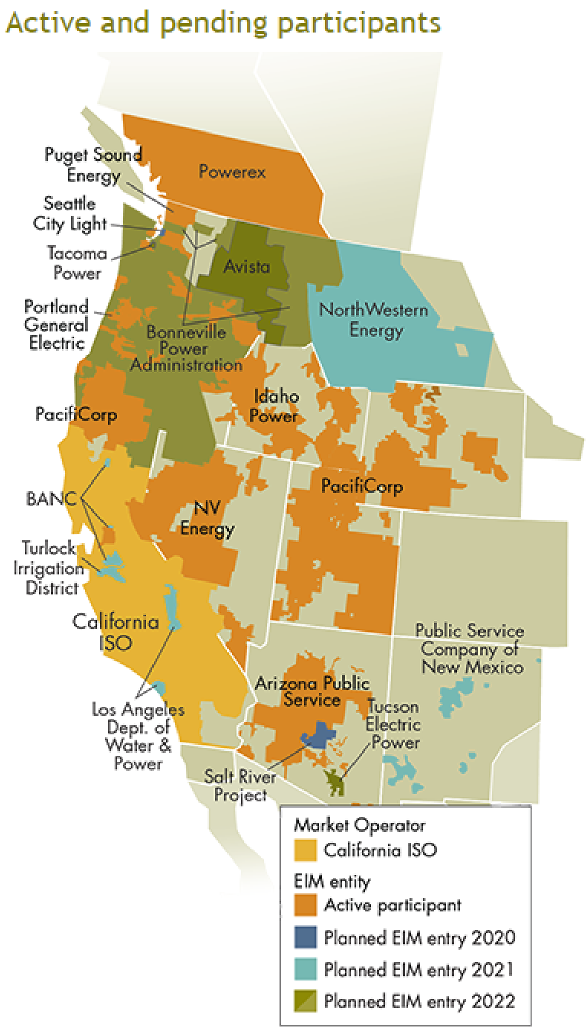
An efficient short-run electricity market determines a market clearing price based on conditions of supply and demand balanced in an economic dispatch. Everyone pays or is paid the same price. The same principles apply in an electric network. (Schweppe, Caramanis, Tabors, & Bohn, 1988)

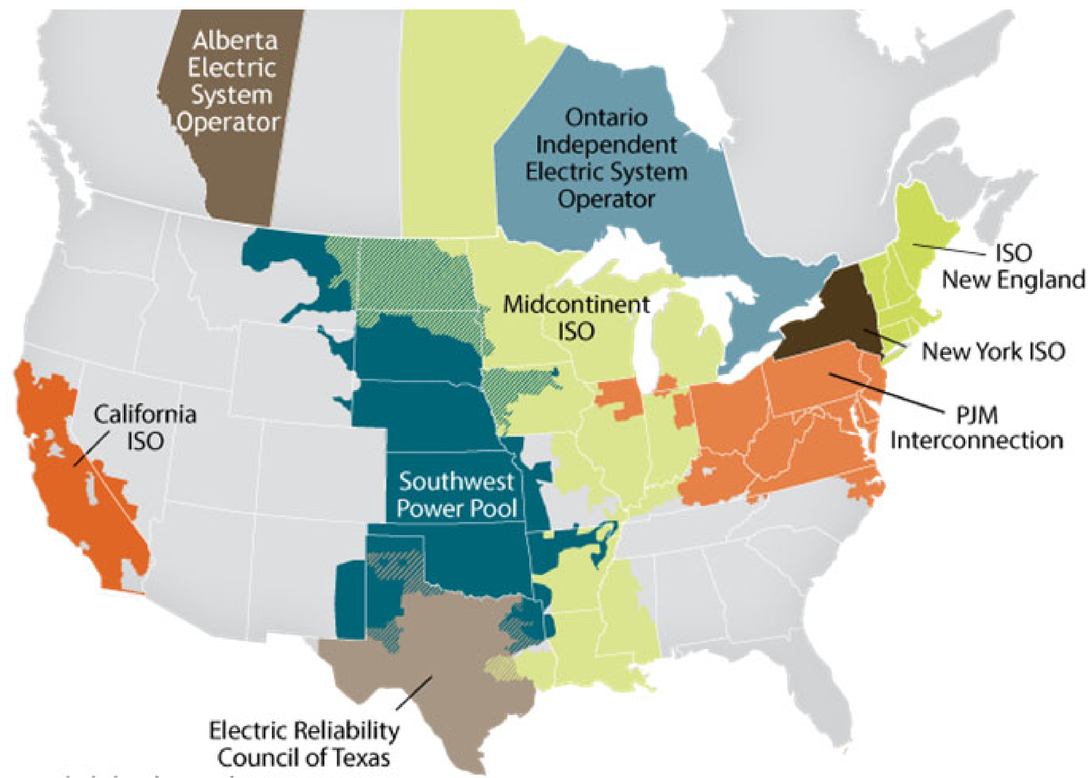




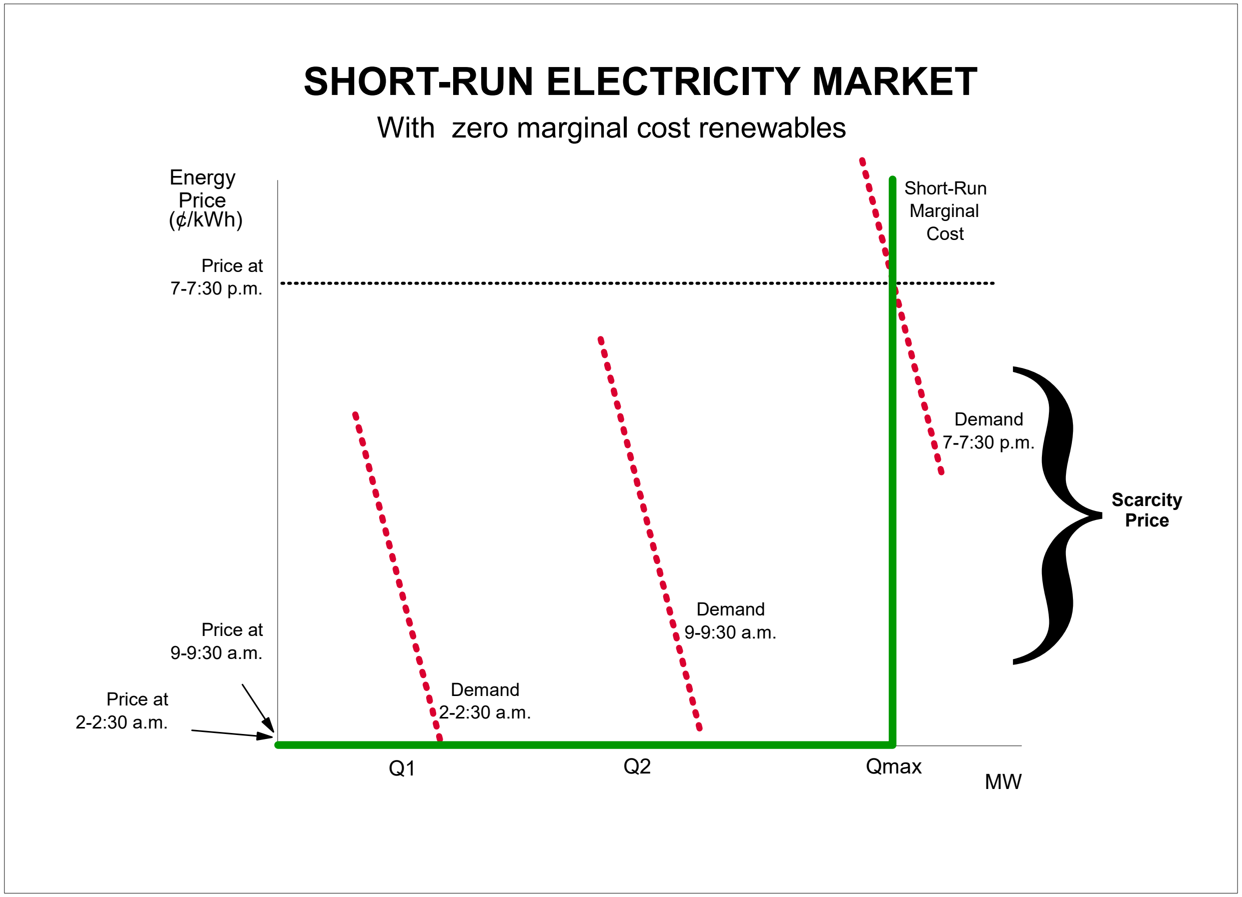
The basic model covers the existing Regional Transmission Organizations and is expanding through the Western Energy Imbalance Market. ([www.westerneim.com)](http://www.westerneim.com)

(IRC Council and CAISO maps)

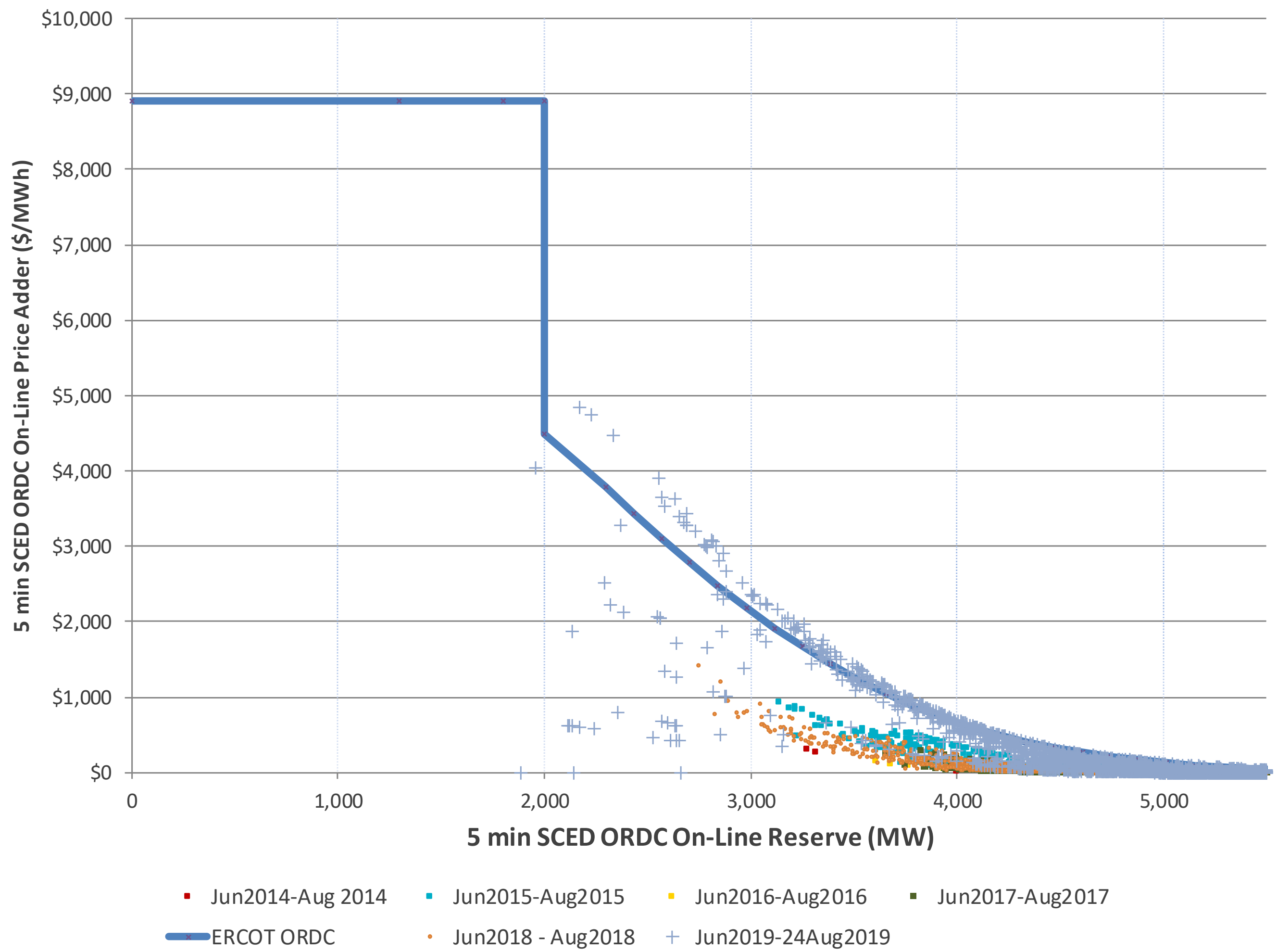




A limiting case illustrates a key issue. Electricity market design with even complete penetration by zero-variable cost renewables would follow the same analysis. But scarcity pricing would be critical to provide efficient incentives.



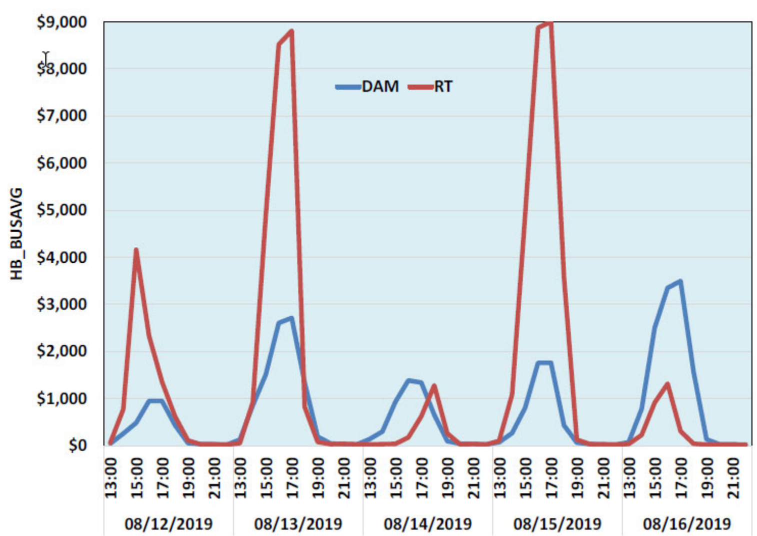
ERCOT launched implementation of the ORDC in in 2014. The summer peak is the most important period. The first five years of results show recent scarcity of reserves and higher reserve prices.



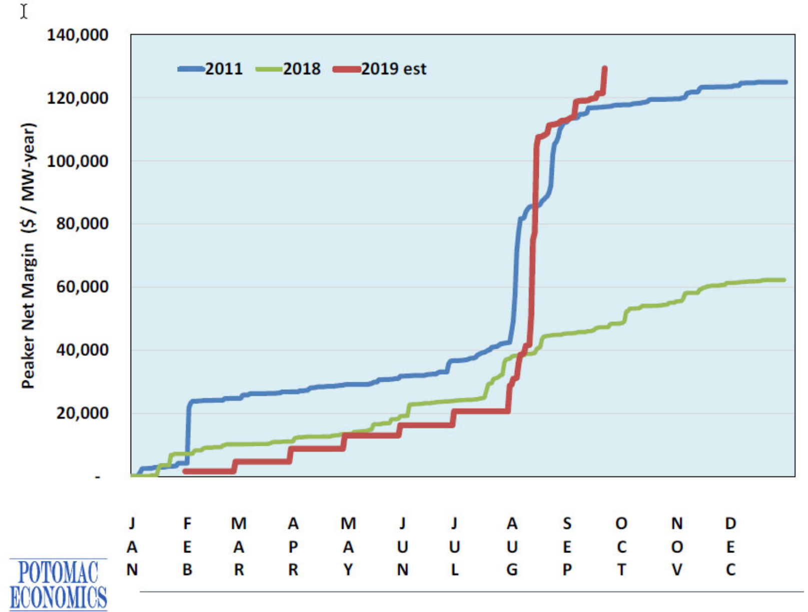
Source: Resmi Surendran, ERCOT, EUCI Presentation, Updated 8/31/2019. The ORDC is illustrative. See also (Hogan & Pope, 2017)

After introduction of the ORDC scarcity prices and the contribution to Peaker Net Margin were low for several years, but this changed in 2019.[1] **The PNM target level is $80,000-$95,000/MW-Yr.** (Potomac Economics, 2019, p. 112)

Day Ahead vs Real-Time Prices



Peaker net margin in 2019 is the highest ever



[1] Beth Garza, "Independent Market Monitor Report," Potomac Economics, ERCOT Board of Directors Meeting Presentation, October 8, 2019.

An ERCOT review of the Summer of 2019 underscored that scarcity pricing was consistent with performance of the system.[2]

Key Observations for Summer 2019

- Early summer was mild, and August was very hot (September was also above normal).
- There were many days with tight conditions, and an Energy Emergency Alert (EEA) Level 1 was declared twice.
  - Emergency Response Service (ERS) deployments prevented the need for EEA2.
- Peak demand day saw higher Intermittent Renewable Resource (IRR) production.
  - As a result, it was not one of the highest-priced days, and there was no EEA.
- Tightest conditions frequently occurred earlier than time of peak demand.
- Resource performance continues to outpace historical patterns.
- Overall, the market outcomes supported reliability needs.
- Even with significant pricing events, there were no mass transitions.

Notably, high prices occurred at the right time, and were not socialized through capacity market charges spread over all load.

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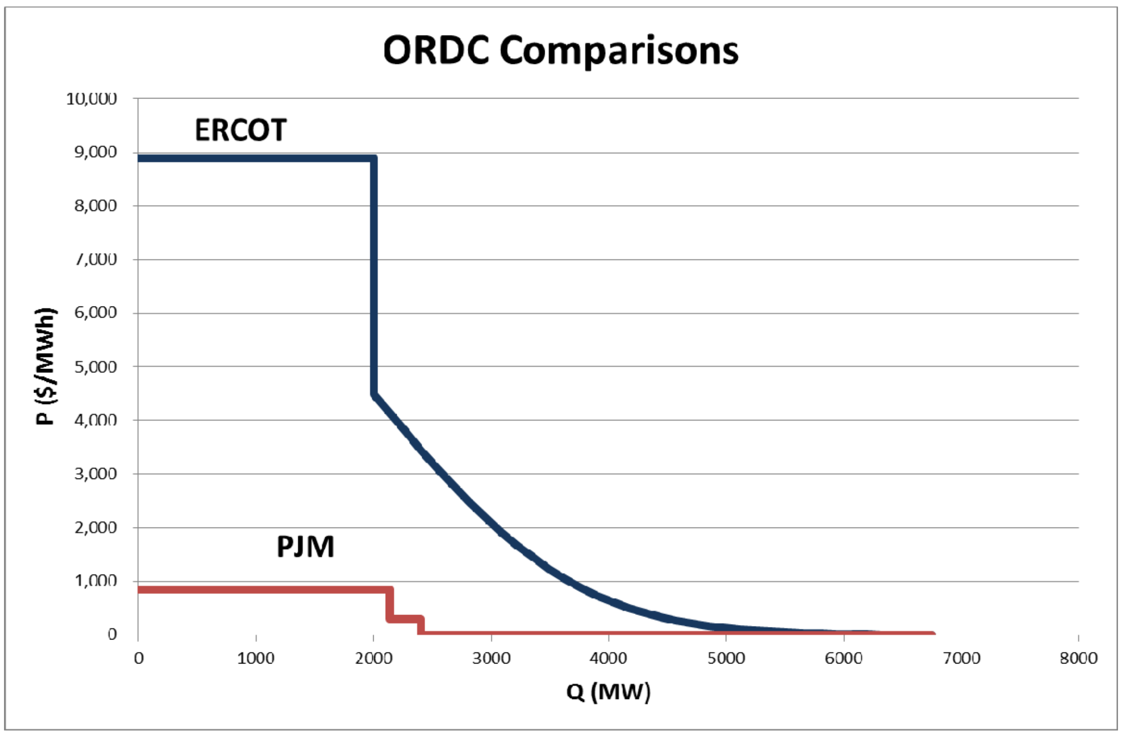
2 Dan Woodfin and Carrie Bivens, "Summer 2019 Operational Review", ERCOT Board of Directors Meeting Presentation, October 8, 2019.

ELECTRICITY MARKETELECTRICITY MARKET   Markets and Scarcity Pricing

Other RTOs have long used ORDCs, but without building the design on basic principles.

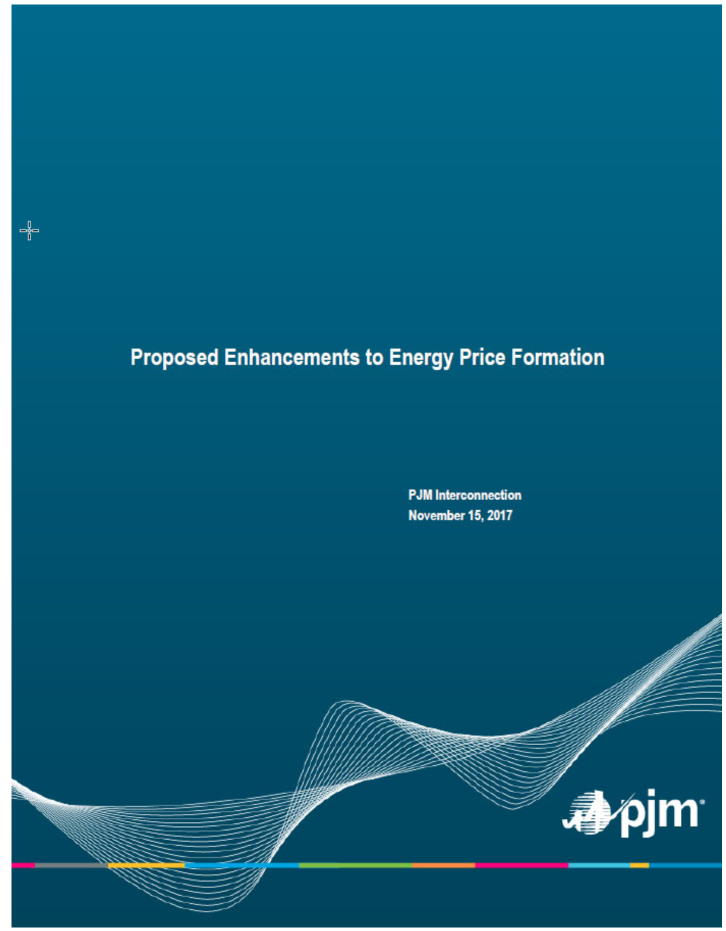
- **Limited to Declared Shortage Conditions.** "The ORDCs PJM currently utilizes were designed under the assumption that shortage pricing would only occur during emergency operating conditions and therefore the curves are a step function." (PJM and SPP, "Joint Comments Of PJM Interconnection, L.L.C And Southwest Power Pool, Inc. Addressing Shortage Pricing," FERC Docket No. RM15-24-000, November 30, 2015.)
- **Based on the Cost of Supply, not the Value of Demand.** "[T]he $300/MWh price is appropriate for reserves on the second step of the proposed ORDC based on an internal analysis of offer data for resources that are likely to be called on to provide reserves in the Operating Day." (PJM, Proposed Tariff Revisions of PJM Interconnection, L.L.C., Docket No. ER15-643-000, December 17, 2014)



PJM has proposed a series of reforms for energy price formation, motivated in part by the impact of increased penetration of intermittent renewable resources. (PJM Interconnection, 2017) (PJM Interconnection, 2019)

"…the continuing penetration of zero marginal cost resources, declining natural gas prices, greater generator efficiency and reduced generator margins resulting from low energy prices have resulted in a generation mix that is differentiated less by cost and more by physical operational attributes." (p. 1)

Figure 9. Demand Curve for Operating Reserves with Minimum Reserve Requirement



"Redefining PJM's ORDCs using this methodology would enhance PJM's shortage pricing mechanism by assigning a value to reserves consistent with their reliability benefit to the system. Additionally, this ORDC model allows reserves to be committed in excess of the nominal requirement when it lowers the LOLP but assures that the cost of such reserves will never exceed the reliability benefit." (p.23)

The extension of market design to distribution systems seems straightforward in principle. (Caramanis, Bohn, & Schweppe, 1982) **But in practice the challenges will be different.**

- **High Voltage Grids (Wholesale Markets)**
  - Small Losses
  - Simpler Voltage Control Challenges
  - Market Design Assumes Sufficient Reactive Power
  - Network Interactions with Thousands of Locations
  - Workable Approximations
    - DC Load Model, at least for local adjustments
    - Nomograms and Interface Constraints
    - Centralized Coordination
    - Long-history with Optimization Models
    - "Dispatch-Based Pricing" Models Accommodate Operator Interventions

- **Low Voltage Grids (Distribution Markets)**
  - Larger Average and Marginal Losses
  - Voltage Control a Central Problem
  - Largely Radial Systems with Millions of Devices
  - Moving from Passive Revelation to Active Participation
  - Less Operating Experience with Optimization Models

Some of the issues (an incomplete list):

- **Coordination**
  - Centralized
  - Decentralized
  - Hybrid Models (Gross Pool versus Net Pool Debate)
  - Operator Interventions

- **Efficiency (Optimization) and Pricing**
  - Dispatch Signals and Settlement Prices
  - Non-Convexities
    - Commitment Decisions
    - Switching Decisions
    - AC Models
  - Uplift (Side Payments and ELMP) (Gribik, Hogan, & Pope, 2007) (Chao, 2019)

- **Intertemporal Optimization and Efficiency**
  - Rolling Update of Dispatch with Look Ahead
  - With Convex Conditions and No Uncertainty: Dispatch Signals = Settlement Prices
  - Non-convexities from Commitment Decisions
    - Dispatch Signals Differ from Settlement Prices (ELMP)
    - Sunk Costs Matter
  - Convexity but with Uncertainty and Intertemporal Updates (Hua, Schiro, Zheng, Baldick, & Litvinov, 2019)
    - Ramping Constraints
    - Dispatch Signals Differ from Settlement Prices
    - Sunk Costs Matter
  - **Reality**: All of the Above, and More (Aggregators?)

The general problem of interest is the multi-period commitment and dispatch problem, in real-time and day-ahead. Assume a DC-Load model with a linear loss approximation. A stylized version of the unit commitment and dispatch problem for a fixed demand $\mathbf{y}$ is formulated in (Gribik et al., 2007) **as:**

Constants:	Variables:
$\mathbf{y}_t =$ vector of nodal loads in period t	$start_{it} = \begin{cases} 0 \text{ if unit i is not started in period t} \\ 1 \text{ if unit i is started in period t} \end{cases}$
$m_{it} =$ minimum output from unit i in period t if unit is on	$on_{it} = \begin{cases} 0 \text{ if unit i is off in period t} \\ 1 \text{ if unit i is on in period t} \end{cases}$
$M_{it} =$ maximum output from unit i in period t if unit is on	$g_{it} =$ output of unit i in period t
$ramp_{it} =$ maximum ramp from unit i between period t-1 and period t	$\mathbf{d}_t =$ vector of nodal demands in period t.
$StartCost_{it} =$ Cost to start unit i in period t	
$NoLoad_{it} =$ No load cost for unit i in period t if unit is on	
$\overline{F}_{kt}^{\max} =$ Maximum flow on transmission constraint k in period t.	

$$v\left(\{\mathbf{y}_t\}\right) = \inf_{\text{g,d,on,start}} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right)$$

subject to

$$m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} \qquad \forall i,t$$

$$-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} \qquad \forall i,t$$

$$start_{it} \le on_{it} \le start_{it} + on_{i,t-1} \qquad \forall i,t$$

$$start_{it} = 0 \text{ or } 1 \qquad \forall i,t$$

$$on_{it} = 0 \text{ or } 1 \qquad \forall i,t$$

$$\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 \qquad \forall t$$

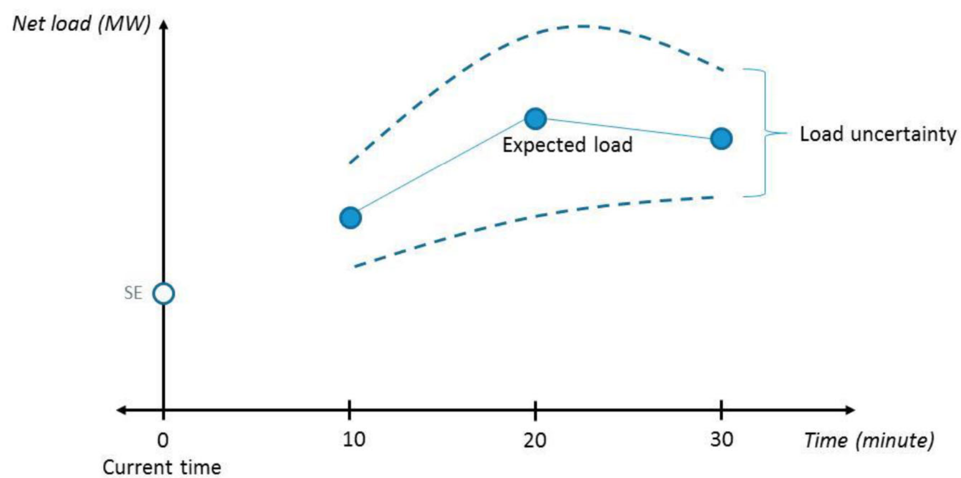
$$Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} \qquad \forall k,t$$

$$\mathbf{d}_t = \mathbf{y}_t \qquad \forall t.$$

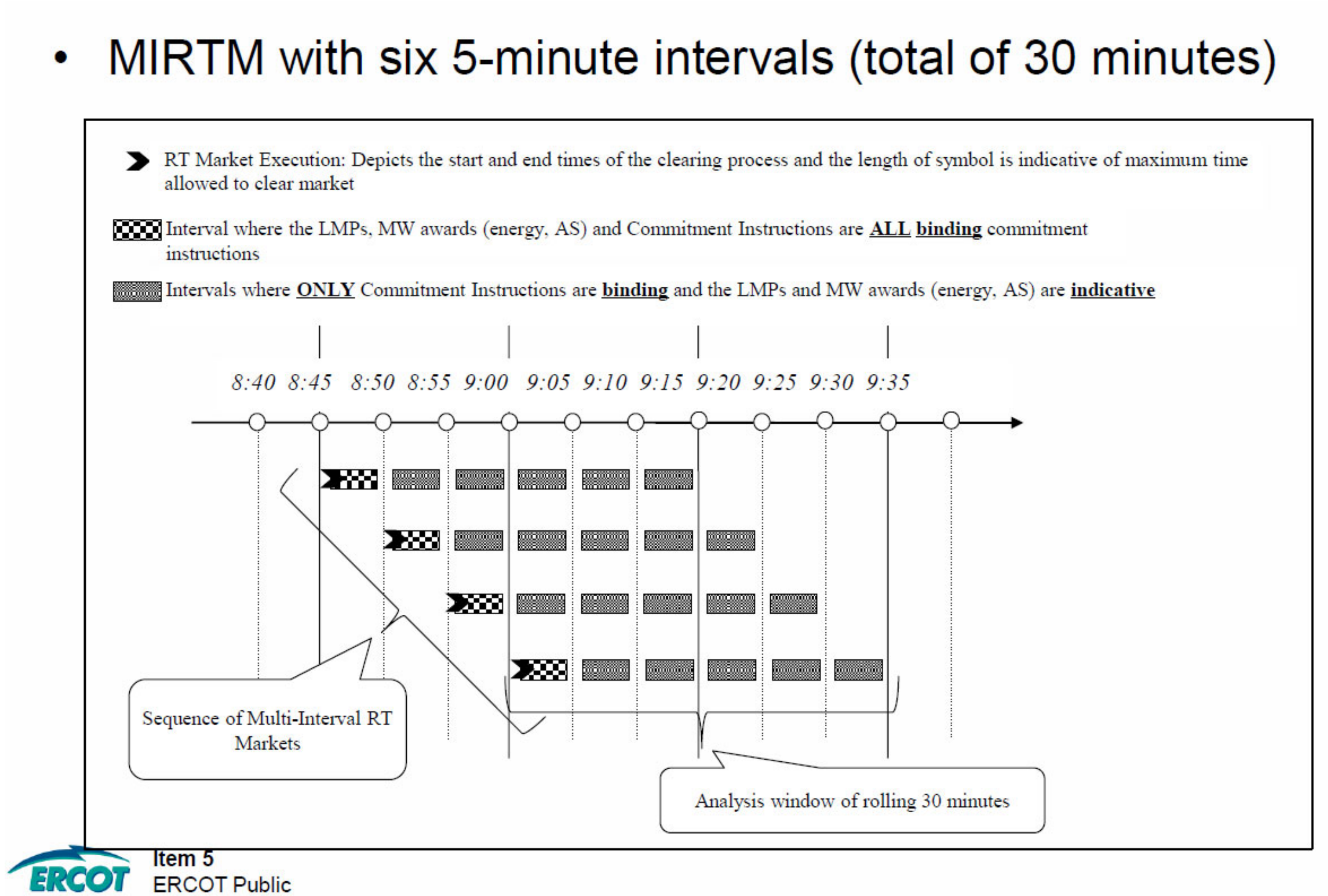
Multi-Period pricing must address both uncertainty and look-ahead dynamics. For a discussion of operating reserves see (Hogan & Pope, 2017)**. The focus here is on deterministic models with rolling updates, not on the related questions surrounding stochastic problems and operating reserves.** (Schiro, 2017)

***Multi-period pricing*. Uncertainty**

- Multi-period pricing is useful for expected load changes but may not help with load uncertainty
  - Load uncertainty for Time 10 is handled by AGC
  - Load uncertainty for Times 20-30 can be problematic (economic dispatch runs the system "as lean as possible")



A real-time dispatch model with multiple periods and look ahead is an approach found in most organized markets:



Joel Mickey, "Multi-Interval Real-Time Market Overview," Board of Directors Meeting, ERCOT Public, October 13, 2015.

A simple version of a real-time dispatch and pricing model with multiple periods might be as in:

$$
\begin{aligned}
&\underset{\mathrm{g,d}}{\mathrm{Min}} \ \sum_t \sum_i GenCost_{it}\left(g_{it}\right) \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i,t \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t
\end{aligned}
$$

This drops all the commitment decisions, treating them as fixed. If the generation costs are convex, then the usual LMPs will support the solution over the full horizon. Hence, the basic pricing model in a dynamic setting does not require any special treatment because of ramping or similar dynamic constraints. (Hua et al., 2019)

Rolling forward to the t* interval, with prior dispatch $g_{i,1}^{*} \cdots g_{i,t^*-1}^{*}$. A Look Ahead (LA) dispatch model has:

$$
\begin{aligned}
&\underset{\mathbf{g},\mathbf{d}}{\text{Min}} \sum_{t \ge t^*} \sum_i GenCost_{it}\left(g_{it}\right) \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i, t \ge t^* \\
&-ramp_{it} \le g_{it^*} - g_{i,t^*-1}^{*} \le ramp_{it} && \forall i, t^* \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i, t > t^* \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \ge t^* \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k, t \ge t^*
\end{aligned}
$$

This model will produce LMP values for the future periods. However, even in the fully convex case with perfect foresight, this model may produce prices that are not consistent over time and do not support the dispatch. (Hua et al., 2019)

Part of the difficulty arises from the generality of convex generation offer functions that can create dual degeneracy in the supply curves. With strictly convex cost functions, implying continuous offer curves rather than step functions, the "time inconsistency problem disappears." (Biggar & Hesamzadeh, 2020)

This clarifies the source of the difficulty. However, given the ubiquitous use of step-function generator offer curves, the price consistency problem remains.

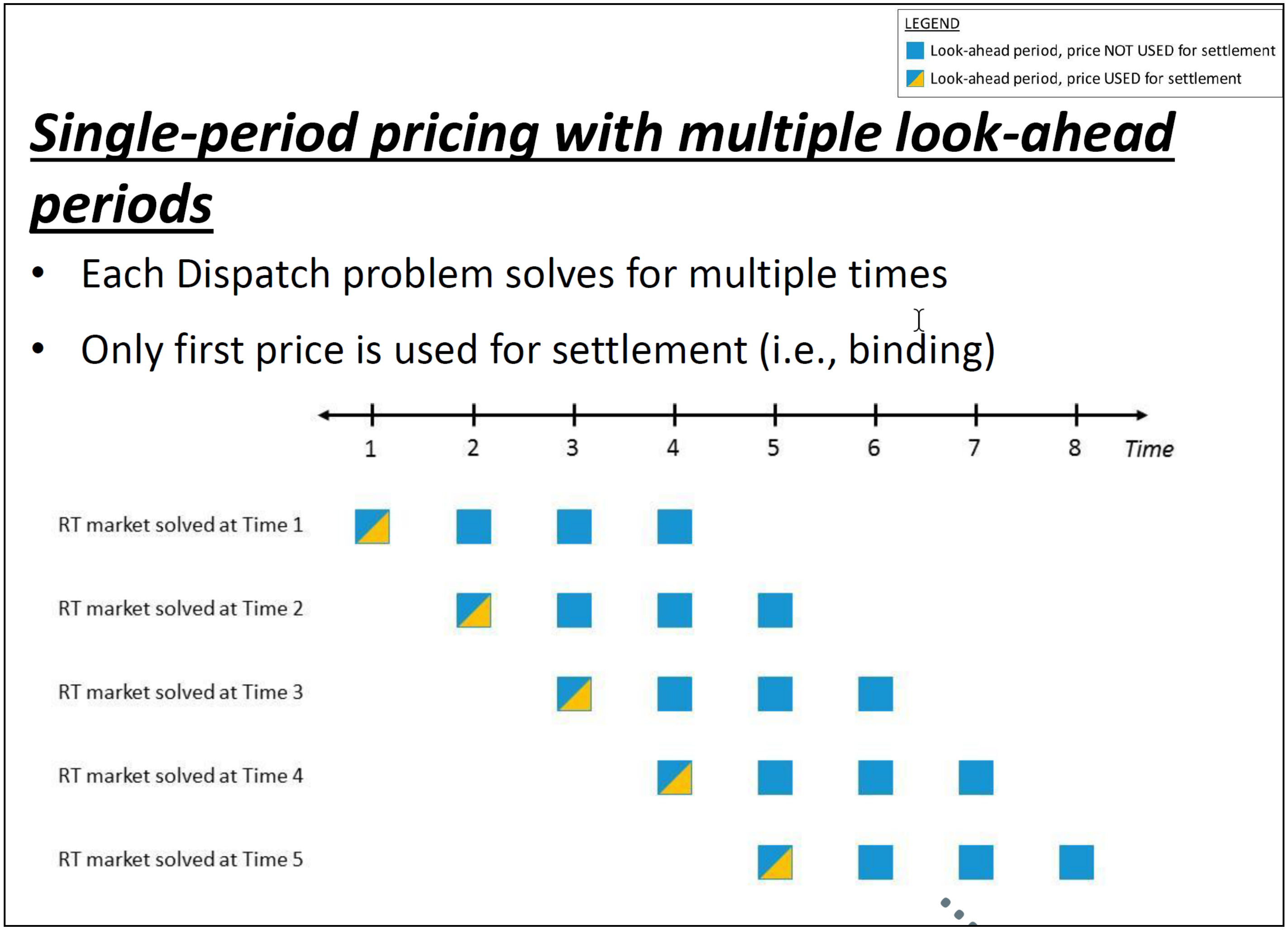
A modified version of a rolling pricing model addresses this price consistency problem in the convex case. The essence of a more general proposal of (Hua et al., 2019) **is to use the dual or shadow prices from the prior LA dispatch model to modify the objective function for the current LA pricing model.**

Accompanying the LA dispatch model, with up and down ramp shadow prices $\mu_{t^*-1}^{ur*}, \mu_{t^*-1}^{dr*}$ **from the prior pricing run, set the (separate but related) LA pricing as:**

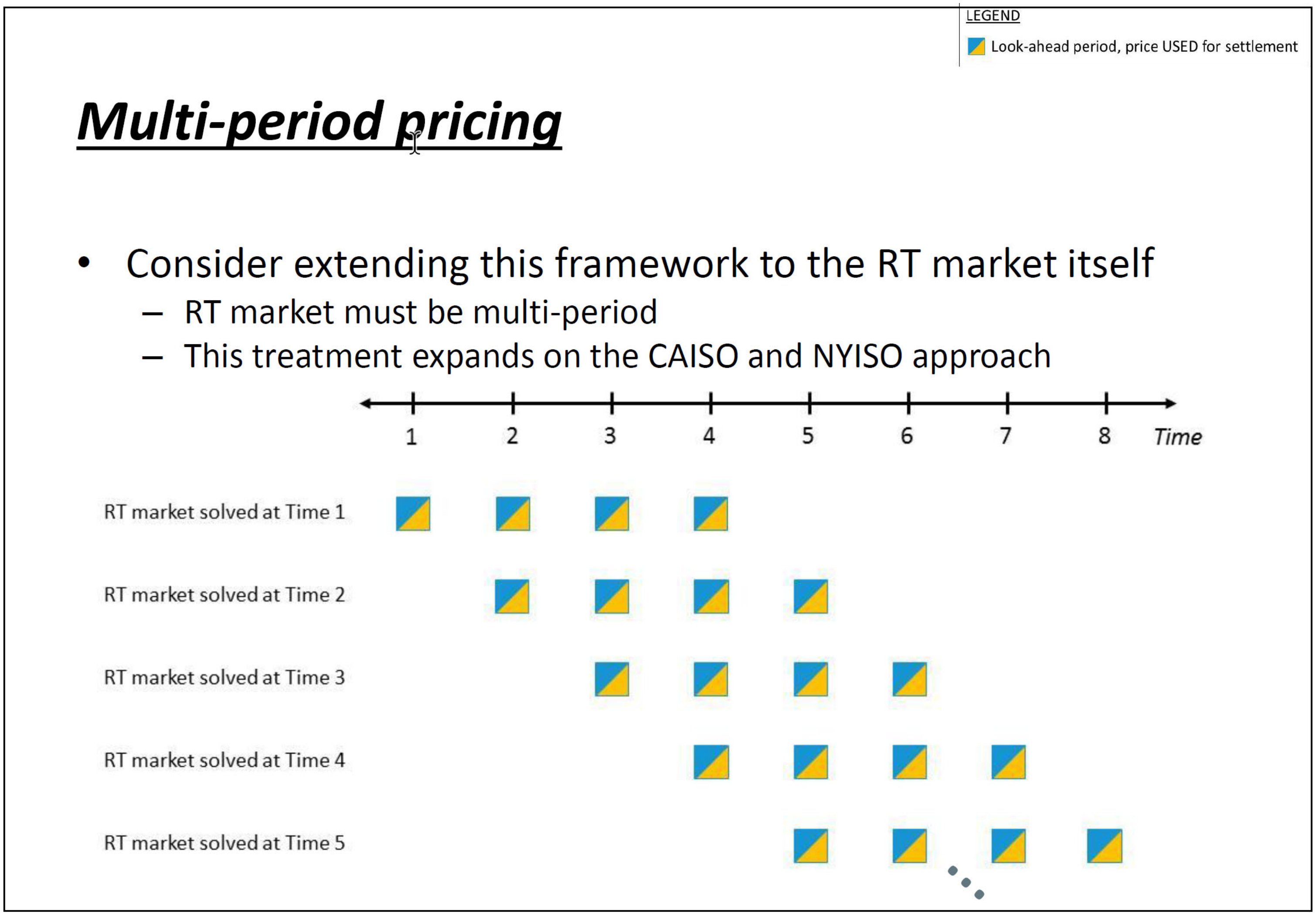
$$
\begin{aligned}
&\underset{g,d}{\text{Min}} \sum_{t \ge t^*} \sum_i GenCost_{it}\left(g_{it}\right) + \sum_i \mu_{i,t-1}^{ur*} g_{it^*} - \sum_i \mu_{i,t-1}^{dr*} g_{it^*} \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i, t \ge t^* \\
&-ramp_{it} \le g_{it^*} - g_{i,t^*-1}^{*} \le ramp_{it} && \forall i, t^* \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i, t > t^* \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \ge t^* \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k, t \ge t^*
\end{aligned}
$$

The shadow price from the prior LA pricing model reflects the opportunity cost over past decisions. This "prices out the past" and preserves price consistency in the perfect foresight case.

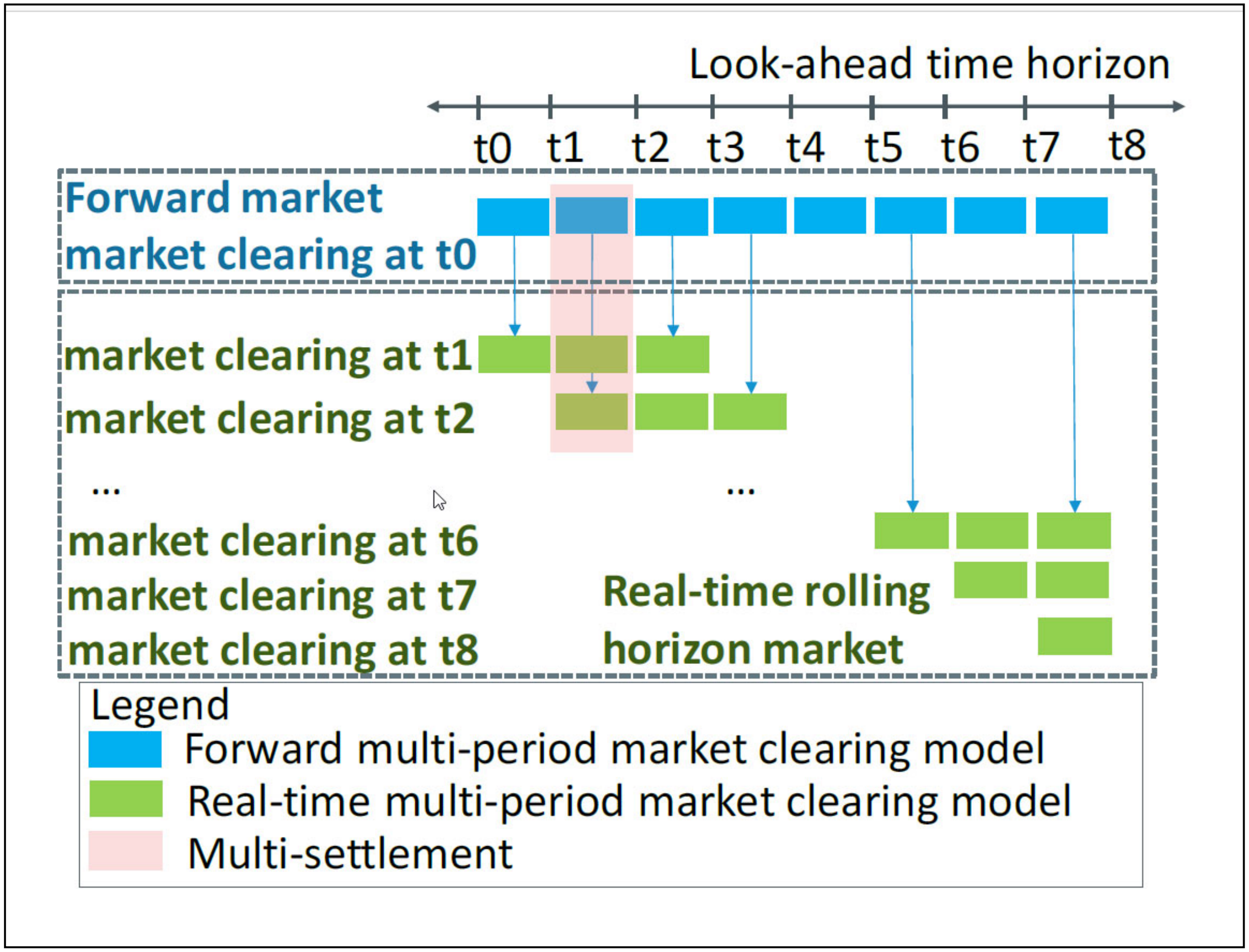
The rolling real-time LA dispatch and pricing will not enjoy perfect foresight. Changing conditions will change the dispatch and prices. This will create an uplift problem to compensate those for whom the realized prices do not support the solution. (Schiro, 2017)



With rolling prices, for the fully convex case, as an alternative to uplift payments, rolling forward markets with repeated settlements would preserve incentives and support the evolving dispatch. (Schiro, 2017)



With rolling dispatch and prices, the end of the horizon presents another problem. For example, if the real-time look-ahead is materially shorter than the day ahead forward market, the rolling model will not replicate the day-ahead forecast even under the same conditions and with perfect foresight. An innovative approach from (Zhao, Zheng, & Litvinov, 2019)**, the ZZL model, includes both constraining and pricing the past and future.**



The ZZL model modifies both the actual dispatch formulation and the associated pricing model. The dispatch model uses the solution from the (longer horizon) day-ahead market to constrain the starting and terminal conditions for the real-time LA dispatch.

Suppose the LA dispatch terminate at t. Then let $g_{i,t^{*}+1}^{DA}$ be the solution from the next period for the day-ahead market. The essence of the rolling ZZL dispatch is**

$$
\begin{aligned}
&\underset{\mathrm{g,d}}{\mathrm{Min}} \sum_{t \ge t^{*}} \sum_{i} GenCost_{it}\left(g_{it}\right) && \\
&\text{subject to} && \\
&m_{it} \le g_{it} \le M_{it} && \forall i, t^{*} \le t \le t^{**} \\
&-ramp_{it} \le g_{it^{*}} - g_{i,t^{*}-1}^{*} \le ramp_{it} && \forall i, t^{*} \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i, t^{*} \le t \le t^{**} \\
&-ramp_{it} \le g_{t^{**}+1}^{DA} - g_{i,t-1} \le ramp_{it} && \forall i, t^{**}+1 \\
&\mathbf{e}^{T}\left(\mathbf{g}_{t} - \mathbf{d}_{t}\right) - LossFn_{t}\left(\mathbf{d}_{t} - \mathbf{g}_{t}\right) = 0 && \forall t \ge t^{*} \\
&Flow_{kt}\left(\mathbf{g}_{t} - \mathbf{d}_{t}\right) \le \overline{F}_{kt}^{\max} && \forall k, t \ge t^{*}
\end{aligned}
$$

Hence, the rolling real-time dispatch model is constrained to match the real-time past and DA future after the LA horizon, which should be the best available estimate of the future conditions.

"If the realization deviates significantly from the [DA] forecast value, then the estimations become inaccurate, [DA] re-optimization should be performed." (Zhao et al., 2019)

The ZZL pricing model proposal modifies both the actual dispatch formulation and the associated pricing model. The pricing dispatch model uses the dual or shadow prices from the (longer horizon) day-ahead market price out the past and the future.

With ramp shadow prices from the day-ahead solution $\mu_{t^*-1}^{urDA}, \mu_{t^*-1}^{drDA}, \mu_{t^{**}+1}^{urDA}, \mu_{t^{**}+1}^{drDA}$, set the (separate but related) LA pricing as:[3]

$$\begin{aligned}
&\underset{\mathrm{g,d}}{\mathrm{Min}} \sum_{t \ge t^*} \sum_i GenCost_{it}\left(g_{it}\right) + \sum_i \mu_{i,t-1}^{urDA} g_{it^*} - \sum_i \mu_{i,t-1}^{drDA} g_{it^*} - \sum_i \mu_{i,t^{**}+1}^{urDA} g_{it^{**}} + \sum_i \mu_{i,t^{**}+1}^{drDA} g_{it^{**}} \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i, t^* \le t \le t^{**} \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i, t^* \le t \le t^{**} \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \ge t^* \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k, t \ge t^*
\end{aligned}$$

Hence, this preserves the computational advantage of a shorter dispatch period by using the day-ahead solution to connect the present to the past and future. With perfect foresight, this achieves price consistency.

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[3] This differs from the LA model of (Hua et al., 2019) in the treatment of the priced-out constraints. For the perfect foresight analysis, these constraints are redundant.

Energy dispatch is continuous but unit commitment requires discrete decisions. Bid-based, security constrained, combined unit commitment and economic dispatch presents a challenge in defining market-clearing prices.

- **Continuous convex economic dispatch**
  - Electric power systems are almost convex, and use convex approximations for dispatch.[4]
  - System marginal costs provide locational, market-clearing, linear prices.
  - Linear prices support the economic dispatch.

- **Discrete, economic, unit commitment and dispatch**
  - Start up and minimum load restrictions enter the model.
  - System marginal costs not always well-defined.
  - There may be no linear prices that support the commitment and dispatch solution.

The various LA dispatch and pricing results depend on the convexity assumption. They do not apply, at least directly, to the unit commitment problem.

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[4] J. Lavaei and S. H. Low, "Zero duality gap in optimal power flow problem," *IEEE Transactions on Power Systems*. (Lavaei & Low, 2012).

Selecting the appropriate approximation model for defining energy and uplift prices involves practical tradeoffs. All involve "uplift" payments to guarantee payments for bid-based cost to participating bidders (generators and loads), to support the economic commitment and dispatch.

Uplift with Given Energy Prices=Optimal Profit – Actual Profit

- **Restricted Model (r)**
    - Fix the unit commitment at the optimal solution.
    - Determine energy prices from the convex economic dispatch.
- **Dispatchable Model (d)**
    - Relax the integer constraints and treat commitment decisions as continuous.
    - Determine energy prices from the relaxed, continuous, convex model.
- **Extended Locational Marginal Pricing (ELMP) Model (h)**
    - Equivalent formulations
        - Select the energy prices from the convex hull of the cost function.
        - Select the energy prices from the Lagrangean relaxation (i.e., usual dual problem for pricing the joint constraints).
    - Resulting energy prices minimize the total uplift.

A formulation that separates out the discrete variables (u) serves to distinguish the modeling approaches.[5] Here assume that the problem is convex but for the integer restriction on the commitment variables. The optimal commitment is u^O.

- **Unit commitment and dispatch**

$$\begin{aligned} v(y) = \underset{(x,u)\in X}{Min} \quad & f(x,u) \\ s.t. \quad & g(x) = y \\ & u = 0,1. \end{aligned}$$

- **Restricted Model (r)**

$$\begin{aligned} v^r(y) = \underset{(x,u)\in X}{Min} \quad & f(x,u) \\ s.t. \quad & g(x) = y \\ & u = u^O. \end{aligned}$$

- **Dispatchable Model (d)**

$$\begin{aligned} v^d(y) = \underset{(x,u)\in X}{Min} \quad & f(x,u) \\ s.t. \quad & g(x) = y \\ & 0 \le u \le 1. \end{aligned}$$

[5] For a further discussion of dual price functions, see (Bjørndal & Jörnsten, 2008, pp. 768–789).

Economic commitment and dispatch is a special case of a general optimization problem.

$$\begin{aligned} v(y) = \underset{x \in X}{Min} \quad & f(x) \\ s.t. \quad & g(x) = y. \end{aligned}$$

From the perspective of a price-taking bidder, uplift is the difference between actual and optimal profits.

Actual profits: $\pi(p,y) = py - v(y)$

Optimal Profits: $\pi^*(p) = \underset{z}{Max}\{pz - v(z)\}$

$$Uplift(p,y) = \pi^*(p) - \pi(p,y)$$

Classical Lagrangean relaxation and pricing creates a familiar dual problem and a characterization of the convex hull $v^h(y)$.

$$L(y,x,p) = f(x) + p(y - g(x))$$

$$\hat{L}(y,p) = \underset{x \in X}{Inf}\{f(x) + p(y - g(x))\}$$

$$v^h(y) = \underset{p}{Sup}\,\hat{L}(y,p) = \underset{p}{Sup}\left\{\underset{x \in X}{Inf}\{f(x) + p(y - g(x))\}\right\}$$

The optimal dual solution minimizes the uplift, and the "duality gap" is equal to the minimum uplift.

$$v(y) - v^h(y) = \underset{p}{Inf}\,Uplift(p,y) = Uplift(p^h, y).$$

In general, the solutions can be such that:

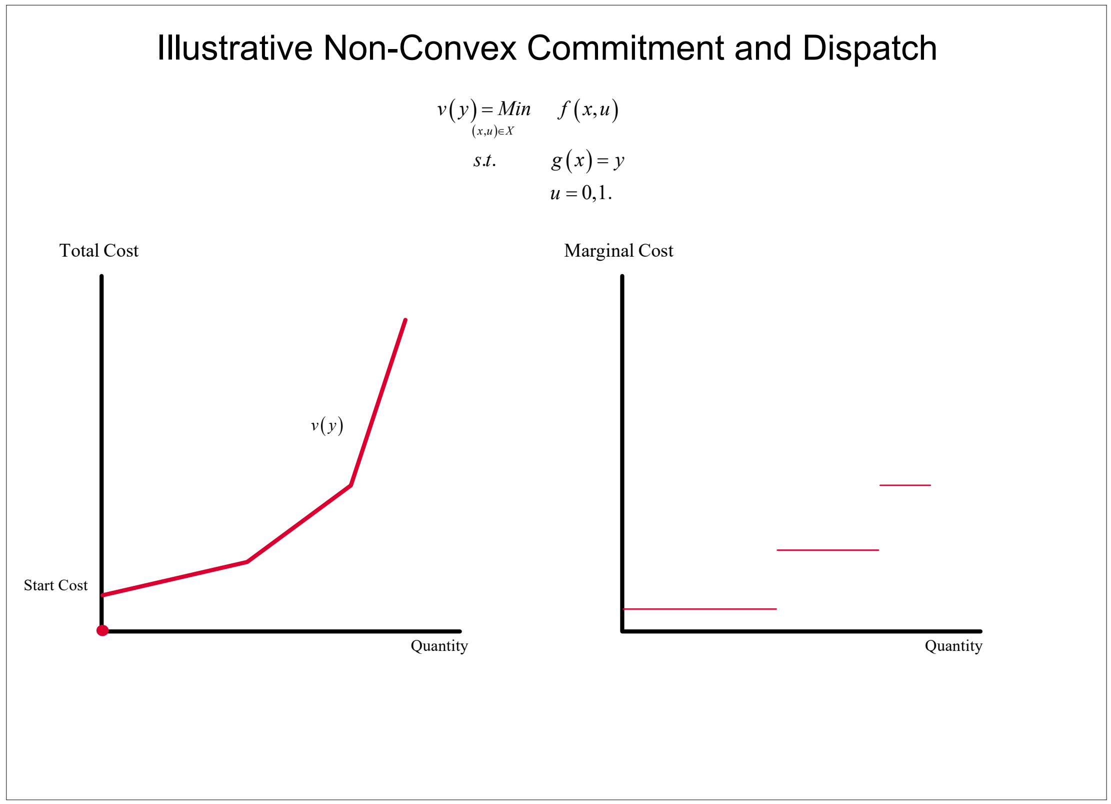
$$v^d(y) < v^h(y) < v^r(y) = v(y),$$

$$p^d \neq p^h \neq p^r.$$

A simple example illustrates connections among the different pricing model formulations.

$$v(y) = \underset{(x,u)}{Min} \quad \sum_{i=1}^{n} c_i x_i + ku$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^{n} x_i = y \\ & 0 \le x_i \le X_i \\ & \sum_{i=1}^{n} x_i \le uK \\ & u = 0,1. \end{aligned}$$



A dispatchable model applies an integer relaxation.

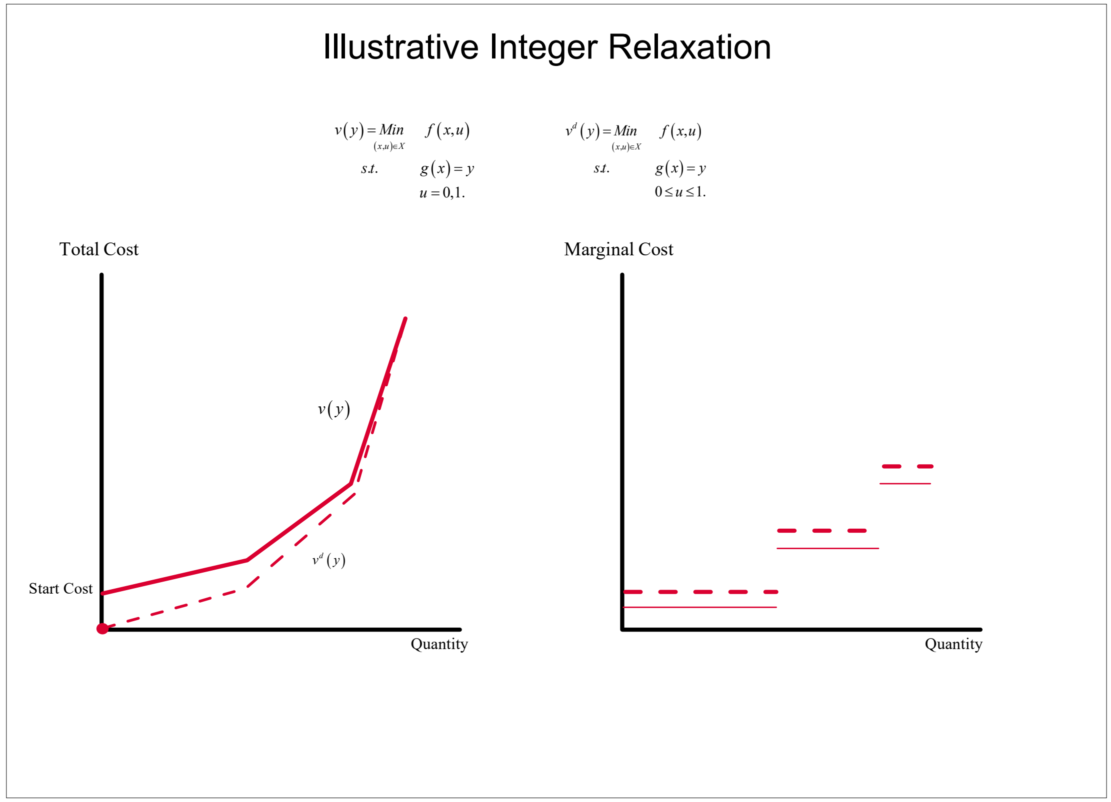
$$v^d(y) = \underset{(x,u)}{Min} \quad \sum_{i=1}^{n} c_i x_i + ku$$

$$\sum_{i=1}^{n} x_i = y$$

$$s.t. \qquad 0 \le x_i \le X_i$$

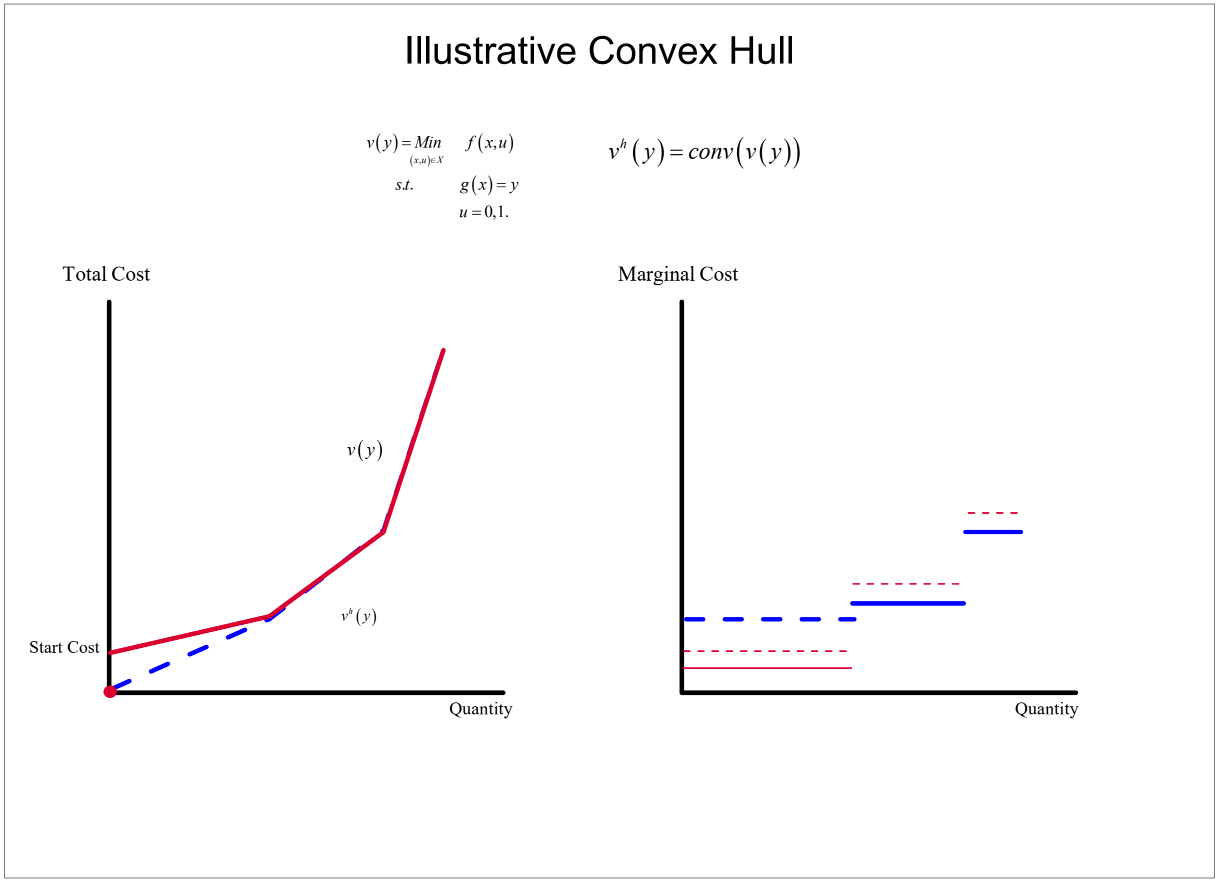
$$\sum_{i=1}^{n} x_i \le uK$$

$$0 \le u \le 1.$$



Illustrative Integer Relaxation

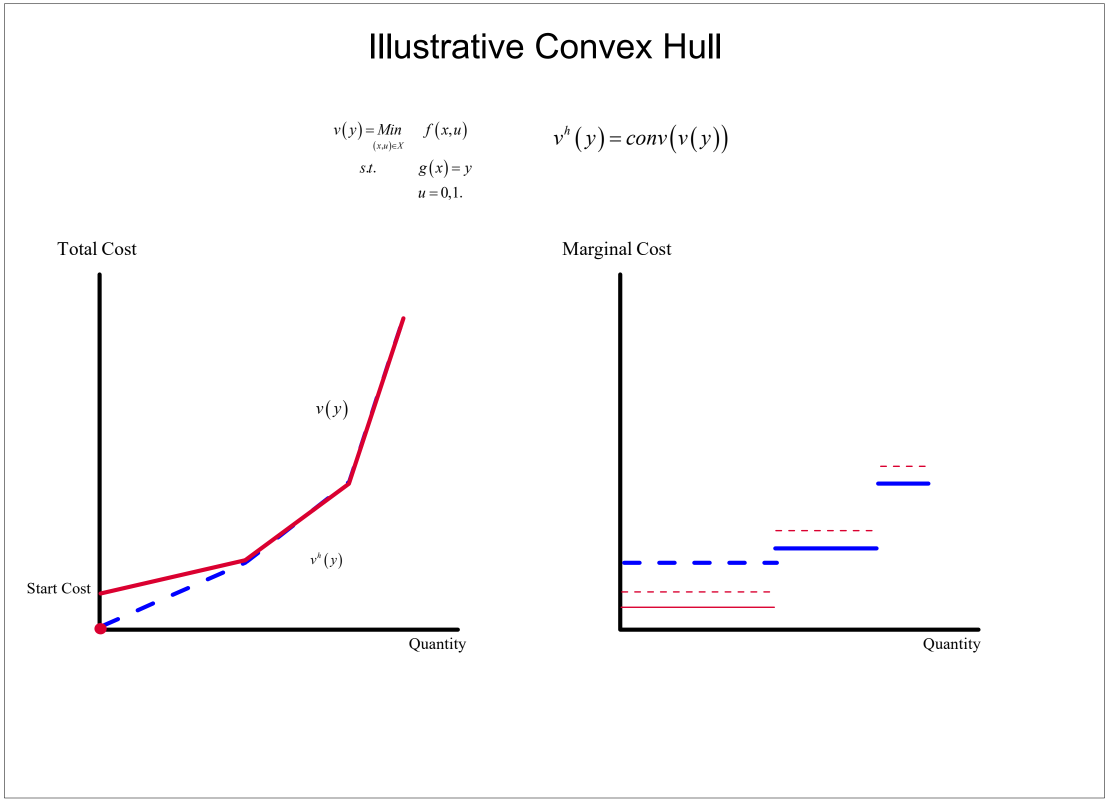
The convex hull or minimum uplift model provides the best convex approximation to the total cost function.



An alternative description of the model with the same solutions provides a different integer relaxation or dispatchable model. (Chao, 2019) **In this case the dispatchable model produces the convex hull. In general, there are many ways to change the formulation of the original model without affecting the dispatch solutions but producing different price approximation.**

$$\begin{aligned} v(y) = \underset{(x,u)}{Min} \quad & \sum_{i=1}^{n} c_i x_i + ku \\ s.t. \quad & \sum_{i=1}^{n} x_i = y \\ & 0 \le x_i \le uX_i \\ & u = 0,1. \end{aligned}$$

$$\begin{aligned} v^d(y) = \underset{(x,u)}{Min} \quad & \sum_{i=1}^{n} c_i x_i + ku \\ s.t. \quad & \sum_{i=1}^{n} x_i = y \\ & 0 \le x_i \le uX_i \\ & 0 \le u \le 1. \end{aligned}$$



The ELMP model applied to the stylized unit commitment problem employs the dual prices from a particular Lagrangean relaxation.

$$
v^h\left(\{\mathbf{y}_t\}\right) \equiv
\sup_p \left\{ \begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{ \begin{array}{ll}
\inf_{\text{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t
\end{array} \right\}
\end{array} \right\}
$$

The ELMP price is determined for all periods as the pricing solution to this problem.

The ELMP is a solution $\mathbf{p}$ for the dual or convex hull problem with the loss and transmission limits included as constraints. A "market-clearing" solution is a solution to the inner problem for given prices $\mathbf{p}$.

$$v^h\left(\{\mathbf{y}_t\}\right) \equiv$$

$$\sup_p \left\{ \begin{array}{l} +\sum_t \mathbf{p}_t^T \mathbf{y}_t \\ + \left\{ \begin{array}{ll} \inf\limits_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\ \text{subject to} & \\ m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\ -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\ start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\ start_{it} = 0 \text{ or } 1 & \forall i,t \\ on_{it} = 0 \text{ or } 1 & \forall i,t \\ \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\ Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \end{array} \right\} \end{array} \right\}$$

A real-time pricing model involves multiple periods and look ahead. Applying an ELMP framework involves choices about what is fixed and what is variable. Natural principles include:

- **Real-time quantity anchor.** Conditioning to reflect evolving economic dispatch and commitment. For example, the pricing dispatch would account for ramping limits that constrain the degree that the pricing dispatch could deviate from the actual dispatch to ensure that the price market-clearing dispatch would always be feasible conditioned on the actual dispatch.
- **Real-time price consistency.** Given perfect foresight, where actual conditions equal the forecast conditions, the methodology produces the same set of prices.

For actual commitment and dispatch, past decisions are sunk and real-time quantity anchors apply.

The pricing model could employ more flexibility. The Restricted model can meet both conditions by always ignoring fixed costs. But ELMP in general incorporates intertemporal constraints and reflects fixed costs of units not committed.

A proposal for real-time price consistency in ELMP is to fix past decisions in the inner "market clearing" solution, as well as fixing the prices. Hence, the conditional market-clearing pricing model at time τ would take the determined prices $p_1^*, p_2^*, \cdots p_{\tau-1}^*$ and market clearing dispatch $z^\tau = \{\mathrm{g}_t, \mathrm{d}_t, \mathrm{on}_t, \mathrm{start}_t\}_1^\tau$ for the prior periods as fixed and solve as the pricing model:

$$
v^\tau\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{ \begin{array}{ll}
\inf\limits_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\
z^{\tau-1} = z^{*\tau-1} &
\end{array} \right\} \\
p_t = p_t^*, t \le \tau - 1
\end{array} \right\}
$$

However, the hoped for price consistency depends on separability across periods. The general problem is not separable, and fixing $z^\tau = \{\mathrm{g}_t, \mathrm{d}_t, \mathrm{on}_t, \mathrm{start}_t\}_1^\tau$ does not ensure price consistency.

A sufficient condition for real-time price consistency in ELMP is that all commitment and dispatch variables that are in the economic dispatch or are assigned an uplift payment from the market-clearing solution be included in the pricing model. This allows for slowly pruning those offers that were not committed in either the economic commitment or the market-clearing commitment and are subsequently excluded from retroactive starts ($Excluded_\tau$). Hence, the conditional dual pricing model at time τ could take as determined prices the prior periods $p_1^*, p_2^*, \cdots p_{\tau-1}^*$:

$$
v^\tau\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l} +\sum_t \mathbf{p}_t^T \mathbf{y}_t \\ + \left\{ \begin{array}{ll} \inf\limits_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\ \text{subject to} & \\ m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\ -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\ start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\ start_{it} = 0 \text{ or } 1 & \forall i,t \\ on_{it} = 0 \text{ or } 1 & \forall i,t \\ \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\ Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\ start_{it} = 0,\, i \in Excluded_\tau & \end{array} \right\} \\ p_t = p_t^*,\, t \le \tau - 1 \end{array} \right\}
$$

The minimum uplift or convex hull prices present a computational challenge.

- **Calculation of the Lagrangean Dual Solution.** Works in theory but is slow to converge and inevitable leads to numerical approximation.
- **New Methods.** Focus on construction of the convex hull.
  - **Primal Convex Hull.** Constructing the convex hulls of the components and then optimizing the resulting problem. Provides exact solutions for a useful class of problems, and reports good approximations in other case. (Hua & Baldick, 2017)
  - **Expanded Unit Commitment.** Adding constraints and variables, constructing a master problem that characterizes the convex hull via Benders Decomposition. Provides exact solution and reports good computational performance. (Ben Knueven, Ostrowski, & Wang, 2018) (Bernard Knueven, Ostrowski, Castillo, & Watson, 2019) (Yu, Guan, & Chen, 2019a) (Yu, Guan, & Chen, 2019b)
- **Approximate ELMP.** Focus on the calculation of the convex hull prices.

Different formulations of the unit commitment problem yield the same convex hull but have different integer relaxations.

Suppose that $GenCost_{it}(g_{it})$ **is convex and homogeneous of degree one.**[6] **This is a very weak condition. Suppose we have a commitment variable and a piecewise representation of generation cost over intervals** $\left[0, X_j\right], \sum_j X_j = M$ **.**

A: $\widetilde{GenCost}(g,z) = \underset{x}{Min}\left\{\sum_j c_j x_j \,\middle|\, 0 \le x_j \le X_j, \sum_j x_j = g, 0 \le g \le zM\right\}$ **is not homogeneous.**

B: $GenCost(g,z) = \underset{x}{Min}\left\{\sum_j c_j x_j \,\middle|\, 0 \le x_j \le zX_j, \sum_j x_j = g, 0 \le g \le M\right\}$ **is homogeneous.**

These functions agree when $z = 0,1$**, so they have the same convex hull.**

- Without ramping constraints, unimodular constraints on commitment variables assure that integer relaxation with model **B** provides convex hull prices. (Chao, 2019)
- With ramping constraints, an expanded unit commitment characterization can employ a variant of model **B** and provide convex hull prices. (Yu et al., 2019b)

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6 Homogeneous of degree one: $GenCost_{it}(\alpha g_{it}) = \alpha GenCost_{it}(g_{it}), \alpha \ge 0.$

The integer relaxation or dispatchable model allows for continuous commitment variables. This is a standard convex optimization problem. Assume the formulation has a convex and homogeneous objective function. (MISO, 2019)

$$
\begin{aligned}
&\underset{\text{g,d,on,start}}{Min} \left(\sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right)\right) \\
&\text{subject to} \\
&m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
&start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
&0 \le start_{it} \le 1 && \forall i,t \\
&0 \le on_{it} \le 1 && \forall i,t \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t
\end{aligned}
$$

The ramping constraints are an important complication.

- **With no ramping constraints.** The LMP prices from the relaxed problem are the same as the convex hull ELMP prices. (Chao, 2019)
- **With ramping constraints.** The LMP prices are good approximate convex hull prices. There are additional valid inequalities that can be included to strengthen the approximation. (Hua et al., 2019)

The relaxed problem is easy to solve, uses the exact formulation of the corresponding dispatch model but treats the commitment variables as continuous.

An adaptation of the sequential model for the rolling LA pricing with the relaxed approximation of the pricing problem presents a relative simply tool. First we fix the prices for prior periods and price out the constraints to include them as part of the objective function. Then we utilize the relaxed model to find the approximate prices:

$$
\begin{aligned}
& v^{r\tau}\left(\{\mathbf{y}_t\}\right) \equiv \\
& \inf_{\mathrm{g,d,on,start}} \begin{pmatrix} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_{t \le \tau - 1} \mathbf{p}_t^{*T} \left(\mathbf{d}_t - \mathbf{y}_t\right) \end{pmatrix} \\
& \text{subject to} \\
& m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
& -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
& start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
& 0 \le start_{it} \le 1 && \forall i,t \\
& 0 \le on_{it} \le 1 && \forall i,t \\
& \mathbf{e}^T \left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
& Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t \\
& \mathbf{d}_t = \mathbf{y}_t && \forall t \ge \tau.
\end{aligned}
$$

This is a much easier problem to solve than the ELMP. Absent ramping constraints, it yields the convex hull prices. With ramping constraints, it should provide a good approximation.

Since the integer relaxation is a convex problem, the various LA pricing and settlement approximations proposed for LMP in the convex real-time model without commitment decisions could be implemented with an appropriate integer relaxation pricing model to account for commitment decisions.

The discussion of pricing enhancements and these alternative models identifies common objections.

- **Not Using Real Marginal Costs.** The shorthand that short-run efficient prices equal marginal costs depends on an underlying assumption of convexity. In the non-convex case, there may be no efficient linear prices that clear the market.

- **Dispatch and Pricing are Inconsistent.** Only under the assumption of convexity will the marginal cost prices from the dispatch also support the dispatch solution and clear the market. The market design seeks an efficient commitment and dispatch solution. The pricing and uplift payment model is related but not identical under conditions where there is a duality gap.

- **The Implied Dispatch in the Pricing Model Creates Congestion.** Absent convexity, and with a duality gap, the dispatch in the pricing model can violate existing or encounter new constraints. This is a feature, but not a bug. The prices are relevant, not the dispatch.

- **The Prices and Uplift Will Create Perverse Incentives.** The purpose of the uplift payments is precisely to remove the most perverse incentives not to follow the commitment and dispatch. The mechanism for pricing and uplift is “almost” incentive compatible in a manner like other “first -price” auction frameworks.

- **Self-Scheduled Units Will Manipulate Prices.** The pricing model only employs bids and offers included in the commitment and dispatch model. Competitive self-scheduled units have an incentive to participate in the dispatch.

- **Prices Will Go Up.** This is an empirical question. If the efficient energy prices under ELMP are higher, that is a solution and not a problem.

Alternative pricing models have different features and raise additional questions.

- **Computational Requirements.** Dispatchable model is the easiest case, ELMP model the hardest. But not likely to be a significant issue. Approximate solutions (e.g., NYISO model) may be workable.
- **Network Application.** All models compatible with network pricing and reduce to standard LMP in the convex case.
- **Operating Reserve Demand.** All models compatible with existing and proposed operating reserve demand curves.
- **Solution Independence.** Restricted model sensitive to actual commitment. Relaxed and ELMP models (largely) independent of actual commitment and dispatch.
- **Financial Transmission Rights.** Transmission revenue collected under the market clearing solution would be sufficient to meet the obligations under the FTRs. However, this may not be true for the revenues under the economic dispatch, which is not a market clearing solution at the ELMP prices, even though the FTRs are simultaneously feasible. The FTR uplift amount included in the decomposition of the total uplift that is minimized by the ELMP prices. This uplift payment would be enough to ensure revenue adequacy of FTRs under ELMP pricing.(Cadwalader, Gribik, Hogan, & Pope, 2010)
- **Day-ahead and real-time interaction.** With uncertainty in real-time and virtual bids, expected real-time price is important, and may be similar under all pricing models.

APPENDIX

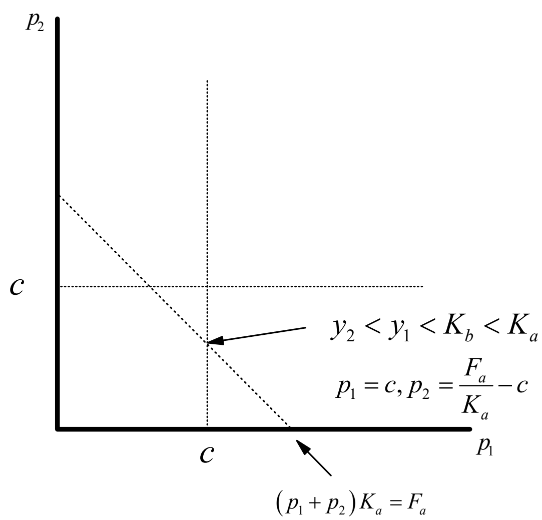
A two period example illustrates the solution and properties of pricing model. The simplified structure with only fixed costs for one plant and variable costs for the other allows us to determine the solution from the graph of critical regions in the dual space of the prices.

Two-Period Price Illustration

Plant a has only fixed costs, b has only variable costs.

$$v(y) = \underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2}}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2})$$

$$\begin{aligned}
&s.t.\\
&0 \le x_{a1} \le K_a u_{a1}\\
&0 \le x_{a2} \le K_a \max(u_{a1},u_{a2})\\
&0 \le x_{b1} \le K_b\\
&0 \le x_{b2} \le K_b\\
&u_{a1} = 0,1\\
&u_{a2} = 0,1\\
&x_{a1} + x_{b1} = y_1\\
&x_{a2} + x_{b2} = y_2.
\end{aligned}$$



Dual Problem

$$v^h(y) = \underset{p_1,p_2}{Max} \left(\begin{array}{l} p_1 y_1 + p_2 y_2 - \\ \left[\begin{array}{l} Max\big((p_1 - c)K_b, 0\big) + \\ Max\big((p_2 - c)K_b, 0\big) + \\ Max\big((p_1 + p_2)K_a - F_a, 0\big) \end{array} \right] \end{array} \right)$$

The dual of this problem reduces to a simple solution.

Two-Period Price Illustration

$$v(y) = \underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2}}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2})$$

$$\begin{aligned}
&s.t.\\
&0 \le x_{a1} \le K_a u_{a1}\\
&0 \le x_{a2} \le K_a \max(u_{a1},u_{a2})\\
&0 \le x_{b1} \le K_b\\
&0 \le x_{b2} \le K_b\\
&u_{a1} = 0,1\\
&u_{a2} = 0,1\\
&x_{a1} + x_{b1} = y_1\\
&x_{a2} + x_{b2} = y_2.
\end{aligned}$$

Dual Problem

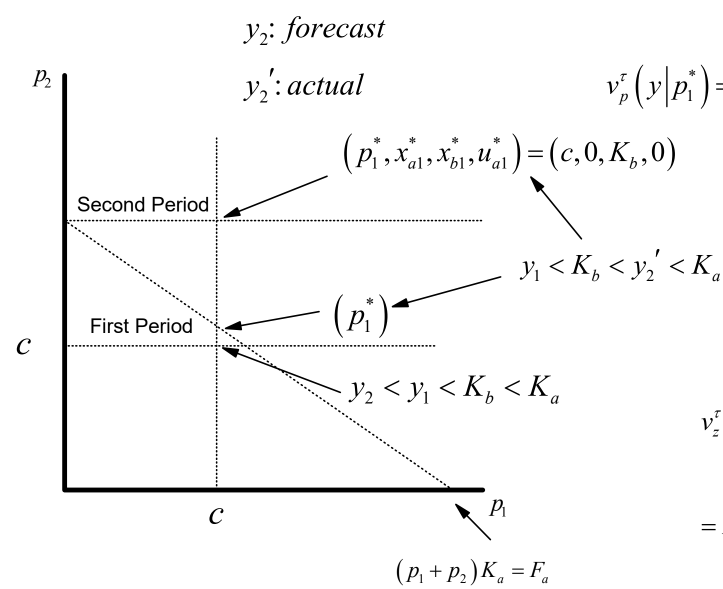
$$v^h(y) = \sup_p \left\{ \begin{aligned}
&py + \underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2},d_1,d_2}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2}) - pd\\
&s.t.\\
&0 \le x_{a1} \le K_a u_{a1}\\
&0 \le x_{a2} \le K_a \max(u_{a1},u_{a2})\\
&0 \le x_{b1} \le K_b\\
&0 \le x_{b2} \le K_b\\
&u_{a1} = 0,1\\
&u_{a2} = 0,1\\
&x_{a1} + x_{b1} = d_1\\
&x_{a2} + x_{b2} = d_2.
\end{aligned} \right\}$$

$$v^h(y) = \underset{p_1,p_2}{Max} \left(\begin{aligned}
&p_1 y_1 + p_2 y_2 -\\
&\left[\begin{aligned}
&Max\left((p_1 - c)K_b, 0\right) +\\
&Max\left((p_2 - c)K_b, 0\right) +\\
&Max\left((p_1 + p_2)K_a - F_a, 0\right)
\end{aligned} \right]
\end{aligned} \right)$$

A related version with different assumptions about the relation of costs illustrates the different solutions that can arise in the conditional dual and conditional market-clearing pricing problems in the second period.

Two-Period Sequential Update

Plant a has only fixed costs,
b has only variable costs.



2nd Period Conditional
Price Problem

$$v_p^\tau\left(y \middle| p_1^*\right) = \underset{p_2}{Max}\left(\begin{array}{l} p_1^* y_1 + p_2 y_2' - \\ \left[\begin{array}{l} Max\left(\left(p_1^* - c\right)K_b, 0\right) + \\ Max\left(\left(p_2 - c\right)K_b, 0\right) + \\ Max\left(\left(p_1^* + p_2\right)K_a - F_a, 0\right) \end{array}\right] \end{array}\right)$$

2nd Period Conditional
Market Clearing Problem

$$v_z^\tau\left(y \middle| \left(p_1^*, x_{a1}^*, x_{b1}^*, u_{a1}^*\right) = \left(c, 0, K_b, 0\right)\right)$$

$$= \underset{p_2}{Max}\left(\begin{array}{l} p_1^* y_1 + p_2 y_2' - \\ \left[\begin{array}{l} Max\left(\left(p_2 - c\right)K_b, 0\right) + \\ Max\left(p_2 K_a - F_a, 0\right) \end{array}\right] \end{array}\right)$$

References

Biggar, D., & Hesamzadeh, M. (2020). *On Multi-Interval Real-Time Markets*. (January), 1–27.

Bjørndal, M., & Jörnsten, K. (2008). Equilibrium prices supported by dual price functions in markets with non-convexities. *European Journal of Operational Research*, *190*(3), 768–789. Retrieved from http://www.sciencedirect.com/science/article/pii/S0377221707006340

Cadwalader, M. D., Gribik, P. R., Hogan, W. W., & Pope, S. L. (2010). *Extended LMP and Financial Transmission Rights* (pp. 1–8). pp. 1–8. Retrieved from http://www.hks.harvard.edu/fs/whogan/CHP_ELMP_FTR_060910.pdf

Caramanis, M. C., Bohn, R., & Schweppe, F. C. (1982). Optimal spot pricing: practice and theory. *Power Apparatus and …*, *75*(9), 3234–3245. Retrieved from http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=4111733

Cervigni, G., & Perekhodtsev, D. (2013). Wholesale Electricity Markets. In P. Rinci & G. Cervigni (Eds.), *The Economics of Electricity Markets : Theory and Policy*. Retrieved from http://www.e-elgar.com/bookentry_main.lasso?id=14440

Chao, H. (2019). Incentives for efficient pricing mechanism in markets with non-convexities. *Journal of Regulatory Economics*. https://doi.org/10.1007/s11149-019-09385-w

Gribik, P. R., Hogan, W. W., & Pope, S. L. (2007). *Market-Clearing Electricity Prices and Energy Uplift*. Retrieved from http://www.hks.harvard.edu/fs/whogan/Gribik_Hogan_Pope_Price_Uplift_123107.pdf

Hogan, W. W., & Pope, S. L. (2017). *Priorities for the Evolution of an Energy-Only Electricity Market Design in ERCOT*. Retrieved from https://scholar.harvard.edu/whogan/files/hogan_pope_ercot_050917.pdf

Hua, B., & Baldick, R. (2017). A Convex Primal Formulation for Convex Hull Pricing. *IEEE Transactions on Power Systems*, *32*(5), 3814–3823. https://doi.org/10.1109/TPWRS.2016.2637718

Hua, B., Schiro, D. A., Zheng, T., Baldick, R., & Litvinov, E. (2019). Pricing in Multi-Interval Real-Time Markets. *IEEE Transactions on Power Systems*, *34*(4), 2696–2705. https://doi.org/10.1109/TPWRS.2019.2891541

Knueven, Ben, Ostrowski, J., & Wang, J. (2018). The Ramping Polytope and Cut Generation for the Unit Commitment Problem. *INFORMS Journal on Computing*, *30*(4), 739–749. https://doi.org/10.1287/ijoc.2017.0802

Knueven, Bernard, Ostrowski, J., Castillo, A., & Watson, J. (2019). A Computationally Efficient Algorithm for Computing Convex Hull Prices. *Optimization OnLine*, 1–19. Retrieved from http://www.optimization-online.org/DB_HTML/2019/08/7327.html

Lavaei, J., & Low, S. H. (2012). Zero duality gap in optimal power flow problem. *IEEE Transactions on Power Systems*, *27*(1), 92–107. https://doi.org/10.1109/TPWRS.2011.2160974

MISO. (2019). *ELMP III White Paper I: R&D report and Design Recommendation on Short-Term Enhancements*. Retrieved from https://cdn.misoenergy.org/20190117 MSC Item 05 ELMP III Whitepaper315878.pdf

PJM Interconnection. (2017). *Proposed Enhancements to Energy Price Formation*. Retrieved from http://www.pjm.com/-/media/library/reports-notices/special-reports/20171115-proposed-enhancements-to-energy-price-formation.ashx

PJM Interconnection. (2019). *Enhanced Price Formation in Reserve Markets of PIM Interconnection, L.L.C., Docket Nos. ER19-1486-000, EL19-58-000*. Retrieved from https://pjm.com/directory/etariff/FercDockets/4036/20190329-el19-58-000.pdf

Potomac Economics. (2019). *2018 State of the Market Report for the Ercot Electricity Markets*. Retrieved from https://www.potomaceconomics.com/wp-content/uploads/2019/06/2018-State-of-the-Market-Report.pdf

Schiro, D. A. (2017). Flexibility Procurement and Reimbursement: A Multi-period Pricing Approach. *FERC Technical Conference*. Retrieved from https://www.ferc.gov/CalendarFiles/20170623123635-Schiro_FERC2017_Final.pdf

Schweppe, F. C., Caramanis, M. C., Tabors, R. D., & Bohn, R. E. (1988). *Spot pricing of electricity*. Retrieved from http://books.google.com/books?id=Sg5zRPWrZ_gC&pg=PA265&lpg=PA265&dq=spot+pricing+of+electricity+schweppe&source=bl&ots=1MIUfKBjBk&sig=FXe_GSyf_V_fcIuTmUtH7mKO_PM&hl=en&ei=Ovg7Tt66DO2x0AH50aGNCg&sa=X&oi=book_result&ct=result&resnum=3&ved=0CDYQ6AEwAg#v=onep

Yu, Y., Guan, Y., & Chen, Y. (2019a). *An Efficient Iterative Algorithm for Convex Hull Pricing Problems on MISO Case Studies*. Retrieved from https://arxiv.org/pdf/1910.12994.pdf

Yu, Y., Guan, Y., & Chen, Y. (2019b). *An Integral Formulation and Convex Hull Pricing for Unit Commitment*. Retrieved from http://arxiv.org/abs/1906.07862

Zhao, J., Zheng, T., & Litvinov, E. (2019). *A Multi-Period Market Design for Markets with Intertemporal Constraints*. 1–11. Retrieved from http://arxiv.org/abs/1812.07034

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