Combined Energy Models

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ABSTRACT

Large-scale energy policy and planning models play an important role in the analysis and development of energy options. The decade of the seventies saw a rapid increase in both the number of such models and the range of their application. Complexity and size make these large-scale energy models expensive to build, difficult to understand, and, therefore, hard to use. The electric power industry, which is affected by nearly every major energy policy decision and which is confronted with its own difficult policy and planning problems, has been at the forefront of the development, analysis, and use of large-scale energy models. The research on the methodology of large-scale energy models has improved the capability to understand and manipulate existing models, and has created the capability for efficiently building new models or adapting old models to the study of new problems.

Most large-scale models of any type are highly structured, and large-scale energy models are no exception. Typically, the model grows through the combination of several smaller models of individual component sectors. These combined energy models build on the extensive work invested in modeling the separate sectors, adding the capability of the combined system to characterize the sometimes unexpected effects of feedbacks through the interactions of the several model components. An examination of the methodology employed in different combined energy modeling efforts reveals a common framework for organizing the models, establishing the properties of the solutions, and manipulating the model in the study of particular policy and planning problems. Within this general framework, we describe the many modeling techniques found in

practice, show how these techniques can be applied to other models, and outline an approach to the development and application of combined models. The results should serve as a guide to the use of the many models developed by and available to energy decision-makers in the electric-power industry, other energy industries, and the public sector.

EPRI PERSPECTIVE

PROJECT DESCRIPTION

EPRI's Supply and Demand Programs have supported the development of a number of Energy Sector Models, covering the supply or demand for specific fuels or energy forms, and Industry Market Models, which include both supply and demand relationships for individual or related fuels. EPRI's Systems Program is charged with interfacing the individual industry supply and demand models into an Energy Systems Model to produce a comprehensive and integrated supply and demand forecasting system. Although similar integrated systems have been developed elsewhere (most notably the Federal Energy Administration's Project Independence Evaluation System Model), the integration of a series of models, each embodying different underlying conceptual structures, is not well understood.

In the first phase of this project the researchers have studied available techniques for interfacing a series of models and arriving at consistent results. Past modeling efforts are reviewed, and fundamental principles, techniques, and concepts for linking submodels are discussed. Guidelines and criteria are developed to judge the appropriateness of various modeling techniques and are then applied to past modeling efforts. Finally, suggestions are made to improve the models that were examined as well as to offer promising avenues for future research.

PROJECT OBJECTIVE

The objective of Research Project (RP) 1016 is to develop fundamental principles, techniques, and concepts for combining the individual industry supply and demand models developed through EPRI-sponsored research into a comprehensive and integrated supply and demand forecasting system.

This project was initiated to provide the methodological foundation for RP1108, the Integrated Forecasting Model (IFM). The IFM is being designed to integrate the diverse models developed by EPRI's Supply and Demand Programs into a consistent forecasting system. The IFM will be used to generate forecasts of energy prices and quantities and will serve as an analytic tool in research and development analysis, such as technology assessment.

PROJECT RESULTS

The examination of a number of large-scale systems models suggests that nearly all of these models can be interpreted as an approximation to a network of process models solved to obtain an optimal solution of the integrated system. Through this general framework the researchers have been able to develop modeling rules for linking component models, properties of the solution of the component models, and a structure for the systematic exploitation of the particular details of the individual models.

The researchers have identified limitations in the area of general price-level formulation and the interpretation of solutions in the presence of complicated taxes and regulations. Even within these limitations, however, their general framework provides operational procedures for building and manipulating large-scale energy models.

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ACKNOWLEDGMENTS

We conducted this research in close collaboration with Lawrence J. Lau in connection with a related effort to develop reduced-form estimates of energy demand models. We also received assistance from James L. Sweeney, Dennis P. Fromholzer, Stephen P. Regulinski, and Patrick C. Coene.

In addition to the normal source materials, we relied heavily on conversations with energy model builders and model users to develop our ideas on the design, interpretation and use of combined energy models. The list of our helpful colleagues is too long to repeat here; it would, for instance, include the participants in the first several working groups of the Energy Modeling Forum. We owe a special debt, however, to Horace W. Brock, Edward G. Cazalet and Dale M. Nesbitt. They demonstrated to us the advantages of a network of process models, as opposed to hierarchical systems, as a framework for describing combined energy models.

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INTRODUCTION

Large-scale energy policy and planning models play an important role in the analysis and development of energy options. The expanded role of energy policy in political and economic events, the rush of new and difficult energy policy questions, and the abundance of information available to describe the energy system created the need and the opportunity for the construction of large-scale energy models. The decade of the seventies saw a rapid increase in both the number of such models and the range of their application.

Complexity and size make these large-scale energy models expensive to build, difficult to understand, and, therefore, hard to use. The electric power industry, which is affected by nearly every major energy policy decision and which is confronted with its own difficult policy and planning problems, has been at the forefront of the development, analysis, and use of large-scale energy models. The research on the methodology of large-scale energy models has improved the capability to understand and manipulate existing models, and has created the capability for efficiently building new models or adapting old models to the study of new problems.

Most large-scale models of any type are highly structured, and large-scale energy models are no exception. During the process of studying the energy system, the analyst isolates relatively homogeneous segments with common characteristics and purposes. Energy demand, for instance, typically is separated into customer classes: residential, commercial, industrial, transportation. Energy supply is classified according to fuel type: coal, oil, gas, nuclear, hydro, geothermal. Electric power generation is separated by region, plant type, and load characteristics. These natural classifications, recognized by all who study the energy system, provide the starting point for modeling. The modeler constructs a separate description of each sector or segment, capitalizing on the special characteristics, data, and theory that apply to that segment. The individual models might be used in isolation, for instance, to analyze a separable problem such as the consumer response to implementation of a load management policy. For many problems, however, the interactions between sectors and across the entire system are an important focus of the problem analysis; a description of the whole, as a large-scale model, is required. In almost all cases, the modeler constructs the large-scale model by combining the individual models of the component systems. The result, a combined energy model, preserves much of the structure of the individual components but, through the interactions of the component and their simultaneous solution, the large-scale model creates new problems of model linkage, complexity, and interpretation.

The purpose of this research is to examine the methodology of combining energy models in order to make that methodology and the models more transparent and usable. The major conclusion of the research is that although there is a great diversity of technique in the construction of the combined energy models, the many different approaches can be related through a natural and general theoretical framework. The theoretical framework--the optimization or equilibration of a network of process models--provides a powerful organaizing device for analyzing existing models, establishing criteria for the consistency of solutions in combined models, developing procedures for standardizing model construction, designing efficient computational procedures, and building on the many different efforts for modeling energy and economic system components. At the same time, the analysis of existing models in the context of the general framework identifies common problems for all energy models; for instance, there are limitations in the explicit modeling of price level formation with resulting implications for the interpretation of equilibrium solutions in energy-economic models.

CRITERIA FOR COMBINED MODELS

The existence of consistent theory improves our ability to construct and use combined energy models. Given a set of separate models of components of the energy system, we could declare any simultaneous solution of these models to be a combined description of the larger energy system. But without some understanding of how the component models interact, it is not possible to guarantee that the linkages are correct. Furthermore, it is not possible to define the properties of the combined solution merely by knowing the properties of the component models. The first requirement for a combined energy model, therefore, is a theory of how to structure the system, how the component models interact, and how the linkages affect the overall solution.

Most combined energy models, because of their complexity, draw on many different data sources to obtain parameter estimates and variable inputs. Correspondingly, they produce information on a great diversity of system components. If the structure of the combined model follows the natural organization of the data, we can simplify the acquisition of these data, the adaptation of the data to the model structure, and the interpretation of the model results. The more the data inputs and outputs conform to natural industry practice, and the less the number of internal transformations of the data to match arcane modeling concepts, the better will be our ability to construct and use a combined energy model.

The use of a highly modular structure, one which perserves the identity, independence, and separability of the individual model components, will improve our capability to use a combined energy model and adapt it to new problems. With a modular design it will be easier to conform to the natural organization of the data. If we preserve the identity of the individual model components, it will be easier for the model users to examine critical assumptions. If the individual model components can be removed and replaced without upsetting the design of the overall combined energy model, it will be easier to incorporate model improvements and to analyze the results. In analyzing counter-intuitive results, for example, it may be critical to be able to expand the detail of one model component without upsetting the structure of the remainder of the combined energy model.

Modular design will facilitate decentralized implementation. In the best circumstances, a combined energy model exploits the expertise of users and analysts steeped in the detail of the individual sectors. With decentralized implementation, the energy demand experts can build the demand model; the engineers can describe the complex technology of electric power generation, crude oil refining, or energy transportation. With each expert confident that he has included his best model in the combined system, we will have increased the credibility of the overall combined energy model. Since the important part of any modeling study is the process of analyzing the problem with the model, not the model itself or any particular set of results, decentralized implementation of a combined energy model would greatly facilitate the ability to use the model to promote improved decision-making.

Computational efficiency is the final criterion for promoting ease of use of combined energy models. If the model is difficult and expensive to run, it will not be run often and its value will be thereby reduced. Hence, any combined energy model must achieve a modicum of computational efficiency. But here we are thinking of computational efficiency in the large, including the time involved in preparing the model, adapting it to new problems, and interpreting the results. This is only one element of the design of a combined energy model and it is much broader than the narrow objective of conserving computer resources.

OPTIMIZATION IN A NETWORK OF MODELS

We can describe most large-scale energy (or energy-economic) models as a network of individual process models. The nodes of the network represent the separate and separable component models of the individual segments or sectors of the energy system. The arcs on the network represent the links between the component models in terms of the flow of goods and services. This simple, natural setting lends itself to easy construction and interpretation of combined energy models. The first detail in the construction of a combined energy model, therefore, is the identification of the linkages in the network through the consistent definition of the variables at the interface of the models.

By establishing the linkages, we sketch the structure of the combined energy model. The next step is to identify the characteristics of a solution of the combined system. Typically, there are many possible solutions for the individual component models which will also satisfy the linking conditions. Loosely speaking, there are many possible paths through the network. In practice, there are many variations in the precise specification of the solution characteristics of a combined energy model, but most models approximate the pure ideal of obtaining an optimal solution to the combined system. The optimality property is defined according to an implicit or explicit objective function as, for example, maximizing the utility of consumers. The solution of the network of models is defined as the solution which does best according to this criterion.

Within this framework of an optimal solution, there is a formal, price-directed decomposition of the linking constraints between the model components. This decomposition defines the properties of the corresponding solution of each component model. Furthermore, it guarantees that a solution to the overall network is obtained with any solution satisfying the optimality conditions for each component model and maintaining the linkages defined by the network. This decomposition leads to a formal equivalence between the optimization approach and an equilibrium interpretation of the solution to the network of models. In short, each model has an optimal solution which creates supply and demand for outputs and inputs. When all supplies and demands are in balance, an equilibrium is obtained for the total network. This equilibrium solution also solves the optimization problem.

This network approach preserves the natural data organization and modular design of the combined energy model. With the application of the optimization principle, there is a consistent theory for defining and interpreting the solutions to the overall combined energy model. Through the decomposition approach we achieve decentralized implementation by separating the characterization of each component and permitting specialized approaches to each sector, exploiting its particular properties. With some care, this decentralized equilibrium or optimization interpretation of the network can be exploited further to define efficient computational procedures for obtaining a solution of the overall system.

SOLVING A COMBINED MODEL

The decomposition of the network of models into a collection of individual components, with restrictions on the solution of the individual components and formal rules for the matching of variables at the model interfaces, presents an opportunity for iterative solution of the combined energy model. The generic algorithm for solving the combined energy model successively visits each node in the network, updates information about inputs and outputs, and obtains new solutions for each of the model components. A large part of the art in building and manipulating combined energy models centers on the design of specific algorithms and the implementation of procedures for obtaining appropriate solutions for the component models. The rules for linking the variables at the model interface are simple and guarantee that a decentralized solution with the correct properties for each component will also provide a solution to the overall combined energy model. The conditions for the solution of each component model derive directly from the optimization interpretation of the overall modeling problem.

A large part of the benefit of the combined energy model derives from this decentralized implementation and exploitation of special approaches to each modeling sector. The overall algorithm can exploit the structure of the model components themselves through the design of the sequence of iterations and the process for updating inputs and outputs for each model component. In addition, the experience with combined energy models has identified a number of computational procedures which do not derive from the theoretical formulation but which can be made consistent with that formulation. These computational procedures, which amount to the expansion of the component model solution procedure to include approximations of the response of the remainder of the network, can be instrumental in quickly finding a solution for the combined energy model.

IMPLEMENTATION

The general framework allows an impressive diversity of modeling approaches in implementation. This diversity is one of the strengths of combined energy models. The methodology and implementation techniques can be adapted as needed to utilize the special characteristics of each component system. The simplest component models are of the fixed coefficient type: there is a linear relationship between inputs and outputs. With this description of the technology, the general theory reduces to a linear equation for describing the flow of goods and services and a simple markup equation for calculating consistent prices.

The generalization of the fixed coefficient model to include a set of relationships between inputs and outputs leads to a formal statement of the component process model as a constrained optimization or mathematical programming problem. The theory of mathematical programming defines the optimality conditions that must be satisfied by the component model. We could, for example, include a linear programming model of a refinery and the general framework would provide a straightforward procedure for incorporating the linear program in the larger combined energy model. During the iterative process of solution, we could utilize the theory of mathematical programming to employ simple and efficient procedures for updating optimal solutions. Often this involves using the structure of the component model in a way that greatly conserves on the amount of information that must be obtained at each iteration.

One powerful and pervasive application of the theory of constrained optimization is in the use of production functions and dual cost functions. Under the optimization assumption, the process model can be characterized according to the reduced form representation of a production function. Under the appropriate conditions, these reduced form representations of the component model are all that is needed to provide the information required during the iterative process for solving the combined energy model. The production function characterizes the output as a function of the input; the cost function describes the price formation or, in an alternative mode, can be used to determine the inputs required to achieve a fixed level of output under a specified price regime.

These theoretically exact descriptions of the nature and solution of the component model are usually approximated, in the interest of simplifying the component model and reducing its data requirements. Grid approximations, sometimes known as extreme point approximations, describe the component model at a selected subset of input combinations. The behavior at points within the grid is then estimated through a variety of interpolation methods. This procedure, for example, is used universally in the linear programming description of large-scale refining models. An alternative approach to approximating the reduced form operations of the component model is through a functional representation. Data on the operation of the component model would be obtained for a range of inputs and outputs and a functional approximation fit to conform to these data. The functional approximation is then used as the true representation of the component model. This procedure is used widely, for example, in econometric models of energy demand which summarize the results of the complex sequence of decisions involved in design and operation of energy-using equipment.

Examination of the practice in the number of combined energy models also reveals different approaches to the solution of the component models to accelerate the convergence to a solution of the combined energy model. Although not necessary for the general theory, experience indicates that some approximation of local equilibrium conditions will enhance the ability to solve the overall model. A number of different approximate approaches can be used for achieving this local equilibrium. At the same time, larger combined models can create new problems in price level determination. In a complete combined model, with a full representation of a closed economy, there is an important indeterminancy in the establishment of the nominal price level. This indeterminancy is not important in the pure ideal of the perfect optimization problem, but it can be important in the realistic approximations that deviate from the ideal framework. The examination of combined energy models within the general framework reveals this important problem and identifies the *ad hoc* approaches employed in most real energy models.

COMBINED ENERGY MODEL CONVENTIONS

The general framework suggests a number of conventions to be followed in building and using combined energy models. The process description and the network linkage suggests a set of standard principles applied in the design of model components and the construction of the interface between different models. These technical details become important during the iterative solution process. There are many essentially equivalent sequences for iterating through the network and updating solutions of component models. One sequence may be preferred to another, however, depending on the ability to exploit the special properties of the component models.

In designing the specialized solution procedures, it is important to remember the basic conventions that guarantee the consistency properties of the overall solution. Most combined energy models are sufficiently complex, and have sufficient deviations from the precise ideal of the overall optimization problem, that we cannot guarantee convergence to a solution. We can, however, design a model in such a way as to preserve a certain fail-safe characteristic. The decomposition approach lends itself to a constructive test of solution. The algorithm will reach natural termination only by demonstrating the existence of a solution.

An important feature of this iterative process is the ability to use the same steps and sequence of calculations in circumstances where the model deviates from the ideal of the pure optimization problem. Every realistic combined energy model includes characteristics which cannot be derived from the general framework. The most important examples of such anomalies are the existence of taxes and regulation of prices. It is well known that the introduction of taxes and price regulation destroys the optimality properties of the equilibrium solution. However, the algorithm derived for the network approach to combined energy models, although suggested by the optimization formulation, does not depend on that formulation in order to be implemented. Each step in the algorithm is well-defined in the presence of taxes or price regulations. The optimization formulation provides a guide to the implementation and interpretation of the combined energy model solution, but the algorithm is not so rigid as to preclude the incorporation of realistic aspects of energy markets.

CASES AND EXAMPLES

There is a large number of examples of large-scale modeling efforts undertaken through the application of the principles of building combined energy models. Two of the earliest and most interesting cases examine the economies of Mexico and the Ivory Coast, with explicit inclusion of separate descriptions of a number of sectors in their economies, including the energy sector in the case of Mexico. These models are interesting in their use of different methodologies to construct the component models. They are also suggestive of the importance of having a consistent theory for constructing a combined energy model.

A stark example of a combined model, applied to economic problems, is the LINK system describing international trade. In this overall model, the component models represent the economies of individual countries. The component models are built, maintained and operated by modelers in the separate countries; the only communication between the models is through exchange of data on international trade. Yet many of the techniques for constructing the interfaces and seeking a solution find natural parallels in combined energy models.

The Mid-Range Energy Forecasting System (MREFS), formerly Project Independence Evaluation System, of the Department of Energy, is a major example of an energy model constructed by linking separate models of individual components. The MREFS model includes econometric models of energy demand, linear programming models of refinery operations and electric power generation, network optimization models of transportation of energy fuels, engineering process models of investment and production for energy supply development, and an iterative procedure for achieving equilibrium of the of the overall network. The specialized algorithms for solving the MREFS model provided the first source of recognition of the power of the special computational procedures that develop and exploit approximations to the overall equilibrium problem.

The abstract structure of the MREFS model, interpreted in the context of the network of process models, shows a remarkable similarity to the SRI-Gulf model and its descendants as developed by Decision Focus, Inc. The creators of the SRI-Gulf model first conceived of the extensive use of a network of process models and utilized this approach in designing that system. Most of the terminology and conventions involved in combined energy models stem from the example and experience with the SRI-Gulf model. The reinterpretation of the model in the context of optimization of a combined energy model provides new insights into the original formulation and suggests the transfer of computational experience from other settings to the solution of this complex system.

One of the most successful examples of the integration of an energy model with a model of the complete economy is found in the work of Hudson-Jorgenson, in cooperation with Brookhaven National Laboratories. Here we see one model, dominated by the methodology and techniques of econometrics, used to describe the bulk of the economy and then linked with a linear programming model of the energy sector defined through the careful engineering analysis of energy technologists.

The same Hudson-Jorgenson model was also linked to the Baughman-Joskow model of the electric utility sector. This is important in demonstrating the flexibility and modularity that comes with the combined energy model approach, and is a case of particular interest to the electric power industry. The Baughman-Joskow model is a sophisticated system with many innovative modeling techniques for approximating the decision processes of the electric power industry. An important element of this model, for example, is the analysis of the capital requirements of the electric power industry. Any such analysis must recognize the large impact that the electric power industry has on the capital markets of the economy as a whole. The integration of the Baughman-Joskow model with the Hudson-Jorgenson model, in the framework of a combined energy model or network of process models, offers the opportunity to analyze this problem in detail.

CONCLUSION

Energy models are powerful tools for studying energy problems. As a practical matter, the only means for constructing a large-scale model is through the careful integration of separate descriptions of components of the overall system. This necessity is a virtue if the combined energy model is developed within the context of a consistent framework that exploits the properties of the overall system structure and makes the model accessible to the users. A careful examination of a range of combined models, both within and without the energy sector, suggests that nearly all of these models can be interpreted as approximations to the pure ideal of a network of process models solved to obtain an optimal solution of the integrated system. This general framework suggests modeling rules for linkages of the component models, properties of the solutions of the component models, and the structure for the systematic exploitation of the particular details of the individual models. At the same time, the analysis identifies limitations in the existing models in the area of general price level formation and the interpretation of solutions in the presence of complicated taxes and regulations. Even within these limitations, however, the general framework of combined energy models provides operational procedures for building and manipulating large-scale energy models.

COMBINING ENERGY MODELS

INTRODUCTION

Large-scale energy policy and planning models play an important role in the analysis and development of energy options. The expanded role of energy policy in political and economic events, the rush of new and difficult energy policy questions, and the abundance of information available to describe the energy system created the need and the opportunity for the construction of large-scale energy models. The decade of the seventies saw a rapid increase in both the number of such models and the range of their application.

Complexity and size make these large-scale energy models expensive to build, difficult to understand, and, therefore, hard to use. The electric power industry, which is affected by nearly every major energy policy decision and which is confronted with its own difficult policy and planning problems, has been at the forefront of the development, analysis, and use of large-scale energy models. The research on the methodology of large-scale energy models has improved the capability to understand and manipulate existing models, and has created the capability for efficiently building new models or adapting old models to the study of new problems.

Most large-scale models of any type are highly structured, and large-scale energy models are no exception. During the process of studying the energy system, the analyst isolates relatively homogeneous segments with common characteristics and purposes. Energy demand, for instance, typically is separated into customer classes: residential, commercial, industrial, transportation. Energy supply is classified according to fuel type: coal, oil, gas, nuclear, hydro, geothermal. Electric power generation is separated by region, plant type, and load characteristics. These natural classifications, recognized by all who study the energy system, provide the starting point for modeling. The modeler constructs a separate description of each sector or segment, capitalizing on the special characteristics, data, and theory that apply to that segment. The individual models might be used in isolation, for instance, to analyze a separable problem such as the consumer response to implementation of a load management policy. For many problems, however, the interactions between sectors and across the entire system are an important focus of the problem analysis; a description of the whole, as a large-scale model, is required. In almost all cases, the modeler constructs the large-scale model by combining the individual models of the component systems. The result, a combined energy model, preserves much of the structure of the individual components but, through the interactions of the components and their simultaneous solution, the large-scale model creates new problems of model linkage, complexity, and interpretation.

The purpose of this research is to examine the methodology of combining energy models in order to make that methodology and the models more transparent and usable. The major conclusion of the research is that although there is a great diversity of technique in the construction of the combined energy models, the many different approaches can be related through a natural and general theoretical framework. The theoretical framework--the optimization or equilibration of a network of process models--provides a powerful organizing device for analyzing existing models, establishing criteria for the consistency of solutions in combined models, developing procedures for standardizing model construction, designing efficient computational procedures, and building on the many different efforts for modeling energy and economic system components.* At the same time, the analysis of existing models in the context of the general framework identifies common problems for all energy models; for instance, there are limitations in the explicit modeling of price level formation with resulting implications for the interpretation of equilibrium solutions in energy-economic models.

*Brock and Nesbitt (1977) exploited the concept of a network of process models in their excellent analysis of large-scale energy modeling methodologies. Cazalet (1977) and later DFI (1978) led the development of this approach in the construction of energy models, including the design and implementation of a modeling software system for a network of process models. T network of process models as described in the present paper subsumes the hierarchical approach to large-scale modeling. See Mesarovic et al. (1970) for an abstract description of the modeling of hierarchical systems.

The report begins with a description of a heuristic set of criteria for combined models. These criteria lead to the general framework of optimization in a network of process models. After describing this general framework, we discuss the practical issues of solving a combined model and using alternative approximations to implement the theoretical ideal. We then summarize the major combined model conventions suggested by the network of process models. This is followed by the description of a few cases and examples to illustrate the applications of the general framework in the context of energy policy and planning models. The concluding section summarizes the report and discusses the use of the main ideas in practical modeling circumstances which violate one or more of the major assumptions.

CRITERIA FOR COMBINED MODELS

A network of process models provides a convenient framework for organizing the architecture of an integrated model. The component models occupy the nodes of the network, and the links between the nodes represent the exchange of inputs and outputs between models; see Figure 1. Modelers often exploit such a network characterization to explain the general structure of a model. This commonplace use of a network as an organizing device is enough to motivate the use of a network of process models as a formal modeling framework. For our purposes, however, we take the network viewpoint further to describe the organization of the data, to establish the rules for the interaction of the component models, and to characterize procedures for the management of the computations. Using a network approach, we can explain how to build and use a combined energy model while meeting the criteria for an effective, usable modeling structure. These criteria, which summarize the major objectives of the model combination process, include: (i) the use of a consistent theory; (ii) the exploitation of a natural data organization; (iii) the preservation of a flexible, modular design; (iv) the use of a decentralized implementation; (v) and the solution through an efficient computational algorithm.

Consistent Theory

The practice of combining energy models is well ahead of the theory. The motivation for the study of the practice stems from the observation that the natural advantages of an eclectic approach, drawing on the best features

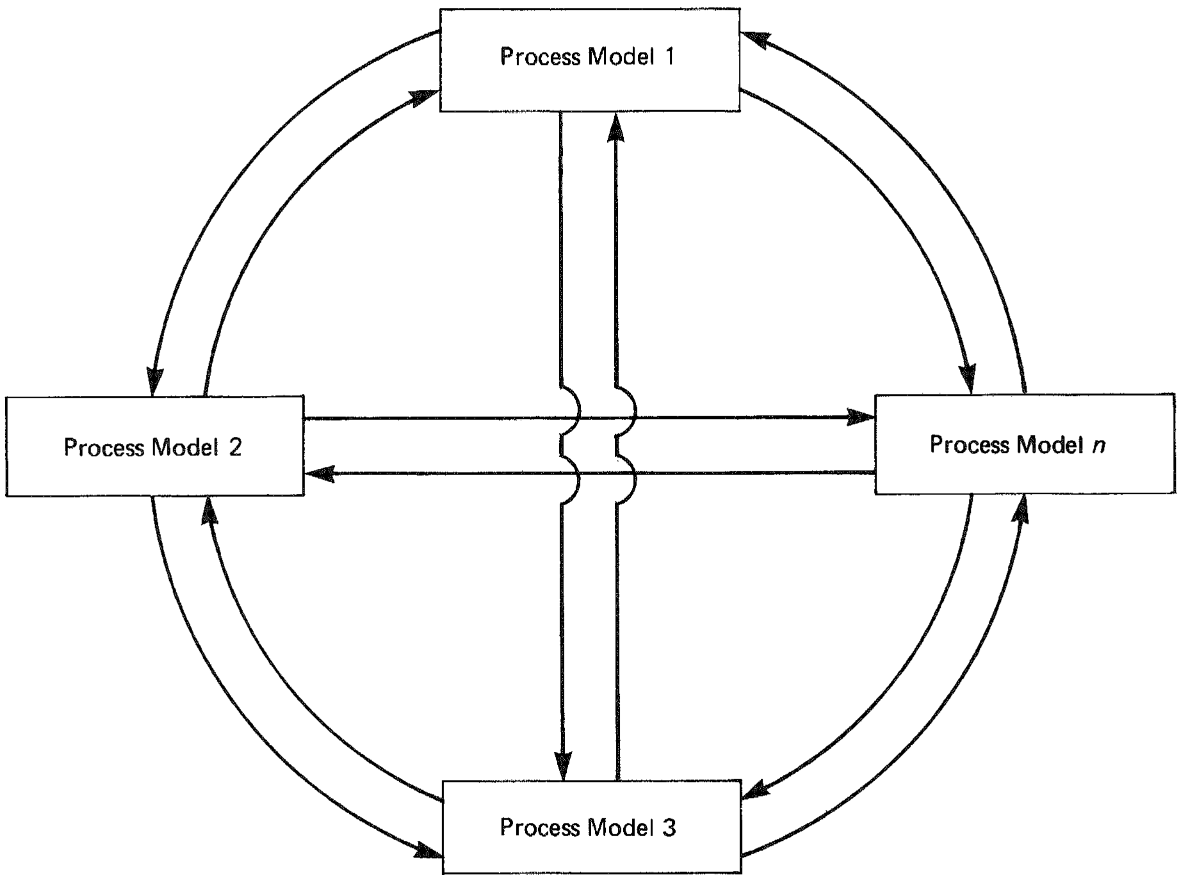


Figure 1. NETWORK OF PROCESS MODELS

of many models, in the construction and use of energy models has led to widespread, successful modeling efforts which, in retrospect, we see include the combination of different, often disparate, energy models.* Hence, the most important use of any general framework is not the establishment of the feasibility of combining energy models, which is well known, but rather the presentation of a consistent theory to explain the common practice. The existence of a consistent theory improves our ability to construct and use combined energy models. Given a set of separate models of components of the energy system, we could declare any simultaneous solution of these models to be a combined description of the larger energy system. But without some understanding of how the component models interact, it is not possible to guarantee that the linkages are correct. Furthermore, it is not possible to define the properties of the combined solution merely by knowing the properties of the component models. The first requirement for a combined energy model, therefore, is a theory of how to structure the system, how the component models interact, and how the linkages affect the overall solution. The theory should tell us what is gained with a combined model, what assumptions we must make, what errors we must correct, and what potential remains untapped.

Natural Data Organization

The value of a model often depends on the ease with which we may use it. This implies a need for efficient computation, but it is even more important that the model exploit the data and describe a problem in a fashion which is accessible and familiar to the potential users. Most combined energy models, because of their complexity, draw on many different data sources to obtain parameter estimates and variable inputs. Correspondingly, they produce information on a great diversity of system components. If the structure of the combined model follows the natural organization of the data, we can simplify the acquisition of these data, the adaptation of the data to the model structure, and the interpretation of the model results. The more the data inputs and outputs conform to natural industry practice, and the less the number of

*For a description and assessment of the state-of-the-art of energy models, see Manne, Richels and Weyant (1979) which includes several examples of energy models built by combining other models into an integrated system.

internal transformations of the data to match arcane modeling concepts, the better will be our ability to construct and use a combined energy model. There will be a close interaction and simultaneity here: the development of the model will lead to an improved understanding of the data needs and the best organization of the information. But the model should be constructed as much as possible to avoid major data conversions by the user. The use of a natural data organization will simplify the provision of the inputs and guarantee that the user can understand the outputs.

Modular Design

One great attraction of a combined energy model is the flexibility inherent in a well-designed system. The use of a highly modular structure, one which preserves the identity, independence, and separability of the individual model components, will improve our capability to use a combined energy model and adapt it to new problems. With a modular design it will be easier to conform to the natural organization of the data. If we preserve the identity of the individual model components, it will be easier for the model users to examine critical assumptions. If the individual model components can be removed and replaced without upsetting the design of the overall combined energy model, it will be easier to incorporate model improvements and to analyze the results. In analyzing counter-intuitive results, for example, it may be critical to be able to expand the detail of one model component without upsetting the structure of the remainder of the combined energy model.

Modularity is near the heart of the eclectic modeling approach. The integration process should be invariant with respect to the internal operations of any component module, depending only on the adherence to rules governing the exchange of information between models. This independence will permit the great flexibility in the design of the component models, exploiting the inherent advantages of particular methodologies; e.g., we may prefer process models of oil refineries, but require behavioral econometric models of consumer choice. And modularity will facilitate the evolutionary development of the modeling system. There can be a quick assembly of models to meet new requirements without requiring a major restructuring of every component of the system.

Decentralized Implementation

If the model is sufficiently modular in design, then we can develop and use the component models in a decentralized manner. We may exploit particular computational advantages of one component model, or delegate its construction and operation to the appropriate user organizational element. Often this means that we may locate different components of the model on different computer systems, and they may interact only infrequently. The evolution of the model can proceed with only periodic coordination, as long as we preserve the information flows and the interfaces with other model components.

This feature may be particularly crucial in the operation of a large model which is likely to cover the interests of many groups that operate normally in a decentralized process. In the best circumstances, a combined energy model exploits the expertise of users and analysts steeped in the detail of the individual sectors. With decentralized implementation, the energy demand experts can build the demand model; the engineers can describe the complex technology of electric power generation, crude oil refining, or energy transportation. With each expert confident that he has included his best model in the combined system, we will have increased the credibility of the overall combined energy model. Since the important part of any modeling study is the process of analyzing the problem with the model, not the model itself or any particular set of results, decentralized implementation of a combined energy model would greatly facilitate the ability to use the model to promote improved decision-making. The model is much more useful, and more likely to be used, if the model structure tends to follow the user organization--it is very unlikely that the organization will be persuaded to adapt to the model.

Efficient Computation

The last criterion deals with the least important aspect of the combined model: its computational efficiency. Within reasonable bounds, the computational cost of a combined model can be limited to no more than a potential nuisance, not a debilitating disease. For large models, the most likely candidates for construction through combination of smaller components, the problems of computation are overshadowed by the difficulties of model formulation, data collection, and solution interpretation. Problems which are of sufficient scope and importance to require the construction of a large model should be able to justify even a large computational budget. And, with the continuing general decrease in computing costs, a relatively small computing budget can purchase a great deal of computational muscle. Hence, even if combined, decentralized energy modeling is no more efficient than other approaches, maybe even a little worse, the other advantages of the integrated, eclectic approach will justify the computational expense. With some care, it may be possible to exploit existing theory to achieve substantial computational advantages within the framework of a network of process models. An objective will be to design a computational procedure which keeps the computational cost as a small part of the modeling budget.

Initially, we take a generous attitude towards the cost of computation. In contrast to the usual motivation for decomposition in model formulation,* which often has the objective of achieving substantial improvements in computational speed, the purpose here is to describe a consistent framework for the development and use of combined energy models. We shall turn later to the question of computational burden. Our attention focusses first on the development of a theory to embrace the eclectic models that have evolved for use in energy planning and policy-making.

OPTIMIZATION IN A NETWORK OF MODELS

We could view a model in many ways. For instance, the collection of equations that comprise a model could be viewed as a large system that must be solved simultaneously. By exploiting the structure of the system, with the decentralized organization of the component models, it may be natural to develop equilibrium concepts to characterize a solution of the system. With the power and generality of a fixed point formulation, we can go further

*For a lucid overview of decomposition approaches to the solution of large-scale optimization models, see Geoffrion (1970).

and structure the problem while seeking or interpreting a solution of the system. And in the most direct cases, the optimization of an objective function might provide a unifying principle for building and using the model. In fact, each of these approaches may be an alternative road to describing the same process or problem, with particular strengths being exploited for different formulations. In part, the facility for combining energy models rests on the ability to see systems of equations, equilibrium systems, fixed point models, or optimization problems as alternative formulations of the same underlying system.

There is a natural duality between the optimization and equilibrium formulations of energy models, and this duality is often at the core of the basic design of energy-economic models; e.g., see Manne (1978). We could use this relationship, which is at the center of the theory of market equilibrium, to describe a process of modeling composition. Much of what follows as a description of combined energy models is an application of the economic theory of markets. Here the actors in the market are not producers and consumers, interacting through exchange transactions; rather, in a combined model, the actors are component models, interacting in a formal algorithm that may mimic the market in the search for an equilibrium that is also an optimum.*

The theory of model decomposition, the separation of a large model into its components, particularly in mathematical programming, provides a body of terminology and concepts that suggests much of what can be done in combining energy models.** In principle, the rules for decomposition should be the reverse of the rules for composition. However, this obvious point does not dispose of the difficulties: in the decomposition case the initial problem is well-posed and the chief difficulty is in finding a procedure for obtaining a solution to a complex but well understood problem. When presented with a collection of models, by contrast, we must address the problem

*The formal equivalence between optimization and general equilibrium can be found in Debreu (1959). Brock and Nesbitt (1977) exploited this duality between optimization and equilibrium to explain the common structure of many energy-economic models.

**See, for example, Geoffrion (1970) or, more recently, Shapiro (1979).

of assuring that the combined model is well defined before turning to the computational process of obtaining a solution. In addition, there are natural approaches to the solution of a combined model that have been pursued successfully in practice, but which might not be apparent if the model is viewed only from the perspective of decomposition theory. Hence, the computational experience with combined models may provide some new insights into the theory of the solution of large-scale optimization models.

The optimization perspective for combined energy models is convenient as an organizing principle. There are many cases, e.g., in the modeling of optimal economic growth, where optimization is an integral part of the problem statement, and the scope of the problem--the full economy--presents the opportunity or the necessity to exploit many different models to describe the many different economic sectors. Furthermore, many combined energy models may rest on optimization concepts in the broadest problem formulation. The adoption of an optimization framework will cover most applications of interest and it streamlines the development of the techniques of model composition. We begin within the context of optimizing modeling in a network of process models and explain the links to equilibrium formulations and the construction of combined models.

An Optimization Framework

A combined energy (or energy-economic) model in an optimization framework is a collection of component models or processes, with the inputs of each process being the outputs of others. All processes must operate simultaneously to achieve the greatest value of some objective function. In this section, we examine such a problem from the perspective of model decomposition. From this we deduce a set of rules for the eventual composition process; rules that provide a consistent theory, natural data organization, modular design, decentralized implementation, and efficient computation.

For ease of interpretation and convenience of terminology, we speak of each component model as a physical process with a set of input and output flows governed by a particular technology. The generic process technology is

$$x_i \rightarrow \boxed{(x_i, y_i) \;\varepsilon\; T_i} \rightarrow y_i \qquad (1)$$

where

x_i : vector of input flows for the i^{th} process,

y_i : vector of output flows for the i^{th} process,

T_i : the set of feasible inputs and outputs for the i^{th} process.

At this stage, the variables x and y and the set T are quite general. For example, there may be many output possibilities associated with the same set of inputs, and the generic process (1) may be incomplete as a component model without the specification of a rule for choosing inputs and outputs.

The many component processes are indexed over i, (i = 0, 1, 2, ..., n). The optimization problem centers on a distinguished component, i = 0; this might be the consumer process for example, and we assume the existence of an objective function defined on the inputs and outputs:

$$x_0 \rightarrow \boxed{\begin{array}{c} u(x_0, y_0) \\ \\ \\ (x_0, y_0) \;\varepsilon\; T_0 \end{array}} \rightarrow y_0 \qquad (2)$$

We link each process to the other processes through the matching of the inputs and outputs. Usually, but not always, this matching will be one-to-one; we will assign each component of the output vector, y_i, to only one component of some input vector, x_j. In practice, no distinction is made between the matched inputs and outputs; e.g., the same variable name and storage location will be used for the crude oil output of the production sector and the crude oil input of the refining sector. But, for the present purposes, it is convenient, in fact essential, to maintain a distinction between these flows and to require that they balance explicitly as part of the linking of the component models.

To represent this linkage, we define an assignment matrix, A, which may be thought of as containing only 1's and 0's, and characterize the balancing of inputs and outputs as

$$y = Ax, \tag{3}$$

where

$$y = (y_0, y_1, \ldots, y_n),$$

and

$$x = (x_0, x_1, \ldots, x_n).$$

Given these component models and definitions, the optimization problem of the combined model is to choose x and y to

$$\underset{x,y}{\text{Maximize}}\ u(x_0, y_0) \tag{4}$$

subject to

$$(x_i, y_i) \in T_i,\ i = 0, 1, \ldots, n, \tag{4.1}$$

$$y = Ax. \tag{4.2}$$

An optimal solution of the combined model requires the determination of feasible inputs and outputs for each process, the balancing of all inputs and outputs, and the maximization of the consumer's utility function. This defines the general framework that will guide the construction of the network of process models and provide the fundamental background for interpreting the combined model solutions.

Decomposition

In principle, we could approach problem (4) directly, and solve it by any of a number of techniques without regard to our several desiderata for the formulation and use of a combined energy model. This, of course, would defeat our intent to solve (4) while preserving the basic structure and individual identity of the component models. If we think of (4) as a large-scale optimization problem, however, we may solve (4) while meeting our criteria. The use of the component model structure is at the essence of large-scale model decomposition. We can apply decomposition techniques to serve two ends: (i) to demonstrate the link between optimization and equilibrium, and (ii) to expose the principles for model composition. We begin with one of the simplest but most powerful decomposition methods, the generalized Lagrange multiplier theorem; see Everett (1963).

Decomposition techniques usually focus on the complicating constraints of a problem. The somewhat artificial distinction between the inputs and the outputs of the processes allows us to decompose problem (4) in a natural way. On first examination, the general technological constaints, (4.1), might be thought to be the complicating part of the problem. And this is true in a certain sense--one objective of the construction of a combined model is to exploit the special techniques most appropriate for dealing with different processes and the complications of the general technological constraints. But for the purposes of integrating the processes, it is the linking of inputs and outputs in (4.2) that presents the main complication and the main opportunity for simplification.

In essence, the decomposition of (4) is achieved by dualizing with respect to (4.2) and exploiting the separability of the resulting problem. For this purpose, we introduce a set of variables, $\lambda = (\lambda_0, \lambda_1, \ldots, \lambda_n)$, as the prices or multipliers for (4.2), and produce the new problem:

$$\underset{x,y}{\text{Maximize}}\ u(x_0, y_0) + \lambda(y - Ax) \tag{5}$$

subject to $\quad (x_i, y_i) \in T_i \quad , i = 0, 1, \ldots, n \quad .$ (5.1)

For a given λ, we define a feasible optimal solution of (5) as $x(\lambda)$ and $y(\lambda)$. The basic result is:

<u>Theorem (Everett (1963))</u>: If for some λ, $x(\lambda)$ and $y(\lambda)$ satisfy (4.2), i.e.,

$$y(\lambda) = Ax(\lambda) \quad ,$$

then $x(\lambda)$ and $y(\lambda)$ are also feasible and optimal in (4).

<u>Proof</u>: Clearly $x(\lambda)$ and $y(\lambda)$ are feasible in (4). But for any other x and y which satisfy the constraints of (4),

$$\begin{aligned} u(x_0, y_0) &= u(x_0, y_0) + \lambda(y - Ax) \\ &\leq u(x_0(\lambda), y_0(\lambda)) + \lambda(y(\lambda) - Ax(\lambda)) \\ &\leq u(x_0(\lambda), y_0(\lambda)) \quad ; \end{aligned}$$

where the inequality follows from the definition of $x(\lambda)$ and $y(\lambda)$ and the observation that the feasible set in (4) is a subset of the feasible set in (5). Hence, $x(\lambda)$ and $y(\lambda)$ are also optimal in (4).

The convenience of (5) is its separability. We can make this feature more apparent by introducing a second vector of multipliers or prices, $\gamma = (\gamma_0, \gamma_1, \ldots, \gamma_n)$, associated with the inputs to each module. This leads to the problem:

$$\underset{x_i,y_i}{\text{Maximize}}\ u(x_0, y_0) + \sum_{i=0}^{n} (\lambda_i y_i - \gamma_i x_i) \tag{6}$$

subject to $\quad (x_i, y_i) \in T_i \quad , i = 0, 1, \ldots, n.$ (6.1)

If we choose γ and λ such that

$$\gamma = \lambda A \quad ,$$

then (6) is equivalent to (5). The optimal solutions of (6) are $x(\lambda,\gamma)$ and $y(\lambda,\gamma)$. By Everett's theorem, a choice of γ and λ such that

$$\gamma = \lambda A \quad ,$$

and $$y(\lambda,\gamma) = Ax(\lambda,\gamma) \quad ,$$

is a solution for the original problem in (4). But now we recognize that for a given selection of γ and λ, the input and output prices respectively, problem (6) is completely separable.

For i = 1, 2,..., n, in the generic process or technology, the response to a given price regime is to

$$\underset{x_i, y_i}{\text{Maximize}} \ \lambda_i y_i - \gamma_i x_i \tag{7}$$

subject to $$(x_i, y_i) \in T_i \quad . \tag{7.1}$$

This is a classic profit maximization problem; i.e., for given prices, choose the inputs and outputs to maximize the value of the outputs less the value of the inputs.

For i = 0, the generic consumer process, the response to a given price regime is to

$$\underset{x_0, y_0}{\text{Maximize}} \ u(x_0, y_0) + \lambda_0 y_0 - \gamma_0 x_0 \tag{8}$$

subject to $$(x_0, y_0) \in T_0 \quad . \tag{8.1}$$

This problem for the generic consumer process is equivalent to

$$\underset{x_0, y_0, \pi_0}{\text{Maximize}}\ u(x_0, y_0) - \pi_0 \quad ,$$

subject to

$$\gamma_0 x_0 - \lambda_0 y_0 = \pi_0 \quad ,$$

$$(x_0, y_0) \in T_0 \quad .$$

This same problem could also be stated as

$$\underset{\pi_0}{\text{Maximize}}\ v(\pi_0 | \lambda_0, \gamma_0) - \pi_0 \quad , \tag{9}$$

where

$$v(\pi_0 | \lambda_0, \gamma_0) = \underset{x_0, y_0}{\text{Maximize}}\ u(x_0, y_0) \tag{9.1}$$

$$\text{subject to} \quad \gamma_0 x_0 - \lambda_0 y_0 = \pi_0 \quad , \tag{9.2}$$

$$(x_0, y_0) \in T_0 \quad . \tag{9.3}$$

These alternative representations of the generic consumer process are equivalent in the sense that:

<u>Lemma</u>: If (x_0, y_0) is a solution to (8), then (x_0, y_0) is a solution to (9) with $\pi_0 = \gamma_0 x_0 - \lambda_0 y_0$. If (π_0, x_0, y_0) is a solution to (9), then (x_0, y_0) is a solution to (8).

This generic consumer process is close to the familiar problem of maximizing utility subject to income constraints. Here we interpret π_0 as the net payments to the consumer process. We can define $\pi_i = \lambda_i y_i - \gamma_i x_i$ as the net payments to the consumer from the other processes.

It is clear that for the appropriate choice of π_0, (9) reduces to the utility maximization problem

$$\underset{x_0, y_0}{\text{Maximize}}\ u(x_0, y_0) \tag{10}$$

subject to

$$\gamma_0 x_0 - \lambda_0 y_0 = \pi_0 \quad , \tag{10.1}$$

$$(x_0, y_0) \in T_0 \quad . \tag{10.2}$$

Furthermore, with additional restrictions on the nature of the consumer process, we can establish a link between the solutions of (10) and the solution of our original optimization problem in (4).

<u>Theorem*:</u> Suppose u is concave, increasing in x_0 and decreasing in y_0, and T_0 is convex. If for some (λ, γ), with $(\lambda_0, \gamma_0) > 0$, and some π_0:

(i) $(x_i(\lambda_i, \gamma_i), y_i(\lambda_i, \gamma_i))$ solves (7), for i = 1, 2,..., n;

(ii) $(x_0(\lambda_0, \gamma_0), y_0(\lambda_0, \gamma_0))$ solves (10) with π_0 and without satiation; i.e.,

$0 \notin \dfrac{\partial v}{\partial \pi_0}$, the null vector is not contained in the sub-differential of v; and

(iii) $\gamma = \lambda A$, and

$y(\lambda,\gamma) = Ax(\lambda,\gamma)$;

then (a) $\pi_0 = \sum_{i=1}^{n} \pi_i$,

and (b) $(x(\lambda,\gamma), y(\lambda,\gamma))$ solves (4).

*For a more general result with quasi-concave utilities, see Debreu (1959).

Proof: First part (a). Let $x = x(\lambda,\gamma)$ and $y = y(\lambda,\gamma)$. Then

$$0 = y - Ax = \lambda y - \lambda Ax = \lambda y - \gamma x = \sum_{r=1}^{n} \pi_i - \pi_0 \, .$$

Next part (b). We proceed by constructing a solution to (9) and (7). By the lemma, this must be a solution to (8) and (7). And by Everett's theorem, a solution to (8) and (7) is a solution to (4).

We note that $v(\pi_0|\lambda_0, \gamma_0)$ is concave, due to the concavity of u and the convexity of T_0, and increasing in π_0, because $(\lambda_0, \gamma_0) > 0$ and u is increasing in x_0 and decreasing in y_0. By concavity, π_0 is a solution to (9) if and only if

$$1 \in \frac{\partial v(\pi_0|\lambda_0, \gamma_0)}{\partial \pi_0} \, .$$

Let $k \in \dfrac{\partial v(\pi_0|\lambda_0, \gamma_0)}{\partial \pi_0}$. By the monotonicity of v and the nonsatiation assumption, we have $k > 0$. Let $u^*(x,y) \equiv \frac{1}{k} u(x,y)$. Then $v^* = \frac{1}{k} v$ and

$$\frac{\partial v^*(\pi_0|\lambda_0, \gamma_0)}{\partial \pi_0} = \frac{1}{k} \frac{\partial v(\pi_0|\lambda_0, \gamma_0)}{\partial \pi_0} \, .$$

Hence,
$$1 \in \frac{\partial v^*(\pi_0|\lambda_0, \gamma_0)}{\partial \pi_0}$$

and therefore, π_0 solves (9) for u^* with

$$x_0^*(\lambda_0, \gamma_0) = x_0(\lambda_0, \gamma_0) \, ,$$

and
$$y_0^*(\lambda_0, \gamma_0) = y_0(\lambda_0, \gamma_0) \, .$$

Furthermore, we have no change in (7), so

$$x_i^*(\lambda_i, \gamma_i) = x_i(\lambda_i, \gamma_i) \quad ,$$

and $$y_i^*(\lambda_i, \gamma_i) = y_i(\lambda_i, \gamma_i) \quad , \quad i = 1, 2, \ldots, n \quad .$$

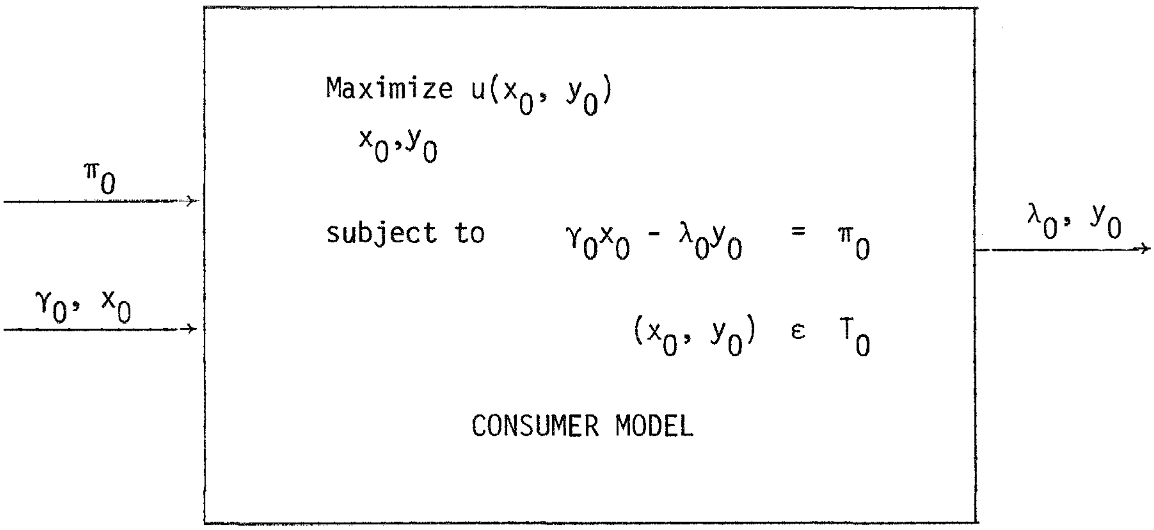
Therefore, $(x(\lambda,\gamma), y(\lambda,\gamma))$ is a solution for (9) and (7); hence, a solution for (8) and (7); and hence, a solution for (4) with u^*. Since multiplying u by a positive scalar does not affect the problem, we have a solution for (4).

Corollary: The prices which yield optimal solutions in (7) and (9) are unique only up to a scalar multiple.

With this background, therefore, we define the generic model of a component process as:

$$\xrightarrow{\gamma_i,\ x_i} \boxed{\begin{array}{ll} \underset{x_i, y_i, \pi_i}{\text{Maximize}} & \pi_i \\ & \\ \text{subject to} & \lambda_i y_i - \gamma_i x_i = \pi_i \\ & (x_i, y_i) \in T_i \\ & \\ \multicolumn{2}{c}{\text{PROCESS MODEL}} \end{array}} \begin{array}{l} \xrightarrow{\pi_i} \\ \\ \xrightarrow{\lambda_i,\ y_i} \end{array} \tag{11}$$

Similarly, we define the generic consumer model as:



(12)

To solve this system, we need to determine a set of input prices, γ, a set of output prices, λ, and the net payment to the consumer, π_0, such that each model is solved separately and the solutions satisfy:

$$y = Ax \quad , \tag{13}$$

and

$$\gamma = \lambda A \quad . \tag{14}$$

Together these conditions imply that

$$\sum_{i=1}^{n} \pi_i = \pi_0 \quad , \tag{15}$$

i.e., the net payments from all processes balance with the net payment to the consumer. This suggests a convenient simplification: use the total of the profits of all processes, $i = 1, 2, \ldots, n$, as the estimate of the net payment to the consumer process.

These generic models, derived from the decomposition analysis, define the properties of the components that might be combined to form a larger energy model. And the conversion of the problem from the form of (4) to the representation found in (11) through (14) provides the framework which meets the several desiderata for combined energy models.

Since the solutions to (11) - (14) yield solutions to (4), the optimization framework provides the consistent theory. The solution of (11) - (14) is an equilibrium problem, and the solutions can be interpreted as optimal in the context of the maximization of utility subject to the limits of the available technology. We treat each process model separately, finding a combination of inputs and outputs that is both feasible and maximizes the profit or net payment to the consumer. Limited by the value of all net payments, the consumer process maximizes the utility defined over inputs and outputs. The equilibrium conditions, (13) and (14), when all prices and quantities balance, identify an optimal solution for the original problem in (4). The complete satisfaction of the equilibrium conditions results in a general equilibrium solution. A partial satisfaction of the equilibrium conditions, leaving slack some constraints on the prices and quantities, results in a partial equilibrium solution. As we shall see, few of the available energy models employ a general equilibrium. Most energy models are partial equilibrium models to one degree or another. But the general equilibrium conditions provide the theory and the standard for evaluating the partial equilibrium approximations.

The organization of the process models into a collection of separable systems yields a natural structure for the organization of data. Each process has a set of well defined inputs and outputs, and the data needed to describe the appropriate technologies can be assembled separately; i.e., the organization of the data in one process is independent of the organization in all others. This permits the independent assembly of the information. And through the appropriate choice of process representations, the architecture of the combined model can be constructed to conform to the existing organizations and data assemblies found in the natural problem setting.

The separability of the component models is the guarantor of modularity in the system design. Once we describe the inputs and outputs, a change in the internal design of any one process leaves unaffected the rest of the combined model structure (but not the solutions). Therefore, it is possible to remove any component model and replace it with another model with the same inputs and outputs. Similarly, the generic structure is not sensitive to the level of aggregation in the processes. For some purposes, it may be convenient to deal with a collection of processes as a single module. For other occasions we may separate the components and treat them individually, according to the same generic rules, without disrupting the remainder of the system.

The modularity allows the decentralized implementation of the models. In the development phase, the only requirement is to preserve the integrity of the interconnections through the definitions of the inputs and outputs. During the computational phase, in the search for a solution, we can treat the components in a decentralized manner, although not necessarily in the fashion requiring the remote communication of separate computer systems. We will investigate the opportunities here when we turn to the subjects of approximations and computational efficiency.

The computational efficiency of the combined model depends on the nature of the algorithm used to search for a solution. It is not obvious that the linking of disparate models is computationally feasible, much less efficient. This is, of course, partly an empirical matter. For the moment, we observe that it is possible to design efficient computational procedures for the integrated model. In some cases, the exploitation of the model structure, which is made more transparent in the network organization, can yield very successful computational methods.

SOLVING A COMBINED MODEL

The detail of the particular algorithms for solving any combined energy model will depend upon the nature of the model and the structure of the components. Later sections of this report investigate some of the major computational options and review the experience on existing combined energy models. For the moment, we can anticipate the detail by describing certain general features that provide a background to motivate the discussion of alternative approaches to the implementation of the generic models.

The central theme of the several computational approaches is one of iteration: the systematic adjustment of the prices and quantities in a search for a solution to the overall system. Each iteration includes a visit to one or more of the nodes of the network of models to update the solution of the component processes. There is a great deal of flexibility allowed in this iteration, including the use of approximate solutions or interim calculations which do not satisfy all of the limitations which must be met by the final solution. These extensions will be developed later in the context of specific applications. But the essence of all the methods can be described in terms of a straightforward family of iteration methods. We differentiate the methods by the classification of the inputs, outputs, and their respective prices into sets of free and fixed variables for the current visit to each process.

Suppose that we are dealing with the i^{th} component model or process. In essence, for each iteration, the elements of the inputs x_i and the outputs y_i are assigned to be either free or fixed variables. This also determines the assignment of the prices to the complementary set; i.e., the prices of the free variables are considered fixed, the prices of the fixed variables are considered free. Assuming that the value of the fixed variables does not change, the free variables are updated such that all the resulting prices and quantities solve the generic problems (11) or (12); i.e., we solve the model such that the resulting quantities represent the optimal inputs and outputs for the associated prices.

For example, we may choose to fix the input prices and the output quantities. The problem, then, is to determine the output prices and the input quantities such that the resulting system satisfies the optimality conditions for the generic problem. If this rule is followed for every node, then the symmetry of inputs and outputs guarantees that every quantity and every price is treated as a free variable in some calculation. With the nodes visited frequently, this simple rule provides a natural mechanism for updating the prices and quantities during the iterative process. And if the iterative process ultimately results in no change in the prices and the quantities, this provides a constructive test of the solution to the equilibrium problem and, therefore, a constructive test of the solution of the overall optimization problem.

Under certain conditions, it may be possible and convenient to reduce further the set of free variables for a given iteration. The chief general rule is that each variable or price must be classified as free often enough to ensure that the variables are updated during the iterative process in order to preserve the validity of the constructive test of a solution. Normally, the choices of fixed and free variables are grouped, with input quantities, output prices, or both being selected as the free variables. The particular choice will often be conditioned on the nature of the component being considered: some variable updates will be easier to solve than others. We shall return to the subject of algorithms later, to make the description more complete and to provide the details for particular applications. But for the motivation of the procedures for implementing the generic models, the background of iteration and the concept of fixed and free variables provide an introduction to the character of the implementation process.

IMPLEMENTATION

The generic processes in (11) and (12) summarize the idealized theory of the component models, but the practice in energy modeling seldom takes the explicit form of a full optimization problem. It is usually possible to relate the actual implementation to this theory, however, and it is helpful to indicate the nature of this relationship in several stylized examples -- to clarify the advantages of the different implementation alternatives and to prepare the way for the later analysis of actual implementations in several existing energy models. This section, therefore, takes up typical situations and explains the simplifications appropriate for computational practice.

Fixed Coefficients

The most frequently occurring processes found in the components linked together in a combined model are the essentially trivial processes modeled as fixed coefficient input-output relationships. For example, a transportation model might represent the process of moving energy products from one location to another. The transfer involves only one option, and the technology is represented by a set of fixed coefficients. Suppose that the material to be transported is oil, which is denoted as z before transport and y after transport. The transportation process might consume capital (K) and labor (L) services as well. By our assumption, the technology set, T, is fully characterized by a set of coefficients, (a_z, a_K, a_L), and the solution is

$$\begin{pmatrix} z \\ K \\ L \end{pmatrix} = \begin{pmatrix} a_z \\ a_K \\ a_L \end{pmatrix} \cdot y \quad . \tag{16}$$

The efficiency of the transport process is established by a_z, with a value greater than one implying the use of energy during transportation.

A price is established for each input and each output; therefore, the generic problem can be represented as

$$\underset{z,K,L,y,\pi}{\text{Maximize}} \quad \pi \tag{17}$$

$$p_y y - p_z z - p_K K - p_L L = \pi \tag{17.1}$$

$$\begin{bmatrix} z \\ K \\ L \end{bmatrix} = \begin{bmatrix} a_z \\ a_K \\ a_L \end{bmatrix} \cdot y \tag{17.2}$$

At first glance, this problem appears to be difficult to deal with; for any set of prices there are either infinitely many solutions or no solutions. Yet in the context of the search for a set of prices that balance the combined model, this problem is trivial.

Recall that during the search for a solution, any visit to this component process will fix some variables and solve for others. If at some iteration the output quantity is fixed, then the technology (16) determines the input quantities; e.g.,

$$z = y\, a_z \ . \tag{18}$$

Furthermore, we recognize that only certain sets of prices will be of interest, as these are the only candidates for a solution to the overall combined model. The relationship that must exist for the prices is

$$p_y = p_z a_z + p_K a_K + p_L a_L \ . \tag{19}$$

If at some iteration the input prices are fixed and the output price is free, then (19) determines the output price. This is the usual situation for such trivial processes at any iteration: fixed input prices and output quantities, and the solution of the model, to determine output price and input quantities, reduces to the evaluation of two linear equations.

Often we can make this special case even simpler through the use of the model in a partial equilibrium setting. Consider a transportation process embedded in a larger model that is being used in a partial equilibrium context. In this case, suppose the deleted interactions, which create the partial equilibrium condition, are those with the rest of the economy in the competition for capital and labor services; in particular, the prices of capital and labor are fixed and the quantities are assumed to be available in unlimited supply at these prices. In this case, the only interesting technology limitation is (18), and the determination of the input quantity involves only the conversion of the output by the appropriate efficiency index. Furthermore, since p_K and p_L are fixed, the costs of the capital and labor inputs can be represented by a simple markup, C.

$$C = p_K a_K + p_L a_L \quad . \tag{20}$$

Then the pricing equation in (19) becomes the standard fixed markup relationship:

$$p_y = p_z a_z + C \quad . \tag{21}$$

This trivial process, either in the form of (19) or (21), recurs frequently in large systems; it will be used later to represent any fixed coefficient model, either as a separate process or as part of a larger model being linked into the combined system.

Mathematical Programming

The generic problem in (11) or (12) is a mathematical programming problem of a general type. With certain regularity conditions, it may be possible to apply standard techniques to the solution of such an optimization problem. Often these techniques can exploit a good starting solution; since the problem must be solved repeatedly during the iteration of the overall model, we can update the solution efficiently.

Consider the case when the technology set, T, is described by a set of inequalities; i.e., (x,y) is feasible if and only if

$$g(x,y) \geq 0 \ , \tag{22}$$

where g(.,.) is a vector valued, possibly nonlinear, function. The inputs and outputs might be further restricted individually; i.e.,

$$x \varepsilon X \ , \quad y \varepsilon Y \ . \tag{23}$$

In this case, the generic problem becomes

$$\underset{\substack{x \varepsilon X,\ y \varepsilon Y \\ \pi}}{\text{Maximize}} \quad \pi \tag{24}$$

$$\text{subject to} \qquad p_y y - p_x x = \pi \ , \tag{24.1}$$

$$g(x,y) \geq 0 \ . \tag{24.2}$$

There are seemingly limitless opportunities to mesh the solution of this problem with the overall iteration of the combined energy model. For example, suppose that the method for solving (24) produces good approximate solutions quickly. Then it may not be necessary to solve (24) completely at each iteration of the full model. As long as the convergence of the full system and (24) occur simultaneously, the ultimate solution should satisfy the overall optimality conditions.

If $g(.,.)$ is linear, and X and Y are polyhedral, then (24) is a linear program and the updating with new prices involves changes only in the objective function. This might be accomplished through parametric procedures, particularly if the problem has some special structure; e.g., as in the case of the fixed coefficient process.

If $g(.,.)$ is differentiable and concave with gradient $\nabla g = (\nabla_x g, \nabla_y g)$, the first order conditions may be solved to obtain a solution in the prices:

$$p_x = \theta \nabla_x g \quad , \tag{25}$$

and

$$p_y = -\theta \nabla_y g \quad . \tag{25.1}$$

where θ is the appropriate lagrange multiplier. Here again, we may exploit a special structure of the problem. We can choose the fixed and free variables in a manner to further simplify the problem. For example, suppose that $(g(.,y))$ is linear in x (i.e., $g(x,y) = Bx - \hat{g}(y)$), X is polyhedral, and the output quantities and input prices are fixed. Then the determination of input quantities, x, is equivalent to solving the linear program

$$\underset{x \in X}{\text{Minimize}} \quad p_x x \tag{26}$$

$$\text{subject to} \qquad Bx \geq \hat{g}(y) \quad . \tag{26.1}$$

We may obtain the output prices from (25.1) with θ taken from (25). Since (26) involves only updating the constraint values in a linear program, we can reduce the overall iteration to a relatively few parametric steps and evaluations of linear functions. This suggests the computational power available with the mathematical programming perspective found so frequently in the construction of energy models.

Production/Cost Function Duality

A similar unifying theory for the generic process exists in the duality of production and cost functions. Consider the simplest case of a process that uses many inputs, the vector x, to produce a single output, y. We assume that the technology set, T, or at least its efficient frontier, can be represented by a linearily homogeneous production function; i.e.,

$$y = F(x) \quad . \tag{27}$$

For fixed output quantity and input prices, the determination of input quantities reduces to

$$\text{Minimize} \quad p_x x \tag{28}$$

$$\text{subject to} \quad F(x) = y \quad . \tag{28.1}$$

Under certain conditions*, there is a dual cost function, c, such that

$$c(p_x, y) = \text{Min} \{ p_x x \mid F(x) = y \} \ .$$

It follows from Shephard's lemma that we can determine the optimal input quantities from the gradient of c (.,y); i.e.,

$$x = \nabla_{p_x} c(p_x, y) \quad . \tag{29}$$

*See Diewert (1974) for a full development of the duality between production and cost functions.

This formulation is familiar to economists as the demand equation for inputs as a function of output and prices. Furthermore, the price of y that will be a candidate for a solution of the combined model is

$$p_y = c(p_x, 1) \quad . \tag{30}$$

Hence, if the cost function is available, we can obtain the input quantities and output prices by evaluating the demand functions, (29), and the unitary cost function, (30).

There are similar results for the generic consumer process. This observation is one link to the many energy models produced by the extensive available econometric research. For example, the essence of econometric models of energy demand is to observe actual behavior (i.e., past observations of x, p_x, p_y and y) and estimate the shape and parameters of (29) and (30). The general framework of a network of process models indicates how we can exploit this large body of work for integration with other types of energy models. Furthermore, the theory provides a framework for modifying the existing econometric work, as necessary, for the introduction of new inputs and outputs.

Approximations

Most energy models characterize the generic problems through the use of standard approximation techniques which may disguise the link to the general formulation. In some instances, data limitations or the need to reduce the complexity of the generic problem dictate the use of approximations. In other cases, the approximation is a computational device during the iteration of the solution algorithms, but the final problem solution does not depend on the approximation. Both cases are important. The first to recognize the underlying nature of the model for integration with other models. The second to permit exploitation of efficient computational techniques. It is useful, therefore, to be able to recognize the major approximation schemes. We can group the usual approximations into two related categories: grid approximations and function approximations.

Grid Approximations* Many modeling applications deal with situations where the technology set, T, is convex, at least up to the degree of precision we can support with the available data. Given two sets of feasible inputs and outputs, it is reasonable to assume that any convex combination of these inputs and outputs is also feasible. This might be the case, for example, if each set of inputs and outputs corresponds to the operation of the process in some particular mode. By operating the process, such as a refinery, for part of the time in one mode and for the remainder of the time in another mode, any convex combination of observed inputs and outputs could be achieved.

Formally, if there is a collection of points $\{(x^j, y^j\}$, $j = 1,\ldots, n$, such that

$$(x^j, y^j) \in T \quad , \quad j = 1,\ldots, m \quad . \tag{31}$$

then for any (x,y) such that

$$(x,y) = \sum_{j=1}^{m} \alpha_j (x^j, y^j) \quad , \tag{32}$$

$$\alpha_j \geq 0 \quad , \quad \sum_{j=1}^{m} \alpha_j = 1 \quad , \tag{32.1}$$

we assume that $(x,y) \in T$.

We often describe the elements of $\{(x^j,y^j)\}$ as the "modes" of the process. Ideally, we might think of them as the extreme points of the technology set, but this is not necessary. If T is compact and convex, then any point in T can be represented as a convex combination of its extreme

*See Geoffrion (1970) for a lengthier discussion of approximations.

points; however, it may be difficult to generate and maintain all the extreme points. In practice, all that is required, and all that is done, is to generate "enough" modes to develop a reasonable representation of T, where reasonable can be defined as a representation large enough to contain a neighborhood of the solution of the combined model. There is usually enough information available to prepare such a reasonable approximation.

The advantage of the approximation is the conversion of the generic problem into a tractable linear program; i.e.,

$$\underset{x,y,\alpha_j}{\text{Maximize}} \quad p_y y - p_x x \tag{33}$$

$$\text{subject to} \qquad (x,y) = \sum_{j=1}^{m} \alpha_j \cdot (x^j, y^j) \quad , \tag{33.1}$$

$$\sum_{j=1}^{m} \alpha_j = 1 \ . \tag{33.2}$$

$$\alpha_j \geq 0 \quad , \quad j = 1, 2, \ldots, m \ . \tag{33.3}$$

This linear program can be solved with standard techniques. And improving the approximation requires no great modification of the problem; once new modes in T have been generated, the expansion of the approximation involves no more than adding new columns to the linear program in (33). This often presents the opportunity for further decomposition; we may solve the full generic problem only when necessary to generate new modes for the approximation in (33). Normally, (33) serves as the interface with the remainder of the models in the full combined system.

Often we recast the approximation in (33) into a more descriptive form, converting the decision variables in the linear program into the natural units of the problem. By normalizing the modes to a unit basis for some key variable, e.g., per unit of operation of the process in that particular configuration, we make the decision variables coincide with the level of the activity. In this case, we replace the convexity constraints (33.2) and (33.3) with more familiar capacity constraints, which leads to a more natural problem formulation and interpretation.

This simple change often facilitates the further decomposition of the construction of the linear programming approximation. This approach is so powerful, in practice, that the linear programming implementation of grid approximations often serves as an integrating framework for the combination of energy models. For example, with the appropriate convexity conditions, we can apply the grid approximation scheme directly to (4), and we could restate all of our theory within this framework. The primary reason for avoiding the restriction to this direction of research is the greater computational flexibility available from formulating the problem as a network of process models.

An important special case of the grid approximation is the use of the linear programming framework in a partial equilibrium setting. On closer inspection, we see the fixed coefficient model in (17) to be a special case of (33). And, just as in the case of the fixed coefficients model, there are many applications of (33) which treat some inputs and outputs as having a fixed price and an unlimited supply or demand.

Suppose, for example, that we treat a subset of the inputs in this fashion, i.e., let $x = (x_1, x_2)$, and treat p_x as exogenous to the partial equilibrium model. The price of x_2 will never change. Then, for each mode, we can identify a cost, c_j, of the exogenous inputs for each unit of operation of that mode; i.e.,

$$c_j = p_{x_2} x_2^j \quad . \tag{34}$$

With this definition, the simplified version of (33) is

$$\underset{x_1, y, \alpha_j}{\text{Maximize}} \quad p_y y - p_{x_1} x_1 - \sum_{j=1}^{m} c_j \alpha_j \tag{35}$$

$$\text{subject to} \qquad (x_1, y) = \sum_{j=1}^{m} \alpha_j \cdot (x^j{}_1, y^j) \ , \tag{35.1}$$

$$\sum_{j=1}^{m} \alpha_j = 1 \ , \tag{35.2}$$

$$\alpha_j \geq 0 \ . \tag{35.3}$$

As before, we can replace the convexity constraints in (35.2) and (35.3) by capacity constraints. In the context of refinery operations, for example, y might represent the refined products, and x, the crude oil inputs; the problem is to maximize the profit: to maximize the revenue on refined products, $p_y y$, less the expenditures on crude oil, $p_x x_1$, and less the cost of operating the refinery, $\sum_{j=1}^{m} c_j \alpha_j$.

In principle, it is always possible to reconstruct the values of the exogenous inputs, x_2. Although this is seldom done in practice, it is important to remember the simplifications in (35) when integrating this approximate model in a larger system where we upgrade some of the previously exogenous variables, x_2, to the status and treatment of the endogenous variables, x_1.

<u>Function Approximations</u> The refinement of the grid approximation may improve the representation of the generic problem, but it also could lead to a rapid growth in the number of input-output vectors to be generated and used. In many cases, however, we can view the correspondence between inputs and outputs as a functional relationship, and, with sufficient regularity, we can approximate the function through a closed-form expression.

This functional representation is the essence of the convenient theory of production functions and their dual cost function, as mentioned above. The theoretical derivation of the production function establishes the conditions for existence; the application of econometrics to empirical data provides estimates of the functional form and its parameters. The econometric work is usually motivated by the use of real-world data to approximate the true, underlying relationships. But, once the approach is noted, it is obvious that we may apply the same ideas and techniques to represent the behavior of a model; we can generate "pseudo-data" from solutions of the generic model and fit a function to the data to serve as an approximate representation of the generic model, for integration into the remainder of the system; see Griffin (1976).

For example, the modes in the grid approximation might be generated from solutions of the generic model at prices $\{(p_x^j, p_y^j)\}$. From the theory of cost functions, it follows that there is some underlying function, c, such that

$$x^j = \nabla_{p_x} c(p_x^j, y_j) \quad . \tag{36}$$

We can use the data from the generic model to estimate c and its gradient as approximate representations of the true model. This simpler function approximation can then be used in the combined model. We might modify this approximate function only when the solution to the combined model is obtained and it is discovered that the functional approximation is not adequate at that point.

With this procedure, often implemented with constant elasticity demand or supply functions, for example, we can manage the interactions with other models. It is as a practical method for maintaining decentralized implementation of the component models. We need not centralize the full generic models, and we can reduce the communication among the models to the periodic updating of the approximate representations. With a modicum of ingenuity, we can select the approximations to be adequate representations of the generic model while maintaining adequate simplicity for the centralized implementation of the equilibrium calculations in the model.

Local Equilibrium Approximations An important special case of the grid and function approximations, applied most often as a computational device for speed and stability in the search for a global equilibrium, is the use of a local equilibrium problem with approximations of the supply and demand functions.

We can solve each component model in a decentralized manner, and the solution for one model will affect the solutions for others. If in the solution for any one model, the reaction of the rest of the system were known, then this reaction could be accounted for in the local problem, and the final equilibrium could be obtained in one iteration. Unfortunately, we usually do not know the full reaction of the rest of the combined model as we change the inputs and outputs of any component model. The essence of the local equilibrium approximation is to estimate the reaction of the remainder of the system, and then to use this estimate during the iterative process to obtain an updated solution for the prices and quantities of the inputs and outputs.

To illustrate this approach*, suppose that for a component model an estimate of the reaction of the remainder of the combined model is available in the form of a set of integrable supply and demand functions,

$$p(x,\bar{x}),\ p(y,\bar{y})\ , \tag{37}$$

*Ahn (1979) provides a detailed examination of this local equilibrium approach to the approximation of energy models and obtains a number of convergence results for the application of this idea to the solution of a class of combined energy models.

relative to some base quantities $(\bar{x},\bar{y})$. The intent is to replace the generic problem (11) with the related local equilibrium problem:

$$\underset{x,y}{\text{Maximize}} \quad \int_0^y p(\hat{y},\bar{y})\, d\hat{y} - \int_0^x p(\hat{x},\bar{x})\, d\hat{x} \tag{38}$$

$$\text{subject to} \qquad (x,y) \in T \ . \tag{38.1}$$

If $p(\cdot,\ \bar{y})$ and $p(\cdot,\ \bar{x})$ are defined properly and the solution to (38) satisfies

$$(x,y) = (\bar{x},\bar{y}) \ , \tag{39}$$

then it is easy to show that $(\bar{x},\bar{y})$ also solves the generic problem with $p_x = p(\bar{x},\bar{x})$ and $p_y = p(\bar{y},\bar{y})$. The advantage of (38) is the recognition, in the component model solution, that a change in the solution to one model will result in a change in the equilibrium prices for the entire combined model. This approximation procedure can accelerate the convergence of the iterative search for equilibrium prices and quantities; see Figure 2. It also provides stability for the local approximation problems and finally, it preserves the constructive test of the equilibrium solution.

Price Level Determination

The multiplication of the utility function by a positive constant does not change the solutions of the model. This fact is equivalent to our corollary and the familiar property of equilibrium prices in a competitive system: if $(p_x,\ p_y)$ is an equilibrium price vector, then $(kp_x,\ kp_y)$ is an equilibrium price vector for any $k > 0$.

This fact suggests the need to fix the price level; i.e., to select a numeraire good. We should note that this requirement arises only in the context of the full implementation of a general equilibrium formulation or, equivalently, in the complete optimization of the combined model. In a partial equilibrium setting, one or more of the prices is

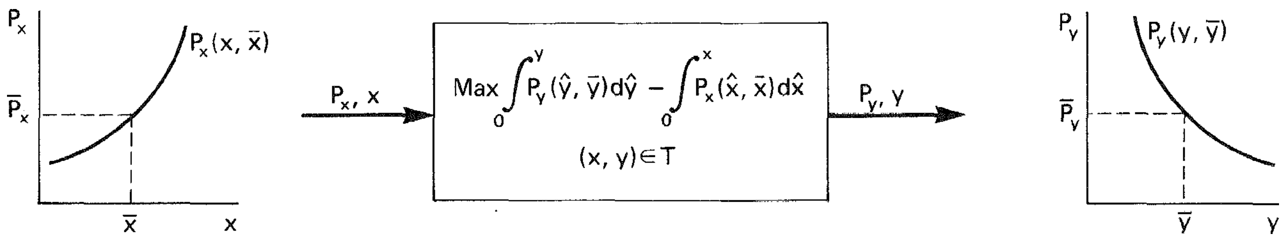


Figure 2. LOCAL EQUILIBRIUM FOR PROCESS MODEL

fixed, and this is usually enough to determine the price level for the combined model by arbitrarily choosing the nominal price for one or more of the goods. In the ideal case, the selection of the price level can be a matter of convenience; only relative prices matter. A common choice is to use labor services or consumption goods as the numeraire and to set the corresponding price at some convenient level.

This arbitrary selection of the price level is often hidden or overlooked in the formulation of energy models. In the pure optimization or equilibrium model of the theoretical ideal, this arbitrary element of the model formulation is of little consequence. The solution is invariant with respect to the price level. But it is a critical element in real energy models which deviate from the assumptions of the theoretical ideal. In most real models the nominal price level has an important effect in determining the solution of the model. In the presence of income taxes based on nominal levels of income, for example, the flow of resources to the government will be sensitive to the price level. Different arbitrary assignments of the price level will produce different solutions to the model. Yet there is little in the way of a theory for determining the proper price level. This critical point identifies an important lack of robustness in policy models and an important area for future modeling research.

In dynamic models, a certain separability over time may produce a type of price level indeterminancy in each period. Suppose that the linking of inputs and outputs must occur in each period. For example, if the inputs and outputs are indexed by t, the corresponding statement of (4) might be:

$$\text{Maximize } \sum_{t=0}^{N} u_t(x_0^t, y_0^t)\, \delta^t \tag{40}$$

$$\text{subject to} \quad \prod_{t=0}^{N} (x_i^t, y_i^t) \in T_i \;, \; i = 0, 1, \ldots, n \;, \tag{40.1}$$

$$y^t = Ax^t, \text{ for all } t \;, \tag{40.2}$$

where δ represents a discount factor. The corresponding representation of (11) would be:

$$\underset{x_i^t, y_i^t, \pi_i^t}{\text{Maximize}} \sum_{t=0}^{N} \pi_i^t \, \delta^t p_\pi^t \tag{41}$$

$$\text{subject to} \qquad p_{y_i}^t y_i^t - p_{x_i}^t x_i^t = \pi_i^t, \text{ for all } t, \tag{41.1}$$

$$\prod_{t=0}^{N} (x_i^t, y_i^t) \in T_i \quad . \tag{41.2}$$

$p_{x_i}, x_i^t \longrightarrow$ PROCESS MODEL $\longrightarrow p_\pi^t, \pi_i^t$; $p_{y_i}^t, y_i^t$

Since the inputs and outputs must match in each period, i.e.,

$$y^t = Ax^t \quad ,$$

it must be that

$$p_y^t y^t = p_y^t A x^t = p_x^t x^t \quad .$$

It follows, therefore, that any solution to the combined model would satisfy the constraint

$$\sum_{i=1}^{m} \pi_i^t = \pi_0^t \quad , \text{ for all } t \quad .$$

Hence, we can replace

$$\sum_{t=0}^{N} (p^t_{x_0} x^t_0 - p^t_{y_0} y^t_0) = \sum_{t=0}^{N} \pi^t_0 \ ,$$

the formal restriction on net payment to the consumer, with the series of constraints,

$$p^t_{x_0} x^t_0 - p^t_{y_0} y^t_0 = \pi^t_0 \ ,$$

for all t. For the dynamic model, therefore, the representation of (12) would be:

p^t_π, π^t_0

$p^t_{x_0}, x^t_0$

$$\underset{x^t_0,\, y^t_0}{\text{Maximize}} \ \sum_{t=0}^{N} u_t(x^t_0, y^t_0)\, \delta^t \tag{42}$$

$$\text{subject to} \quad p^t_{x_0} x^t_0 - p^t_{y_0} y^t_0 = \pi^t_0 \ , \text{ for all } t \ , \tag{42.1}$$

$$\prod_{t=0}^{N} (x^t_0, y^t_0) \ \varepsilon \ T_0 \ . \tag{42.2}$$

CONSUMER MODEL

$p^t_{y_0}, y^t_0$

Since we can scale constraints (42.1) without affecting the solution to the model, it is clear that this version of a dynamic model permits the arbitrary determination of one price in each time period; we choose p^t_π to be the present value of the marginal utility of income in period t in order to maintain a consistent scaling in (41). We might, for instance, choose the price of labor and let it grow at some arbitrary rate of inflation, thereby guaranteeing a certain level of inflation for the average price level. This is the approach taken, either implicitly or explicitly, in most existing energy-economic models.

COMBINED ENERGY MODEL CONVENTIONS

The general framework of a network of process models suggests a number of conventions to follow in building and using combined energy models. These technical details become important during the iterative solution process; there are many essentially equivalent sequences of moves through the network while revising the solutions of the component models. One sequence and mode of solution may be preferred to others, however, depending on our ability to exploit the special properties of the component models. In designing the specialized solution procedures, we must remember the basic conventions that guarantee consistency of the solution. Most combined energy models are sufficiently complex and contain enough deviations from the precise ideal of the full optimization problem that we cannot guarantee convergence to a solution. However, if we use reasonable care in the search for a solution, we can preserve a certain fail-safe characteristic. The iterative algorithm will reach natural termination only by finding a solution, and the termination criteria amount to a constructive test of an equilibrium solution.

The conventions to follow are actually a set of principles and guidelines to remember in the formulation of a combined energy model:

Point of Measurement

An obvious rule is to maintain a consistent set of definitions for the inputs and outputs that are linked at the model interfaces. Recall that we made an artificial distinction between the variable names for the outputs, y, of one component model and the inputs, x, of another component model. Nothing happens to the flow during the transition from one component to another; hence, they must maintain a common definition and common point of measurement for the quantities and the prices.

It is easy to recognize this need. In practice, it is not so easy to follow this convention. Modelers tend to be vague about the precise definition and point of measurement of the inputs and outputs of their models. When measuring the demand for energy, for instance, we may not be especially precise about whether it is energy delivered to the wholesaler or the energy delivered to the final consumer. The prices may differ at these two points of measurement, but the quantities will tend to be very nearly the same. Hence, a great deal of ambiguity might be allowed without affecting the usefulness of the model for stand-alone analysis. But this minor deficiency becomes a major problem when we turn our attention to the integration of the model into a larger system. One of the most troublesome, tedious tasks in building a combined model is in finding the precise definitions of the point of measurement of inputs and outputs in order to make the correct linkages.

Often the measurement problem is so difficult that a precise matching cannot be made. Suppose, for example, that the absolute values of prices for some variables have been suppressed, and only index numbers remain. Then it may not be possible to construct precisely the absolute price measurement that will be needed for a model linkage. A common practice when faced with this difficulty is to construct a bridging function. Given data on the price of a commodity measured at one point, and the price index for the same commodity measured at another point, we estimate a functional relation between the two and use this functional relationship as a bridge for the interface between the component models. See Preston (1975) for a description and extensive use of this procedure.

Flows and Stocks

The variables (x,y) represent the flows between the process models of the combined system. Although the formulation in (4) is general enough to allow the variables to represent almost anything, with the technology set appropriately defined to link inputs to outputs, most of our modeling applications involve variables which are quantity flows, not stocks. Hence, when discussing capacity or capital inputs, we refer to capital services, a flow, not the capacity itself, a stock. The price of the variable is then the rental price of capital, not the present value of the capital stock. Although this is not a hard and fast rule, it is usually easier to think of the technology constraints within individual component models as capturing the transformation between stocks and flows, and we treat all variables linked between models as flows.

Value Flows

The decomposition of the optimization model in (4) introduced the prices for the flows on each link, and created the value links between models, the payments to consumers, π_i. These value flows and prices have particular characteristics derived from the optimization formulation, yet we seldom find these special characteristics in real models of the energy-economic system.

The prices enter only in the accounting equations for determining the objective function of the generic process models and the income constraint of the generic consumer model. There are no other constraints on the prices in determining the solution of any component model. Hence, any restrictions on the prices, as, for instance, in representing the effect of price regulation, constitute deviations from the optimality conditions. These deviations compromise and complicate our interpretation of the model in the optimization setting.

The value flows, π_i, are accounting flows which have natural interpretations as the profits or rents paid to the consumer by each industry or sector of the economy. In the real world and in real models, we have elaborate institutional mechanisms for allocating these payments and balancing the flows across industries and across consumers. The optimization formulation, with the single consumer, does not include these institutional arrangements. Therefore, the more that we modify our modeling of these payments to accommodate some feature of the real world, the more the optimization formulation may be compromised. This is a theoretical point which affects the normative character of the combined energy model. However, it is not a serious point when we consider the descriptive role of the model. As we shall see below, we may follow the linkage rules and computational procedures built upon the optimization framework even though the component models may deviate from the strict formulation in terms of constraints on prices or the treatment of value flows. The common experience is that we do not affect the computational efficiency of the solution algorithms even though we are now searching for an equilibrium that may not be an optimum. This robust feature is one of the attractive characteristics of the combined energy model formulation and use in a network of process models.

Optimality Conditions

The process network framework describes a solution of the combined model as a decentralized set of optimization problems. This solution is characterized by a set of inputs and outputs for each component model which jointly satisfies the optimality conditions in every component model. In many applications, we must remember this narrow objective. We are looking for a final solution to the optimality or equilibrium conditions. Our framework tells us nothing about the nature of solutions during the search for a final solution, nor does it suggest that every optimal solution for a component model will also be an acceptable solution for the overall system.

The reasons for highlighting this point are illustrated best by reconsidering the fixed coefficient model. The feasible set in this model is an unbounded ray defined by the linear relationship between inputs and outputs. This model has a bounded solution only for those prices for which the value of the objective function is identically zero over the feasible set. Hence, if the prices do not satisfy equation (19), then the formal solution to the optimization problem will be unbounded. Similarly, for the set of prices where the profits are identically zero, any feasible solution will also be optimal. We could not assume an arbitrary set of prices and attempt to solve this model; nor could we pick an arbitrary feasible pair of inputs and outputs that have zero profit.

We would use the fixed coefficient process model only in the context of searching for a solution that might satisfy the overall optimality conditions of the combined model. In our example, fixing the output quantities and solving for the input quantities, or fixing the input prices and solving for the output price, provide natural ways to update the optimal solution while linking to other components to determine output quantity demanded and to determine prices of the quantity inputs. In formulating any component model, therefore, we must ensure that the final solution satisfies the component model optimality conditions with the given set of prices, but we must also structure the computational procedure to search for an optimal solution that will balance the remainder of the system.

Algorithms

Given the modular structure of the combined energy model, the most natural approach to the search for a solution of the system is to move through the network of process models, solving and resolving the optimality conditions for each of the component models while checking and updating the values of the variables on the connecting links. The visitation sequence (the order in which we solve the component models) and the procedures for obtaining the local optimal solution for each component model are the targets of the modeler's skill and artistry. There is no apparent limit to the permutations and variations in technique that could be applied to capitalize on the particular structure of any combined model.

Iteration The common feature in solving any component model is to fix some of the quantities and prices, keeping them at the current estimate of the solution, and then to update the remaining free variables in order to satisfy the optimality conditions. We might, for example, fix the output quantities and the input prices, and solve for the input quantities and output prices. In some circumstances, where we might wish to avoid a laborious search for a simultaneous solution of a component model lacking a convenient structure, we may keep all variables except input quantities fixed and employ a simple procedure for updating the solution for the input quantities. Later, in the same component model, we would fix all variables but the output prices and then use a simple procedure for updating the output prices. During the sequence of iterations, the output prices and input quantities may never be a solution to the local optimality conditions. Only with the final stage of iteration will we achieve complete equilibrium in the full model.

Typically, the more variables that are fixed, the easier it will be to solve the component model for the remaining free variables. But the more variables that are free, the more likely it will be that we will rapidly improve the solution in the search for an overall optimum in the network of process models. There is a trade-off here between the computational work of updating the solution of the component models and the number of iterations required of the overall system. In the work with the DFI (1978) modeling software, for example, the trade-off is made in favor of iteration of the model in order to avoid complicated calculations at any component model. The DFI approach distinguishes between "down" iterations in which only the input quantities are treated as free variables, and "up" iterations, in which only output prices are treated as free variables. For many component models, these computations involve no more than the evaluation of a function. These iterations can be performed easily even though the simultaneous solution for output prices and input quantities would require an iterative search within the component model itself. We present below some examples of such process models.

The only rigorous requirement for the design of a solution algorithm is that each variable be treated as a free variable for at least one component model in at least one mode of solution, and that this component model be visited in this mode often enough to guarantee that the free variable is updated every time we test for a solution. This is the minimum requirement to maintain the constructive test of solution of the model. And it is a very weak requirement; it permits great flexibility. In solving an energy-economic model, for example, we might be very concerned about complicated interactions in the dynamics of energy pricing, but less concerned about the interactions with the feedback effect of the changing prices of capital and labor services provided by the remainder of the economy. In this event, we might concentrate the iterative search on the sensitive components of the combined energy model, and update only periodically the prices and quantity flows representing the interaction with the remainder of the economy.

Relaxation The straightforward pursuit of the iterative process, sequentially visiting the component models while updating the demands for inputs and the prices of outputs, is the network process model generalization of the simple cobweb search suggested by Figure 3. Given the symmetry between inputs and outputs, this rule guarantees that every variable is a free variable during the solution of a component model. For a simple, well-behaved model, this process might converge and converge rapidly; but we know that there is no guarantee of convergence. However, we may borrow from directional search algorithms in solving optimization problems and use the new estimate of the values of the free variables as defining a direction in which the solution should move. If we believe this direction is determined by the local properties of the model given the current estimate of the solution, then it is reasonable to expect that moving the full distance might take us well beyond the region where the local approximation is justified. This opens the door for relaxation methods. Relaxation approaches amount to moving only part of the distance in order to calculate a new local approximation to the problem and a new direction for improvement. Again, there are many opportunities for the design of the "step-size" coefficients that control how fast the solution moves and how fast we approach convergence of the model. For any real, large-scale combined energy model, we typically have enough information to design this relaxation process with some degree of efficiency.

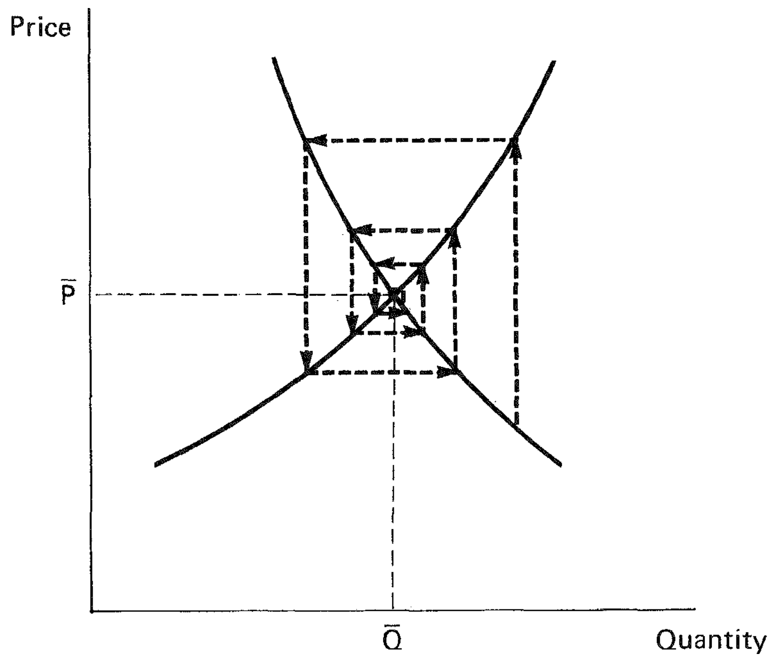


Figure 3. COBWEB SEARCH FOR EQUILIBRIUM

The first and most important piece of information will be a good estimate of the solution. The efficient computation of optimal solutions for a large-scale model owes more to a good initial guess than to any clever algorithm promising ultimate convergence. In a real application, we will have estimates of solutions to similar problems, knowledge about the nature of a possible solution, and experience in past attempts to solve the model. We will use all this experience to design the relaxation procedures. If we start near a good solution, we do not want to move too far away too fast. By sticking to the minimal formal rules for the constructive test to the solution, we can uncover surprises we had not anticipated, if the solution is far from our guess, or we can use the formal algorithm to tidy up our initial guess. During the iterative process, we may obtain only approximate solutions to the component models, trying to converge in the local problems while adjusting the parameters to converge in the global problem as well. The published experience with the computational results of models formulated as networks of process models, e.g., in Cazalet (1977) or in Hogan et. al. (1978), suggests that few if any of the component models will be so non-linear or so unstable to present major difficulties in seeking a solution. For situations where the simple cobweb iteration and relaxation procedures are not sufficient, one of the most promising approaches is to expand upon the local approximation of the combined system in the exploitation of local equilibrium solutions.

<u>Local Equilibrium</u> The work of Ahn (1979) provides a theoretical analysis of the computational procedure used in the Project Independence Evaluation System (PIES). Ahn suggested a generalization of that approach that we could apply to networks of process models. In a sense, this approach calls for the solution of each component model through the use of a second-order approximation of the remainder of the combined energy model. Ahn showed the close relationship of this procedure to Jacobi iteration and Newton-like methods for the solution of non-linear equations.

The local equilibrium approach centers on the use of an approximate representation of the reaction of the remainder of the combined energy model to changes in input-output quantities or prices of any of the component processes. Suppose we have an initial estimate of the solution for a component process, $(\bar{p}_x, \bar{x})$ and $(\bar{p}_y, \bar{y})$. This is an estimate of the solution for (11). Using this point as a reference, we construct a set of integrable functions, $p(y,\bar{y})$ and $p(x,\bar{x})$ that we use to approximate the demand from the rest of the system for the output y at price p_y and the supply from the rest of the system of the inputs x at price p_x. The essential property of these functions must be that $\bar{p}_x = p(\bar{x},\bar{x})$ and $\bar{p}_y = p(\bar{y},\bar{y})$. Using these approximate functions, we replace the generic optimization problem in (11) with the new local equilibrium problem

$$\underset{x,y}{\text{Maximize}} \quad {}_0\int^{y} p(\hat{y},\bar{y})dy^r - {}_0\int^{x} p(\hat{x},\bar{x})d\hat{x}, \qquad (x,y) \in T \ . \tag{43}$$

The solution to this problem is the intersection of the approximate supply and demand curves, i.e., a local equilibrium accounting for the technological relation between inputs and outputs; see Figure 2.

If the approximate reaction functions are an exact representation of the behavior of the rest of the combined model, then we need to solve this local equilibrium problem only once to obtain the solution to the overall model. But even if the approximations are not exact, we can use this local equilibrium problem to estimate the new improved solution for the component model. It is evident that any solution to this local equilibrium problem is also a solution to the generic optimization problem (11) with the choice of prices equal to $p(x,\bar{x})$ and $p(y,\bar{y})$. Hence, if we update the prices and the quantities, we will preserve the property of the algorithm that the interim solution satisfies the optimality conditions for the component process model. If at some iteration the solution does not change from the initial estimate at that iteration, then we have preserved the constructive test of a solution to the overall problem. The advantage of using the local equilibrium approximation is that it incorporates more information and, much like penalty methods for optimization algorithms, tends to prevent the model from deviating substantially from an interim solution. The component model solution recognizes the effect of substantial change on the remainder of the system.

This approach was used extensively in the solution of the PIES model. In that case, the local equilibrium method was an integral part of guaranteeing an outcome that was not a corner solution of the linear program. It was a surprising result that the method also greatly accelerated the speed of convergence of the overall network of models. Ahn's theoretical analysis describes the properties of the model which guarantee this rapid convergence.

Endpoints

Any real large-scale model will have a finite planning horizon. The problems of providing an optimal representation of the endpoint effects transfers directly from the overall combined model to the design of the component models. The methodology of combined energy models and the network of process models formulation provide no special resolution to these problems, nor do the problems seem more serious than in any other modeling context. We follow Grinold (1978) in recommending the dual equilibrium approach for the characterization of endpoint conditions. In essence, the last period decisions must include a description of the optimal choices for the infinite future. We characterize this future by assuming that prices grow at a constant rate, typically the rate of inflation. Using this assumption, it is usually possible to evaluate the residual stocks for the end of the finite planning horizon. We can couple the dual equilibrium method with the selection of a planning horizon sufficiently far in the future to leave the initial decisions relatively unaffected. See Grinold (1978) for further details.

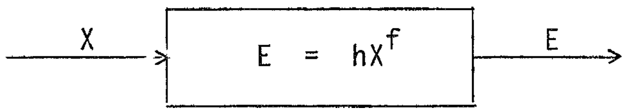
Descriptive Models

Most, maybe all real combined energy models violate one or more of the assumptions of the strict normative framework of optimizing in the network of process models. The regulation of prices, the introduction of taxes, or ad hoc representations of processes which are difficult to model or where the data are unavailable, lead to models which might be a good description of the real world but which take us outside the realm of the formal adherence to the optimizing assumptions. In the strictest sense, we cannot interpret the solution of the model as equivalent to a solution of problem (4).

In an operational sense, however, the principles and procedures identified in the combination of energy models within the framework of optimization in a network of process models may still be applicable to the descriptive model. The iteration through the network and the search for a solution can proceed by updating the quantity flows and prices on the connecting links. The solution of the component models can be an optimization problem with a few additional constraints on prices and quantities. We can still apply the relaxation techniques and approximation methods of the local equilibrium solutions. In short, we motivated the iteration through the network of process models by recognizing that it could yield a solution to the optimization problem. But there is nothing in the operational implementation of these ideas that depends upon the optimization formulation. In the broader context of an equilibrium framework, we can iterate the system until we obtain a balanced solution. We can still exploit the computational techniques developed for the optimization problem. We will not be sure of convergence, and we will not have the optimization interpretation of the final solution, but we do not need to abandon the approach because of special problems in any or even all of the component models. The network framework is robust with respect to problem formulation. And, for many examples of implementation, e.g., the experience with the PIES model, the computational benefits based on the normative framework carry over into the solution of the descriptive model. We can think of our theory of a network of process models as a guide to the formulation and solution of combined energy models. We do not need to think of it as a rigid mold that must determine the shape of our models.

An Example

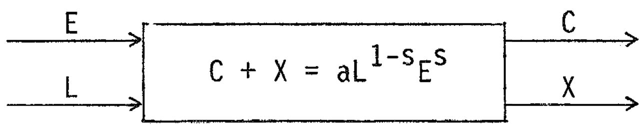
We can illustrate these conventions and guidelines through the examination of a simple, stylized example of a combined energy model. Suppose that we have three component models: energy, industry, and a consumer. The energy sector demands goods and services, X, and produces energy, E. There is a constant elasticity of energy output with respect to the input of goods and services. The energy process model might be:



ENERGY SECTOR

where $h > 0$ and $0 < f < 1$.

The industrial sector utilizes energy, E, and labor, L, in order to produce goods and services for the energy sector, X, and goods and services for the final consumption, C. We assume that the production technology is summarized by a Cobb-Douglas production function:



INDUSTRY SECTOR

where $a > 0$ and $0 < s < 1$.

Finally, we characterize the preferences of the consumer in terms of a utility function over consumption goods and labor provided to the industrial process:

$$\xrightarrow{\;C\;} \boxed{\text{Maximize} \quad u \ln(C) - bL^2} \xrightarrow{\;L\;}$$

CONSUMER SECTOR

where $u > 0$, $b > 0$.

This is a single period model, but it is only for the sake of simplicity of exposition that we restrict ourselves to these few flows and these stylized technological descriptions.

The counterpart to the optimization problem in (4) becomes:

$$\underset{C,L,X,E}{\text{Maximize}} \quad u \ln(C) - bL^2 \tag{44}$$

$$\text{subject to} \qquad C + X = aL^{1-s}E^s \quad ,$$

$$E = hX^f \quad .$$

Using our results and the general framework of the network of process models, we can formulate this combined energy model according to the design of Figure 4. We introduced a price for every physical flow, and created the links for the value flows accounting for the payments to the consumer, π_X and π_E. With a single period model, the price for the value flows, P_π, plays no role in the solution of any component model. (Recall that this scaling factor will be important in dynamic models if we wish to control the numeraire price in each period.)

Suppose for this problem that we elect to use labor as the numeraire good, setting the price of labor equal to one. For the consumer process, the optimality conditions are:

$$\frac{u}{C} - P_\pi P_C = 0 \quad ,$$

$$-2bL + P_\pi P_L = 0 \quad .$$

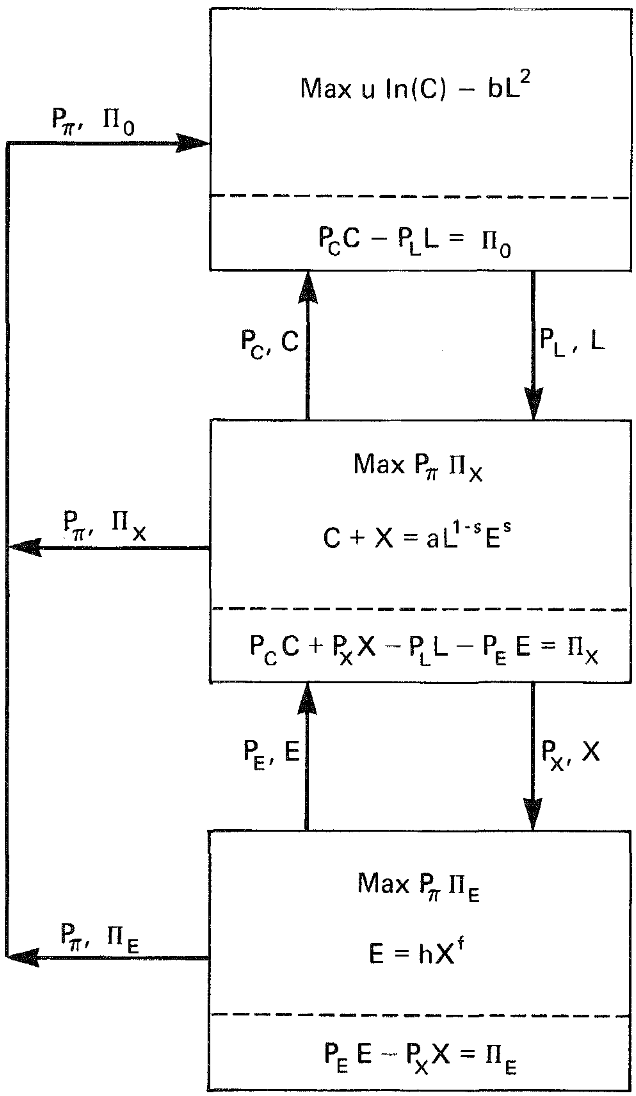


Figure 4. EXAMPLE COMBINED ENERGY MODEL NETWORK FORMULATION

If we assume that the output quantity and input prices are fixed each time we solve the consumer process model, then we can solve for the input quantities and output prices. These solutions are:

$$
\begin{aligned}
P_L &= 1 \quad , \\
P_\pi &= 2bL \quad , \qquad (45) \\
C &= \frac{u}{2bLP_C} \quad .
\end{aligned}
$$

The optimality conditions for the industrial process model are:

$$
\begin{aligned}
P_C - \lambda &= 0 \quad , \\
P_X - \lambda &= 0 \quad , \\
-P_L + \lambda \frac{(1-s)aL^{1-s}E^s}{L} &= 0 \quad , \\
-P_C + \lambda \frac{saL^{1-s}E^s}{E} &= 0 \quad ,
\end{aligned}
$$

where λ is the lagrange multiplier for the production constraint.

Again, fixing output quantities and input prices and solving for input quantities and output prices yields*:

*Let $Y = C + X$. Then $P_Y = P_C$. Now P_Y is equal to the unit cost function and, by Shephard's lemma,

$$\frac{\partial \ln P_Y}{\partial \ln P_L} = \frac{P_L L}{P_Y Y} \quad ,$$

and

$$\frac{\partial \ln P_Y}{\partial \ln P_E} = \frac{P_E E}{P_Y Y} \quad .$$

$$
\begin{aligned}
P_C &= P_X = dP_L^{1-s}\, P_E^{1-s} \quad , \\
L &= (1-s)(P_C C + P_X X)\, P_L^{-1} \quad , \\
E &= s(P_C C + P_X X)\, P_E^{-1} \quad , \\
\text{where} \qquad d &= a^{-1}(1-s)^{-(1-s)} s^{-s} \quad .
\end{aligned}
\tag{46}
$$

Finally, for the energy model, the optimality conditions are:

$$
\begin{aligned}
P_E - \lambda &= 0 \quad , \\
-P_X + \lambda f h X^{f-1} &= 0 \quad ,
\end{aligned}
$$

where λ is now the lagrange multiplier for the energy constraint.

Again, solving for output price and input quantities yields:

$$
\begin{aligned}
X &= \left(\frac{E}{h}\right)^{1/f} \quad , \\
P_E &= \frac{P_X X^{1-f}}{fh} \quad .
\end{aligned}
\tag{47}
$$

---

*(cont'd.)

From the optimality conditions, we have

$$
\frac{P_E E}{P_Y Y} = s \quad , \quad \frac{P_L L}{P_Y Y} = 1 - s \quad .
$$

Therefore, ln P_Y is linear in ln P_L and ln P_E. To determine the coefficient d, note that

$$
ad(P_L L)^{1-s}(P_E E)^s = P_Y Y \quad .
$$

Therefore, $\quad ad(1-s)^{1-s}(s)^s = 1 \quad .$

In a real model, equations (45), (46) and (47) might be the original formulation of the component models. But we see they are equivalent to (11) and (12). In the case of the industrial and the energy process models, these equations amount to nothing more than a demand function and a cost function. We evaluate the demand function for fixed output and we compute the cost function given the input prices. This equivalent characterization of the network of process models would appear as in Figure 5.

Given an initial estimate of the solution to this combined model, we could visit the component models sequentially, and obtain new estimates of the optimal outputs, inputs and prices. Following the cobweb approach, we could continue this iteration until the inputs, outputs and prices did not change for a full cycle through the network. This condition would be a constructive test of the solution.

The highly non-linear character of the energy process model, particularly for low values of the elasticity, f, of output with respect to the input, will make this component model troublesome in a straightforward cobweb iteration. We can illustrate the application of the local equilibrium approach by modifying this component model to include approximations of the response of the rest of the system to a change in the supply of energy or a change in the demand for goods and services delivered to the energy process model. Suppose we use constant elasticity approximations to characterize this reaction of the remainder of the model. Given an initial estimate of the solution, $(\bar{P}_X,\bar{X})$ and $(\bar{P}_Y,\bar{Y})$, the approximating functions would be:

$$P_X(X,\bar{X}) = \bar{P}_X\left(\frac{X}{\bar{X}}\right)^{\delta} ,$$

$$P_E(E,\bar{E}) = \bar{P}_E\left(\frac{E}{\bar{E}}\right)^{-\beta} ,$$

where β^{-1} is the elasticity of demand for the rest of the system and δ^{-1} is the estimated elasticity of supply for the remainder of the system.

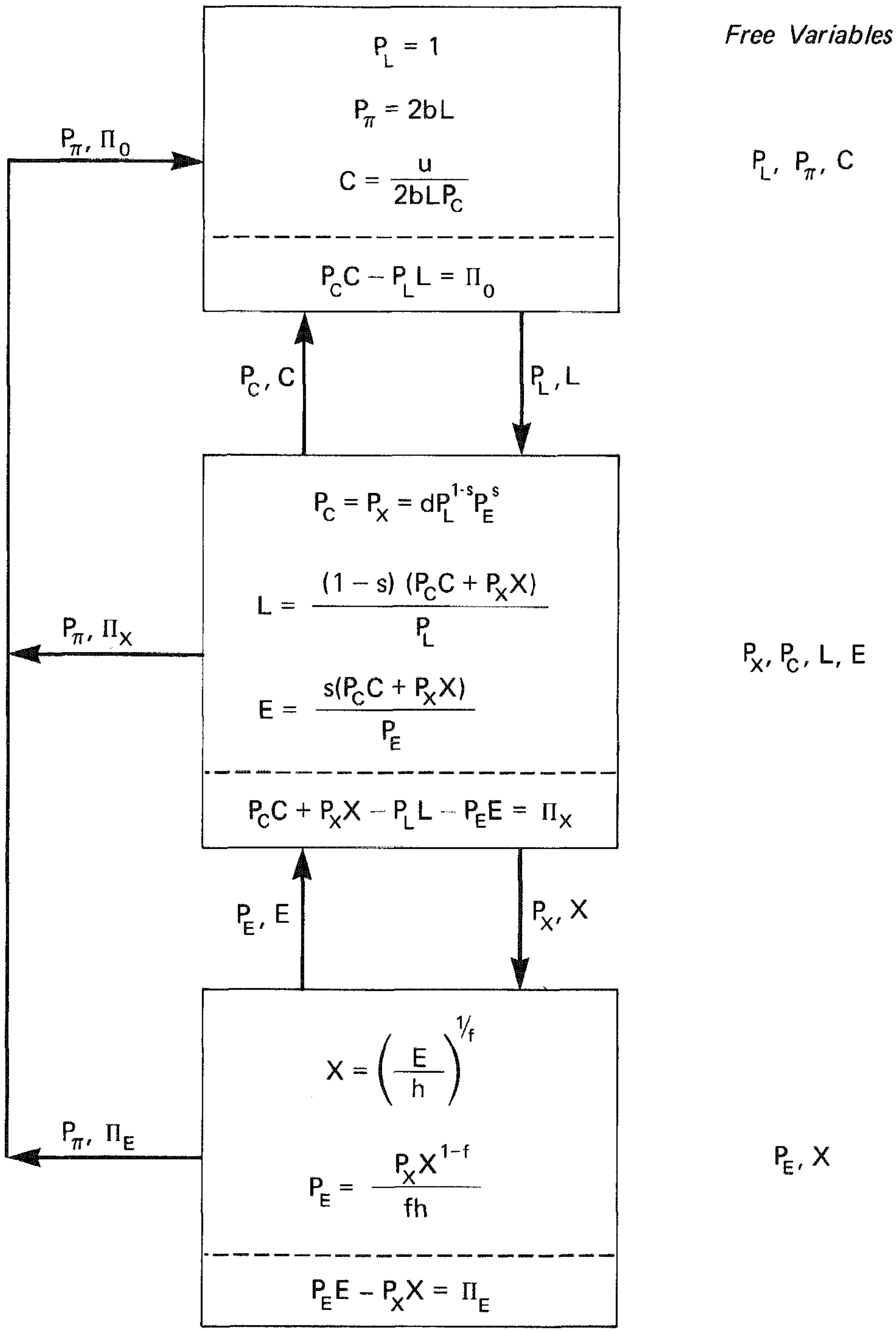


Figure 5. EXAMPLE COMBINED ENERGY MODEL NETWORK

The cobweb solution approach corresponds to an assumption that β equals infinity and δ equals zero. Since the goods and services provided to the energy sector are a small part of the total output of the industrial process model, we would expect the true value of δ to be close to zero and, therefore, we elect to use the value of δ equal to .05. Pretending that we know little about the demand side response, we assume that energy demand is relatively price inelastic, with a price elasticity of .5; this corresponds to a β of 2.

With these approximating functions, the optimality conditions for the local equilibrium problem for the energy process model are:

$$P_E(E,\bar{E}) = \lambda \quad ,$$

$$P_X(X,\bar{X}) = \lambda f h X^{f-1} \quad ,$$

where λ is the lagrange multiplier for the energy constraint.

The solution of these optimality conditions for the prices and quantities is:

$$X = \left[\frac{\bar{P}_E(\frac{h}{\bar{E}})^{-\beta} fh}{\bar{P}_X(\bar{X})^{-\delta}} \right]^{\frac{1}{\delta+\beta f+1-f}} \quad ,$$

$$E = hX^f \quad ,$$

$$P_E = \bar{P}_E(\frac{E}{\bar{E}})^{-\beta} ,$$

$$P_X = \bar{P}_X(\frac{X}{\bar{X}})^{\delta} .$$

Figure 6 summarizes this revised version of the combined energy model.

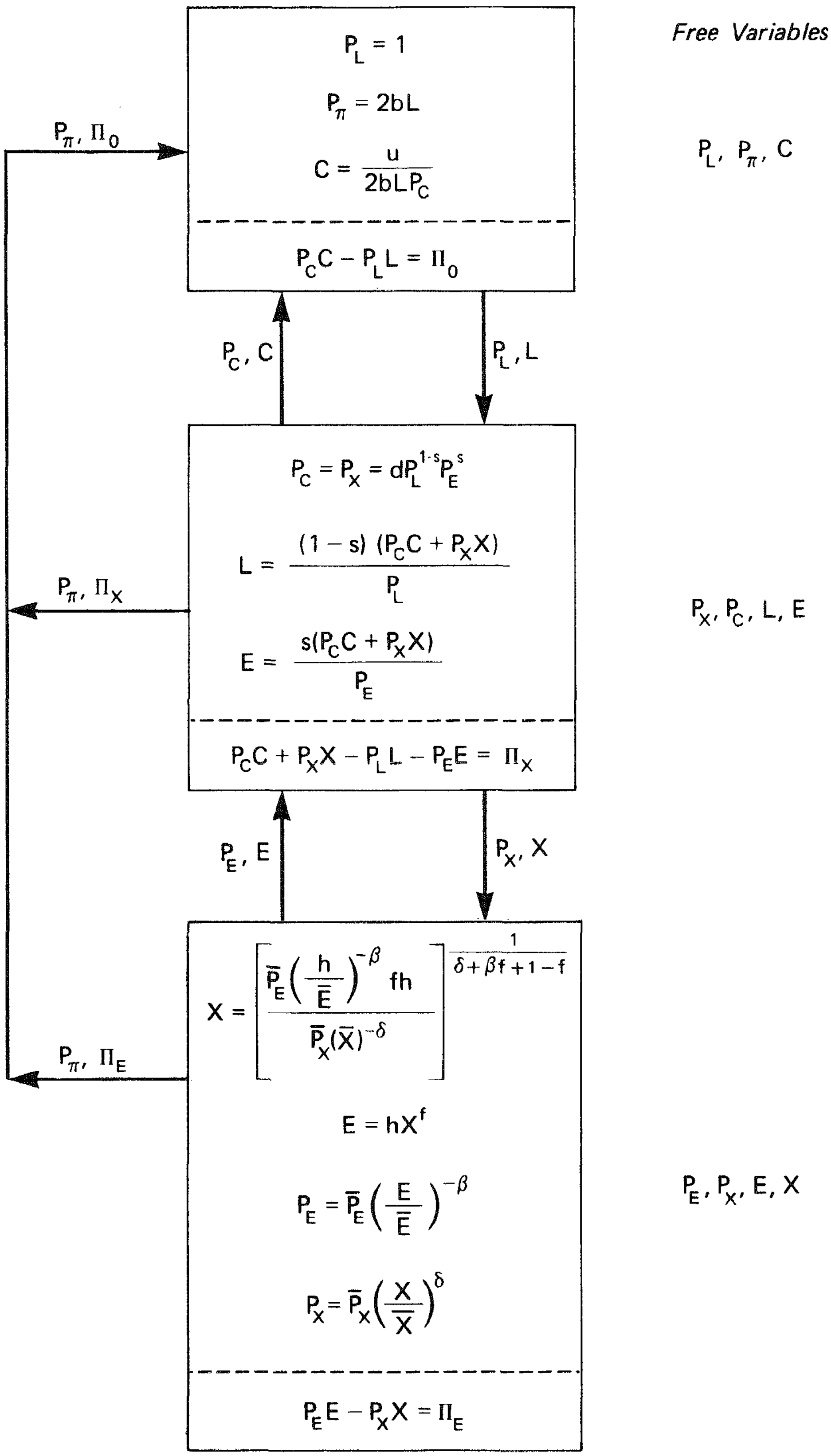


Figure 6. EXAMPLE COMBINED ENERGY MODEL NETWORK LOCAL EQUILIBRIUM APPROXIMATION

Table 1 contains the parameters and initial solution estimate for a number of experiments with the combined energy models of Figure 5 and Figure 6. In solving the model as identified in Figure 5, we used the simple cobweb procedure, without any attempts to design optimal relaxation steps. Likewise, for the problem in Figure 6, we used the cobweb approach with the full adjustment of all the prices and quantities every time we solve the local equilibrium of the energy model. Each problem was solved for three different values of the coefficient f, the elasticity of energy output with respect to the input of goods and services. The three values correspond to increasingly non-linear relationships with f equal to .9, .5, and .25 respectively.

Table 2 summarizes the computational results of the six different solutions. We see that the simple cobweb model worked only in the case of f equal to .9. The cobweb model was converging slowly in the case of f equal to .5. It oscillated wildly in the case of f equal to .25. The use of the local equilibrium approximation for the energy sector alone achieves convergence in all three cases. The extension of the local equilibrium approximation to the consumer and industrial models presumably would further improve the numerical stability in this example. Table 3 summarizes the first few iterations of the problem in Figure 6.

CASES AND EXAMPLES

There is a large number of examples of large-scale modeling efforts undertaken through the application of the principles of building combined energy models. The detailed discussion of these models can be found in the available reference material. Here we outline the interpretation of these cases and examples within our framework for combined energy models. Two of the earliest and most interesting cases examine the economies of Mexico and the Ivory Coast, with explicit inclusion of separate descriptions of a number of sectors in their economies.

TABLE 1

EXAMPLE PROBLEM:
PARAMETERS AND STARTING POINTS

Parameters

With $C = 2200,\ E = 80,\ X = 2Ef,\ L = 2040 + X$

then $f = .9,\ .5,\ .25$

$s = .07$

$a = (C+X)/(L^{1-s}E^{s})$

$b = .00024$

$d = a^{-1}(1-s)^{-(1-s)}s^{-s}$

$u = 2200$

$h = E/X^{f}$

Good Starting Point

$C = 2200,\ E = 80,\ X = 2Ef,\ L = 1500$

$PC = 1,\quad PE = 2,\ PX = 1,\quad PL = 1$

Bad Starting Point

$C = 2200,\ E = 80,\ X = 2Ef,\ L = 1500$

$PC = .5,\quad PX = .5,\ PE = 1,\ PL = 1$

TABLE 2

EXAMPLE PROBLEM:
NUMBER OF ITERATIONS FOR THREE PLACE ACCURACY

Method	Elasticity f	Good Start	Bad Start
Cobweb (Figure 5)	.9	27	26
	.5	>50	>50
	.25	Diverged	Diverged
Cobweb With Local Equilibrium (Figure 6)	.9	26	28
	.5	13	11
	.25	27	32

TABLE 3

SAMPLE ITERATIONS FOR FIGURE 6
FOR BAD START, f = .9*

	Start	C1**	I1	E1	C2	I2	E2	C3	I3	E3	Solution***
C	2200	6111	•	•	868	•	•	3369	•	•	2140
X	144	•	•	562	•	•	86	•		231	145
E	80	•	417	273	•	43	50	•	137	122	80
L	1500	•	5541	•	•	1354	•	•	3183	•	2125
PC	.5	•	.95	•	•	1.01	•	•	.99	•	1.0
PX	.5	•	.95	1.02	•	1.01	.92	•	.99	1.04	1.0
PE	1.0	•	•	2.33	•	•	1.75	•	•	2.18	2.0
PL	1.0	•	•	•	•	•	•	•	•	•	1.0

* • means no change.

** Ci: Consumer model at iteration i
Ii: Industry model at iteration i
Ei: Energy model at iteration i

*** After 28 iterations.

A stark example of a combined model, applied to economic problems, is the LINK system describing international trade. In this overall model, the component models represent the economies of individual countries. The component models are built, maintained and operated by modelers in the separate countries; the only communication between the models is through exchange of data on international trade.

The Mid-Range Energy Forecasting System (MREFS), formerly Project Independence Evaluation System, of the Department of Energy, is a major example of an energy model constructed by linking separate models of individual components. The MREFS model includes econometric models of energy demand, linear programming models of refinery operations and electric power generation, network optimization models of transportation of energy fuels, engineering process models of investment and production for energy supply development, and an iterative procedure for achieving equilibrium of the overall network.

The abstract structure of the MREFS model, interpreted in the context of the network of process models, shows a remarkable similarity to the SRI-Gulf model and its descendants as developed by Decision Focus, Inc. The creators of the SRI-Gulf model first conceived of the extensive use of a network of process models and utilized this approach in designing that system. Another of the most successful examples of the integration of an energy model with a model of the complete economy is found in the work with Hudson-Jorgenson model, in cooperation with Brookhaven National Laboratories. Here we see one model, dominated by the methodology and techniques of econometrics, used to describe the bulk of the economy and then linked with a linear programming model of the energy sector defined through the careful engineering analysis of energy technology.

The same Hudson-Jorgenson model was also linked to the Baughman-Joskow model of the electric utility sector. This is important in demonstrating the flexibility and modularity that comes with the combined energy model approach, and is a case of particular interest to the electric power industry.

From Mexico to the Ivory Coast

The studies on Mexico, in Goreux and Manne (1973), and on the Ivory Coast, in Goreux (1978), represent instructive examples of the evolution of large-scale combined models used to describe developing economies. The World Bank conducted both studies to build and apply detailed models of these two national economies to aid in development planning. Although both models cover the energy sectors in the respective economies, our present interest in the models is methodological, not substantive. In each case the models of the economies grew by building separate models of the component sectors. The several component models link to create combined models of the Mexican and Ivorian economies. In the case of the Mexican study, the integration of the models was not part of the original system design, and a number of conceptual and methodological problems arose. This experience was reported as a benefit in the Ivory Coast study for which the integration of the component models was part of the original system design.

The Mexican study began as an effort to build a number of related, but separate models of key sectors of the Mexican economy:

i.) DINAMICO; a dynamic model of the macroeconomy;
ii.) ENERGETICOS; a dynamic model of the energy sector;
iii.) CHAC; a static model of the Mexican agricultural sector;
iv.) PACIFICO and BAJIO; models of agricultural regions;
v.) INTERCON; a regionalized model for investments in electricity generating plants and transmission lines.

The modelers built these component models with the apparent intention of using them separately. But they soon recognized that the inputs of one model were the outputs of other models; they could learn more about the behavior of the Mexican economy if the models were linked together. But the integration of the models was constrained because the design of the component models was fixed before the integration was envisioned.

The case studies describe a number of alternative approaches to the partial linkage of the component models. For example, the linkage between DINAMICO and CHAC was nearly complete. Viewing CHAC as a more detailed description of the agricultural sector in the macroeconomic model, the modelers built a grid approximation of CHAC and embedded this approximation in DINAMICO. Using the shadow prices for capital, labor, and foreign exchange, the modelers solved the CHAC model for twenty-four different factor price combinations. The optimal solution in each case was treated as an input-output vector for aggregation into the linear programming representation of DINAMICO. Assuming that the feasible set in CHAC was convex, and limiting the choices in DINAMICO to convex combinations of the twenty-four feasible points, this process yielded a nearly complete integration of CHAC and DINAMICO. A similar grid approximation served to link the regional submodels of PACIFICO. Thirty-two price vectors generated thirty-two solutions for each of the five submodels of PACIFICO. These feasible points combined in the optimization problem to produce an integrated model for the northwest region of Mexico.

In contrast, the linkage between CHAC and BAJIO was far less complete, with only a one-way connection, from CHAC to BAJIO; this amounts to treating BAJIO as a disaggregation of CHAC aggregate results, not CHAC as an aggregation of the detailed BAJIO results. The asymmetry between linkages was repeated in the combination of DINAMICO, ENERGETICO and INTERCON. There is a complete linkage between ENERGETICO and DINAMICO through the insertion in DINAMICO of a grid approximation of ENERGETICO. The prices from ENERGETICO are passed to INTERCON, but there is no feedback from the electricity sector model to the aggregate energy sector model.

The experience with the model of the Mexican economy bore fruit in the design of the model of the Ivory Coast. Now the intent to link the models was part of the original planning of the modelers. They built the component models as detailed systems that would be aggregated into a larger description of the macroeconomy. Although we could cast the resulting model in terms of a network of process models, to be solved through an iterative search, this is not the approach used by Goreux et al. Rather they adopted the optimization framework of (4) and built up the model as an aggregate linear programming approximation to the nonlinear optimization of the utility of consumption.

The central model is a dynamic optimization model with four five-year time periods. The central model includes a collection of compact representations of six satellite models:

(i.) Rural-South;
(ii.) Rural-North;
(iii.) Livestock;
(iv.) Special Projects;
(v.) Urban; and
(vi.) Education.

There were different approaches to linking these several models. Each of the three rural sector models, for example, (Rural-South, Rural-North, and Livestock) was a dynamic linear programming model with several hundred constraints. Most of these constraints, however, concerned limits on the availability of sector specific resources. Therefore, to construct a summary representation of the rural models, the modelers solved each for three alternative central resource price vectors and generated three alternative central resource technology vectors. They included in the central model these feasible solutions to the specific rural models along with a convexity constraint to preserve the feasibility of the grid approximation.

A different approach applied for the Urban sector. Production in the Urban sector followed a fixed coefficient technology with an input-output matrix $\overline{A}$. Thus, the supply-demand balance for urban sector goods could be expressed as:

$$(I - \overline{A})\, x - Fy = 0,$$

where x are supply activities and y are demand activities. Further, the modelers represented the resource constraints of the urban model as

$$Rx + Ky \leq \overline{K} ,$$

where $\overline{K}$ was the stock of resources. These two relations represent the full urban sector technology. This led to a reduced form for the urban model of:

$$R(I - \overline{A})^{-1} Fy + Ky \leq \overline{K}$$

which contains less than half the number of constraints of the full urban model. This compact system could enter directly in the central model. Similar approaches were used for the remaining sector to produce a fully integrated model.

The Mexican and Ivory Coast examples represent not only evidence of the use of combined models to build large-scale systems but also of the flexibility of grid and function approximation techniques. When applied directly to (4), along with the great power to build adequate linear approximations to nonlinear optimization problems, the optimization framework offers attractions as an integrating device. In fact, we can embed this integrating mechanism within our network of process models. Whenever there is a hierarchical structure, grid approximation and linear programming optimization will be a competitive alternative to the simple iteration through a network of completely separate process models.

Project Link

The Project LINK system is a stark example of a combined model applied to the analysis of international economic problems. The LINK system arose because of an opportunity and a need created by the extensive work in the development of national econometric models for the major countries of the developed world. Being national models, these systems typically take as exogenous some or all of import-export quantities and import-export prices. The solution of the national economic model depends, often crucially, on the assumptions regarding these exogenous trade variables. But when operated independently, there was no mechanism to insure that the assumptions made for one model were consistent with those made for others. There was a clear need to develop a mechanism for balancing the import and export flows in an international trade model.

Since the exports of one country are the imports of another, the existence of a large number of national economic models created the opportunity to link these models together and balance the flows in an international trade study. The LINK project teams seized this opportunity. Using existing models where possible, and building special models where existing models were not available, the LINK teams constructed a comprehensive combined model of world economic activity. By 1974 the model grew to include thirty-one nations or regions. For twelve of these nations, the major developed market economies of the west, full-blown structural models described the activities in the respective economies. For the remaining regions of the world, smaller, reduced-form models represented the demand for imports and the supply of exports. The interested reader can find the details of these models in Ball (1973), with an overview of the operation of the system and its applications in Hickman (1974).

The Project LINK model is an extreme version of a combined model because the twelve major systems representing the developed market economies are built and maintained by research teams located in or near each of the subject countries. Hence, the component models of the combined system represent complete modularity and decentralized implementation. The only communication between the models is in terms of the quantity flows of exports and imports and their associated prices. The central team iterates between these models in the search for an equilibrium solution that includes a set of prices balancing the quantity flows of exports and imports for each of several traded commodities. In the earliest implementations of Project LINK, the models even resided in the subject countries. But later, when all the models existed in one location, the only communication was still through the model of the trade market.

The schematic in Figure 7 summarizes the general network structure of the Project LINK model. As is evident from this diagram, a major simplification of the model is to treat all export and import flows as passing through a central

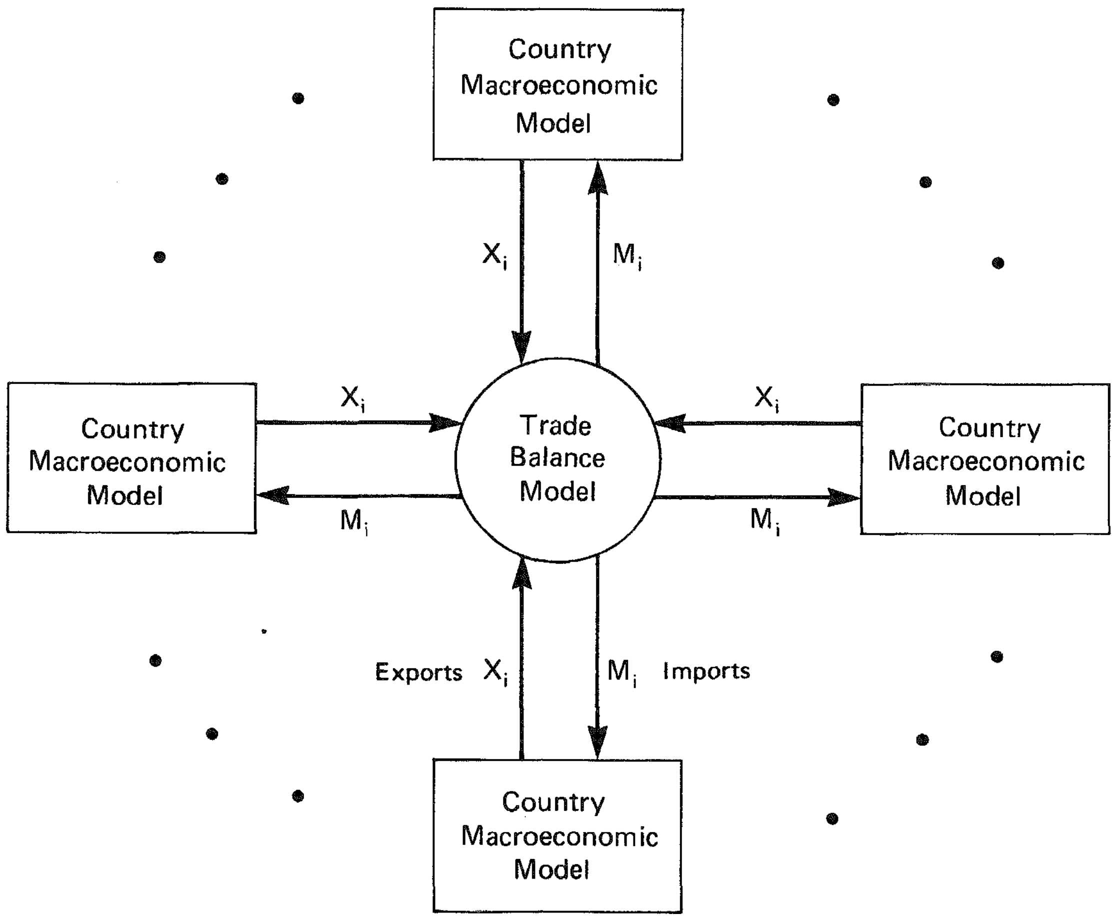


Figure 7. SCHEMATIC NETWORK OF LINK COMBINED INTERNATIONAL TRADE MODEL

trade balancing model. Bilateral flows between countries are modeled only through this indirect process. For our purposes, we can view the content of the trade balance model as a production function with inputs consisting of the country exports of any one of the several commodities and outputs consisting of the country imports of the same commodity. The production function and the prices of exports determine the allocation of the aggregate demand for exports into the demand for the exports of each particular country for each particular commodity. Normalizing ex post for the total quantity of exports, this amounts to determining the shares of total market allocated to the exports of each country. The shares are a function of initial shares and prices. This simplification allows the aggregation of exports and imports from each national model and avoids the complexity of establishing the more complete bilateral trade conditions for the full international market.

The procedure for solving the LINK model, through the trade balancing model, amounts to a direct application of a variant of the iteration of the network model. Starting from an initial estimate of country export prices and country import demand, the trade model is solved to determine the allocation of export demand to each country and the price of imports for each commodity. Then taking the price of imports and the demand for exports as given, the thirty-one regional models are each solved to determine export prices and import demands. And the process repeats through the trade balance model. Given a good initial estimate of the solution, the procedure converges rapidly to a balanced trade general equilibrium solution for the international economy.

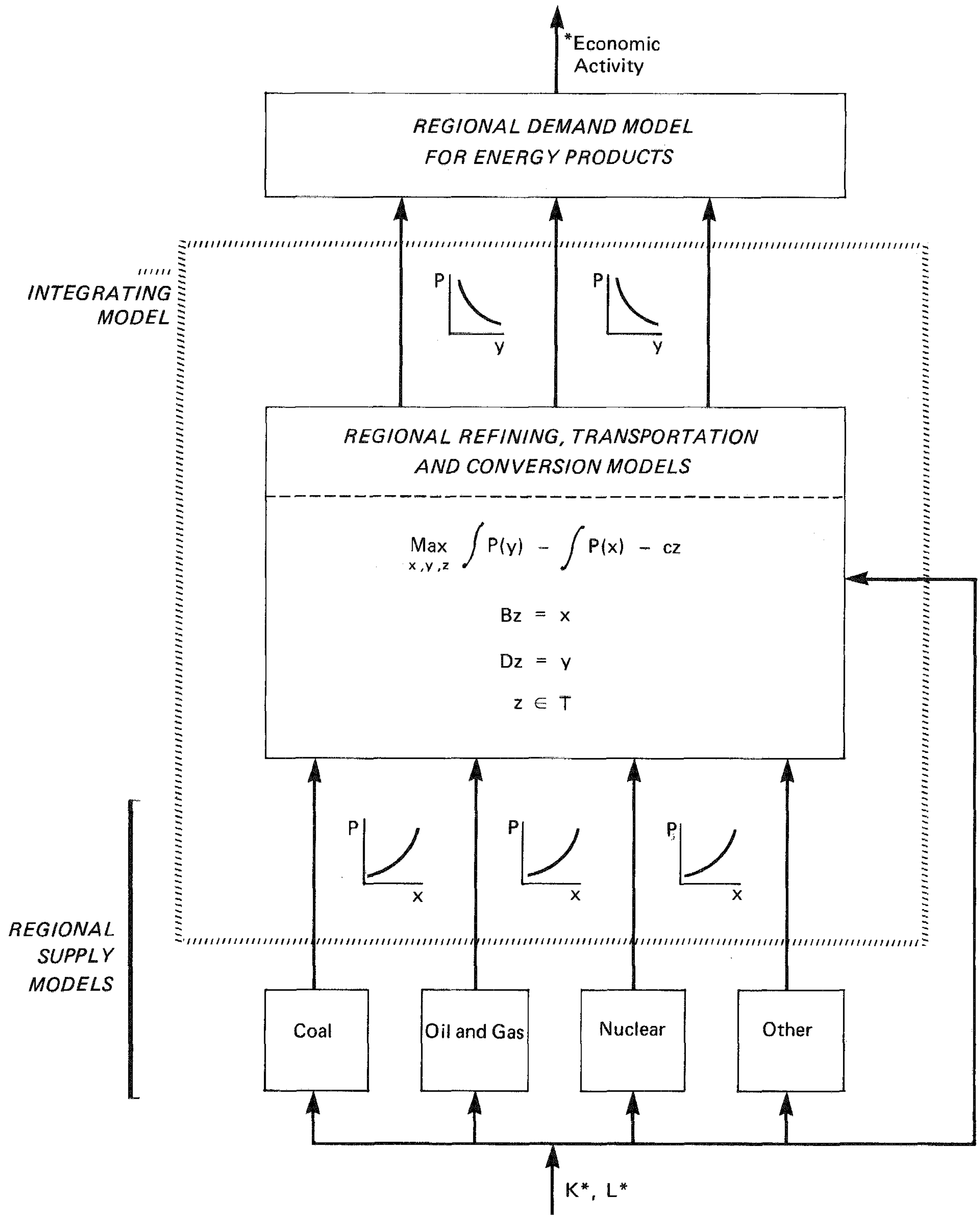
Although not strictly an energy model, the methodology employed in Project LINK is almost identical to that outlined in the present report. The favorable implementation experience provides an encouraging analogy for the study of energy-economic models. And the experience with algorithms for combined energy models, e.g., the local equilibrium approximation, may offer improvements for the LINK system.

Mid-Range Energy Forecasting System

The Energy Information Administration of the Department of Energy maintains and operates over sixty different models applicable to the energy system; see EIA (1978). Many of these models are partial equilibrium descriptions of sectors of the energy system. There are separate models of coal supply, oil and gas supply, electric power conversion, refinery operations, and the demand for energy products by region and over time. We can use these models in isolation, but the most interesting applications examine the competition between alternative sources of energy supply in meeting a range of energy demands. To analyze such problems, the Energy Information Administration uses the Mid-Range Energy Forecasting System (MREFS), a combined energy model built up from the component supply and demand models.

This modeling system, formerly known as the Project Independence Evaluation System (PIES), is described in Hogan (1975) and Hogan et al. (1978). Figure 8 provides a schematic diagram of the system, emphasizing the network structure and the use of the principles of combined energy models. The combined system is a partial equilibrium model in that we assume the prices of capital and labor services are fixed with infinite quantities available at these prices. Likewise, the aggregate level of economic activity is fixed no matter what the price of energy products. Within these few limitations, the PIES model finds a complete static inter-spatial and inter-product equilibrium for the energy sector.

The component models for regional energy supplies, refining, transportation, conversion, and energy demand have different methodological orientations and different means of implementation. The supply models are region specific, dynamic models simulating the behavior of profit-maximizing producers operating in a competitive environment. The details of these models are contained in the documentation of the Energy Information Administration and are not of interest for our immediate purpose. What is important is that under reasonable assumptions about the level of activity and the possible prices, these models can be



KEY: K,L: Capital / Labor Services

*: Partial Equilibrium through fixed prices or quantities

Figure 8. SCHEMATIC NETWORK FOR PROJECT INDEPENDENCE EVALUATION SYSTEM COMBINED MODEL.

solved under a range of price levels to describe an upward-sloping supply curve for the energy materials produced in each year.

The regional demand models are behavioral models of consumer choices among fuels when confronted by the mix of prices for energy products. These models were estimated econometrically from historical data using a dynamic structure to describe the intertemporal response of consumers to changes in energy prices. Again, the details of the component models are described in the documentation of the Energy Information Administration. Under a given set of assumptions about economic activity and over a range of prices, these models can be solved to generate a sequence of demand curves for energy products consumed in each period. These curves take the form of a constant-elasticity representation of energy demand as a function of all prices. Hence, if y is the vector of product demand and P_y is the vector of product prices, we assume that the demand can be described by:

$$\ell n\ y = a + E\ \ell n\ P_y \quad ,$$

where E represents the matrix of elasticities derived from the dynamic demand model.

The refining, transportation, and energy conversion (i.e., electric utility models), are process models characterized through a grid approximation. The refining model, for example, consists of a series of modes of operation. Each mode is an extreme point of the feasible set of refinery inputs and outputs. The transportation links are represented explicitly. The electric utility model consists of a series of models much like the refining sector, with the input being the energy products consumed by electric utilities and the outputs being electric power in base, intermediate and peak segments of the load curve. These refining, transportation, and conversion models, and their grid approximations, are part of a cost-minimizing linear program. Given the inputs x and their prices P_x; the outputs y; and the activity levels for the refining, transportation, and conversion operations, z; this linear program can be represented generically as:

$$\begin{aligned} \underset{x,\, z}{\text{Minimize}} \quad & p_x x + cz \\ \text{subject to:} \quad & Bz = x, \\ & Dz = y, \\ & z \varepsilon T, \end{aligned}$$

where T is the polyhedral set describing all feasible modes of operation of the refining, transportation, and conversion activities, and c is the cost of each activity.

We can interpret this problem as the determination of the least-cost operation of the energy system in order to meet the final level of energy demand. Given fixed output and fixed output prices, this would be equivalent to maximizing the profits of the energy sector. The dual solution to this linear program will yield the marginal cost of meeting the energy demand, and these marginal costs can be interpreted as the prices of energy products.

In principle, we would iterate this model between the supply system and the demand system to update estimates of energy material inputs and prices and energy product demands and prices, while maintaining the optimal solution to the refining, transportation, and conversion models. An equilibrium solution, where the demands from the regional demand model at the prices generated by the marginal cost of the linear program balance within the technological constraints given the prices and supply of the energy materials, could be interpreted as an equilibrium solution for the energy sector.

The resulting model is highly modular. Over the first five years of use, the energy modelers improved or expanded nearly every component of the original system while maintaining the basic design of the combined model. Because of this modularity, the implementation is decentralized, with separate teams maintaining the component models around the natural organization of industry and government data. The PIES model meets our several criteria. Most surprisingly, this large combined energy model lent itself to an efficient solution procedure.

An attractive feature of the PIES model, from the perspective of combined energy models, was in the extensive use of the local equilibrium approximations in order to characterize the solution to the integrating model in Figure 8. In essence, the supply curves from the component regional supply models and the demand curves from the regional demand models were incorporated in the linear program to solve the related problem:

$$\begin{aligned} &\max_{y,\,x,\,z} \quad \int P(y) - \int P(x) - cz \\ &\text{subject to:} \quad Bz = x \\ &\qquad\qquad\quad\; Dz = y \\ &\qquad\qquad\quad\; z \varepsilon T. \end{aligned} \tag{47}$$

In the implementation of this integration model, the approximation of the supply curves for the regional production of energy materials is extensive enough to be exact up to the level of accuracy of the supply model data. Hence, there is only one approximation constructed for these detailed models. For the demand model, however, the simplified approximation of the constant-elasticity demand curve ignores the interaction between energy products. This approximation, therefore, must be adjusted during the iteration of the integrating model in the search for an equilibrium solution. The algorithm proceeds by solving the optimization problem (4) to produce new estimates of the prices for product outputs. The product output estimates are recalculated using the full constant-elasticity model to generate a new approximation of the demand model. This step repeats with a new optimization of the local equilibrium problem and a new estimate of the product prices.

Eventually, the new estimates of the product prices equal the old, and, as we have seen, this produces a solution which satisfies the equilibrium conditions and, in the ideal normative case, would be a partial solution to the optimization problem. The computational experience in the PIES model is that this iterative procedure, which involves adjusting the prices and quantities for approximately 100 regional energy demands and solving at each iteration a linear program with more than a thousand constraints and several thousand variables, leads to acceptable convergence of the price and quantities in the demand regions after no more than ten to twenty iterations of the full system.

The PIES model is a descriptive, not a normative model. We can motivate the organization of the PIES network and the computation of the local equilibrium problem by calling upon our normative principles. But the actual PIES model deviates substantially from the assumptions necessary to interpret the final solution as optimal in the sense of utility maximization for an individual consumer. We know, for example, that the constant elasticity demand function used to represent demand behavior cannot be derived from utility maximization of an individual consumer; see Hogan (1975). Likewise, the refining, transportation and conversion activities in the integrating model are supplemented by a number of modifications to include the effects of regulation and taxation. These additional changes are easy to include within the network model and the general solution algorithm. In fact, it is apparent that the use of an optimization formulation to solve the integrating model at each step is merely a convenience. The optimization of the integrating model plays no role in the interpretation of the solution as an equilibrium. The descriptive modifications of the normative framework are easy to incorporate and have no significant effect on the implementation or performance of the solution algorithm. This robustness of the formulation and computations preserves the modularity of the combined model.

SRI-Gulf

The developers of the SRI-Gulf model began with a need to analyze alternative investments in synthetic fuel technologies. Recognizing the importance of fuel competition in determining the mix of energy demands and in estimating the value of any particular technology, the modelers soon moved to the construction of detailed, partial equilibrium description of the energy sector. This large-scale modeling effort is one of the earliest complete models describing the energy system. It is also one of the first energy models to adopt formally the network of process models approach to the characterization of the interaction among fuels across sectors. Cazalet (1977) and his co-workers saw the energy system as a network with energy goods flowing along the links between a sequence of a few generic processes.

Figure 9 contains an example segment of the SRI-Gulf model. The direction of flow of energy goods is from the bottom to the top of the network. The network contains three generic types of processes: allocation processes, conversion or transportation processes, and primary resource processes. The price-sensitive demands at the top of the network establish the link with the remainder of the economy. This demand curve is assumed to be fixed in the partial equilibrium implementation. Likewise, there are a series of flows, not shown in this network, of capital services, labor services and materials from the remainder of the economy for which the price is held constant and the supply is assumed to be infinite. Within these few constraints, the SRI-Gulf model determines an intertemporal, interspatial and interfuel equilibrium of the energy system.

The details of each generic process model vary according to the particular fuel being modeled or the type of conversion processes considered. But all fall into the three broad categories. In each case, models within these broad categories differ only in the description of the technology relating the inputs and outputs. All the processes attempt to approximate the results of profit maximization by a competitive producer constrained by the technology of the process model.

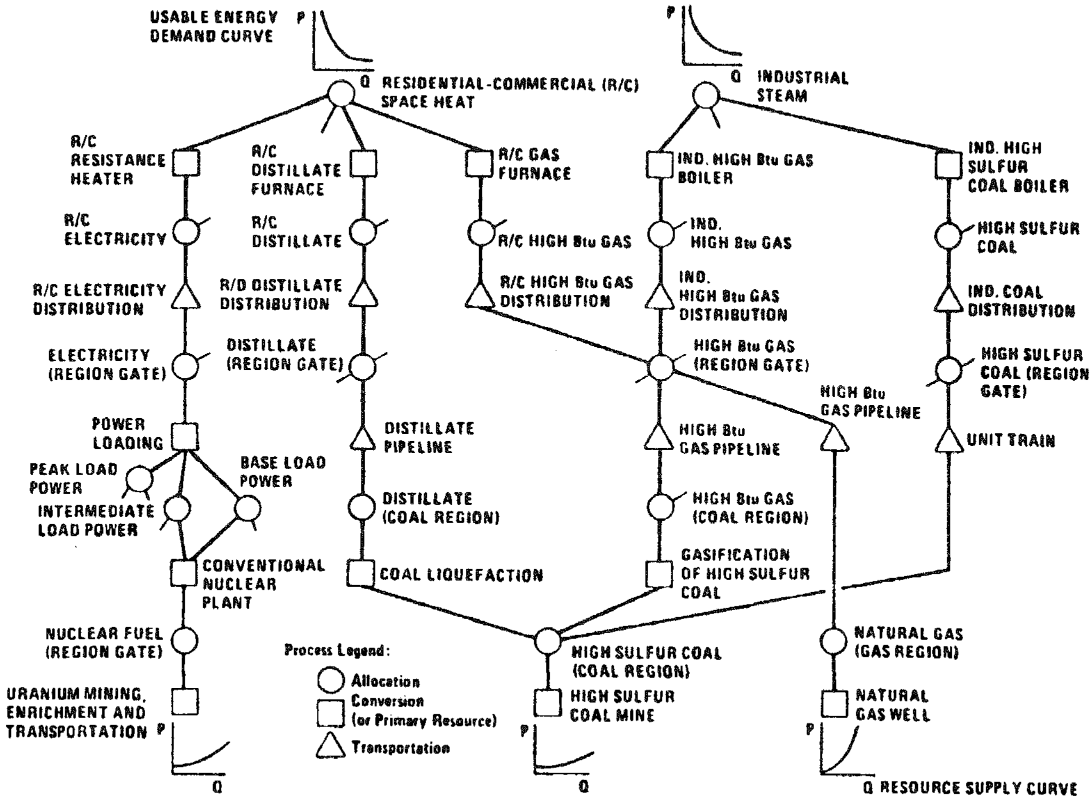


Figure 9. SCHEMATIC DIAGRAM OF THE NETWORK STRUCTURE OF THE SRI-GULF MODEL

The allocation model aggregates several inputs to produce a homogeneous output which is the sum of the demands for the products from that node. The implicit assumption is that there is a production function which determines the output as a function of the inputs and a corresponding cost function which can be utilized to determine the price of the output as a function of the prices of the input. Cazalet (1977) does not describe the detail of the underlying technology for this price function but, rather, develops an ad hoc approximation to the cost function and its derivatives. This approximation takes the form of a market share function where the market share, the ratio of the individual input to the aggregate output, is inversely related to the price of the input. Hence, if q is the aggregate output of the process and q_j is the input of the jth product, with price p_j, then:

$$\frac{q_j}{q} = \frac{p_j^{-\gamma}}{\sum_i p_i^{-\gamma}} \tag{49}$$

This market allocation function is a simplification of that in the SRI-Gulf model. The full implementation in the model includes price premium adjustments and lagged demand responses to changes in prices. But (49) captures the essence of the modeling approach to the disaggregation of any total demand into several input components. This aggregation rule, with different values of the elasticity parameter, γ, repeats throughout the network.

Given this allocation rule, the SRI-Gulf model computes the corresponding output price as the quantity-weighted average of the input prices. This sharing equation, of course, is only an approximation to some underlying demand derived from a consistent cost and production function characterization of the process technology. Given the appropriate econometric results for demand equations at any node in the network, therefore, it would be an easy matter to replace the form of this equation in the allocation node without upsetting the remainder of the combined model.

The conversion and transportation models also vary in the detail of their parameters. But all can be described as versions of a fixed coefficient model. Efficiency factors convert the energy inputs to energy outputs. The consumption of capital services, labor services and materials at fixed prices adds a mark-up to the cost of the input energy. By changing the efficiency and the cost parameters, we can describe a large number of different conversion processes. When considering the interaction with the allocation processes which provide the fuel substitution effects, a collection of conversion and allocation processes can combine into a rich description of the energy sector.

Again the full implementation in the SRI-Gulf model is more sophisticated than this simple abstraction suggests. Most conversion models, for example, include consideration of capacity expansion requirements, the effects of taxation and depreciation on the charges for capital services, and allow for adjustment lags in the speed of capacity expansion. However, these refinements do not upset the basic interpretation of the structure of the conversion processes nor do they change the nature of the algorithm used to search for a solution. It is clear, therefore, that any conversion process can be modified by providing a more elaborate description of the relationships between the inputs and the outputs. We could, for example, borrow from the PIES or the Ivory Coast models to include a linear programming description of the conversion processes for electric utilities and refineries. By introducing not one but many modes of operation and allowing convex combinations in the choice of at least cost mix, we could include substitution effects through a grid approximation of the conversion models. Given a procedure for solving the linear program at each iteration, we could embed these more elaborate models within the remainder of the SRI-Gulf model.

The resource process models capture the essential dynamics of the energy supply problem in allocating depletable sources of energy, with increasing production costs, over the planning horizon of the energy model solution. The resource model contains a description of the capital services, labor services, and materials required to extract the marginal unit of a depletable resource such as oil. Given the fixed prices for these inputs, the model is equivalent to a description of the marginal cost of extraction as a function of cumulative production. Confronted with a time profile of demand, y_t, the resource process model solves the problem:

$$\min_{x_t} \sum_t x_t \, C(Q_t)$$

$$x_t \geq y_t$$

where C is the marginal cost function and $Q_t = Q_{t-1} + y_t$. It is straightforward to show that if there is production at every period, the marginal cost of production, and therefore the output prices, in this model will satisfy the equation:

$$P_t = C(Q_t) + \max_{\tau > t} \frac{P_\tau - C(Q_t)}{(1 + r)^{\tau - t}} \quad .$$

where $C(Q_t)$ is the marginal resource cost of production in period t given the profile y_t, P_t is the resource price; and r is the producer's discount rate. The price and the marginal resource cost will be equal in the last period of the planning horizon (assuming that C(Q) is monotonically increasing and finite everywhere). In other periods, the difference between the price and the marginal resource cost is the economic rent associated with the depletable nature of the resource.

We recognize this cost minimizing problem as equivalent to profit maximization with fixed levels of output. And the resource model fits nicely into the normative framework of the combined energy model as a network of process models. Here again the modular characteristic of the SRI-Gulf network framework would allow us to substitute alternative resource pricing models without upsetting the nature of the remainder of the system. In the actual implementation, the costs and production rates are subject to behavioral lag and technological change effects which permit a richer description of the behavior of the energy system. But these are refinements that do not upset the basic interpretation of the combined model.

This early example of the flexibility of a network formulation of combined energy models has been very successful in use. Analysts have used the SRI-Gulf model extensively in the study of alternative energy problems, inserting new modules to describe special characteristics, such as regulation, for different decision applications. In this usage, we have accumulated a great deal of computational experience with this model. Cazalet (1977) summarizes:

> ...satisfactory convergence is achieved in ten to thirty iterations for a sensitivity case...and thirty to sixty iterations for a nominal case beginning with poor initial estimates of prices in quantities. Generally, the number of iterations is not affected by the size of the model, but is strongly affected by the character of the process relations and the associated input parameters...About 2700 processes are contained in the current model.

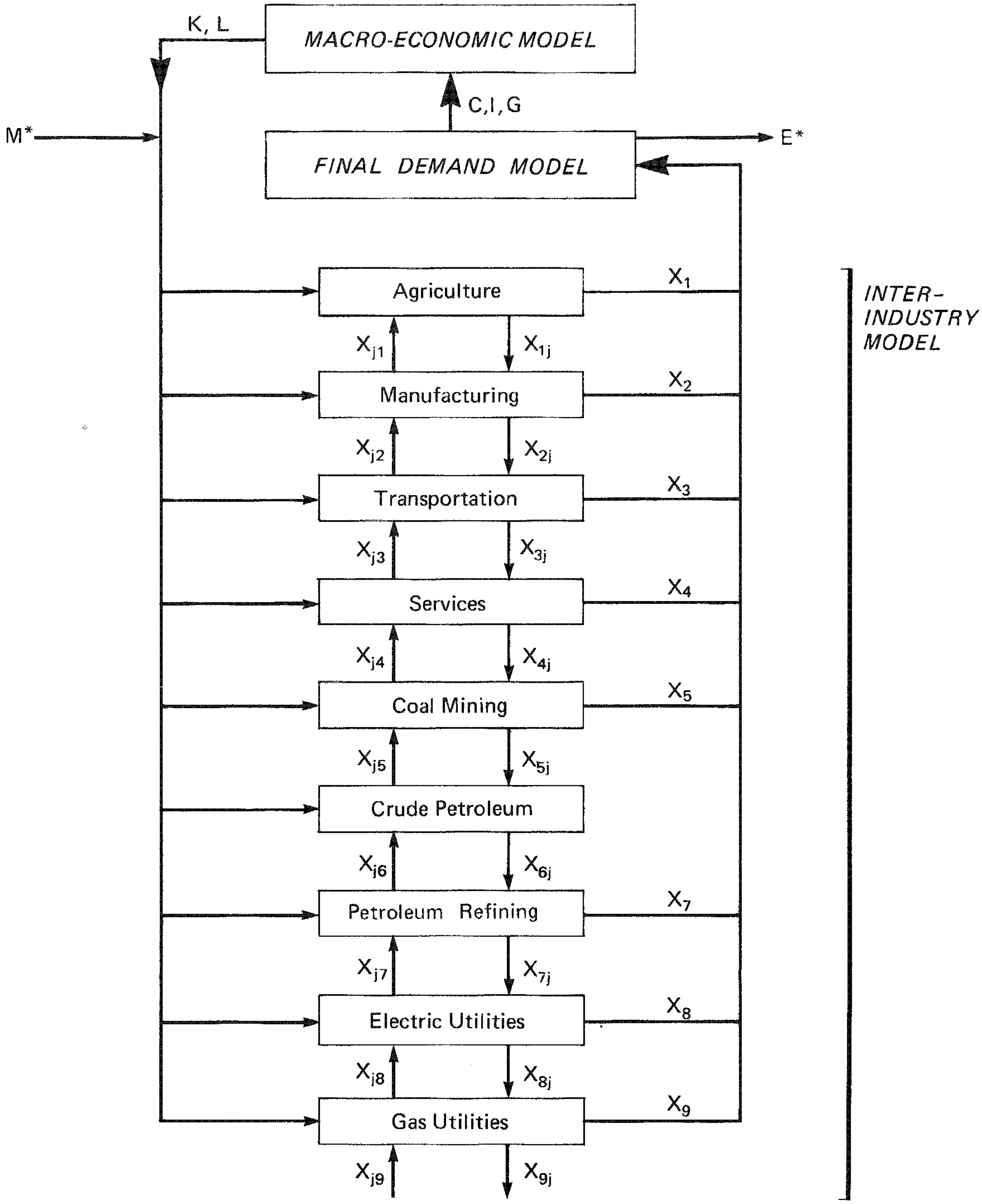
The basic algorithm used to solve the SRI-Gulf model is a variant of the method described above. Starting at the top of the network, the algorithm moves down through the process nodes, taking prices and output quantities as fixed and solving for input quantities at each stage. This sequence continues until the bottom of the network, with the resource models, is reached. The resource models take the demand as fixed and solve for the supply prices. The algorithm then moves up the network taking quantities and input prices as fixed and calculating output prices. This continues until the top of the network is reached at which time new demands are computed and the iteration starts anew. The model builders exploit relaxation techniques to tune the model to achieve rapid convergence in sensitivity tests. The success of this model, in its modularity, in its relative ease of description, and in the relatively inexpensive computational costs have led to the great interest in its use. It led to our adoption of the network approach to describing energy models.

The developers of the SRI-Gulf model have spawned two large efforts to produce specialized software built around the network approach; see Sussman and Rousseau (1978) and DFI (1978). This software capability should simplify the formulation and solution of combined energy models. The testing of this on new problems with new components models is a next step in the research on combined energy models.

Combining H-J with Energy Models

The model of the American economy described in Hudson and Jorgenson (1974) was an innovation in the development of neo-classical, general equilibrium descriptions of the economy with an explicit treatment of the energy sector. This model was still evolving several years after its first formulation. Produced under the sponsorship of the Ford Foundation, the model has been applied widely including studies by the Edison Electric Institute (1976), the Energy Research and Development Administration (1975 and 1976) and later the Department of Energy. In these applications the model has been combined with other energy models, providing more detailed representations of the energy sector.

The sketch in Figure 10 summarizes the conceptual outline of the Hudson-Jorgenson model in its earliest version. A macroeconomic model of aggregate consumption, investment, saving and labor, describes the consumer's behavioral balancing of the demand for goods and services and the supply for labor and savings. This macroeconomic component is based on a model of consumer utility maximization, modified to include the effects of taxation and government expenditures. Exports and imports are handled exogenously. This macroeconomic



KEY: X_{ij}:	Output of sector i for sector j
X_i :	Output to final demand
C, I, G:	Aggregate Consumption / Investment / Government
K, L:	Capital / Labor Services
M, E:	Imports / Exports
*:	Partial Equilibrium through fixed prices or quantities

Figure 10. SCHEMATIC NETWORK FOR HUDSON-JORGENSEN MODEL, WITH QUANTITY FLOWS.

model was estimated econometrically and serves as the main dynamic link of the Hudson-Jorgenson system.

Given the aggregate level of demand for consumption, investment, government expenditures, and exports, the next model disaggregates these flows into the demand for the outputs of various industrial sectors. This final demand model is based on the assumption that there is an aggregate function relating the input from the several industrial sectors to the four aggregate outputs. This function has a dual function over the prices. This dual function can be estimated econometrically and used to derive the final demands for input goods as a function of the demand for aggregate outputs and the price of inputs.

With the final demand for the outputs of the nine industrial sectors included in the model, the inter-industry model solves for the supply activities, the inter-industry flows of goods and services, and the demand for capital services, labor services, and imports to the producing sectors. The major innovation of the Hudson-Jorgenson model was in describing explicitly the substitution possibilities in each of these industrial sectors. Each sector is modeled as a technology summarized by a production function, homogeneous of degree one. The production function takes as inputs the flows of the other industrial sectors, plus capital services, labor services, and imports. For each production function there is a dual cost function, homogeneous of degree zero in the prices, and homogeneous of degree one in output, which determines the price of outputs as a function of the prices of inputs. Hudson and Jorgenson approximated the logarithm of the cost functions as a quadratic in the logarithms of the prices, and as linear in time. This is the "translog" cost frontier. The interaction between the logarithms of prices captures the substitution possibilities between input factors. The linear factor in time allows for the specification of a constant exponential rate of technological change. Each of these translog cost functions was estimated from empirical data using econometric methods.

The general form of the translog cost function is:

$$\ell n P_y = \alpha_0 + \sum_j \alpha_j \ \ell n \ P_j + \tfrac{1}{2} \sum_i \sum_j \beta_{ij} \ \ell n \ P_i \ \ell n \ P_j + \delta t \ .$$

where P_y is the unit cost function.

The key to understanding the use of these cost functions lies in Shephard's lemma; see Diewert (1974). We know that the shares of expenditures on inputs, x_i, for each output, y, must be the logarithmic derivatives of the cost function, i.e.,

$$s_i = \frac{P_i \ x_i}{P_y \ y} = \frac{\partial \ell n P_y}{\partial \ell n P_i} = \alpha_i + \sum_j \beta_{ij} \ \ell n P_j$$

Hence, with the parameters α_i and β_{ij} available from econometric evidence, the solution of the component model for input demands involves only the evaluation of the function:

$$x_i = s_i \frac{P_y \ y}{P_i} \qquad .$$

And the calculation of output prices reduces to the evaluation of the unit cost function.

This interindustry model can be solved in connection with the final demand model to find an overall equilibrium of the system. Hudson and Jorgenson solved this model as a system of simultaneous equations, using Newton-like methods of solving non-linear equations. However, the network structure of Figure 10 and the description of the simple calculations required to calculate input quantities or output prices for each component model suggest the possibility of applying a network iteration type algorithm to the solution of this system.

This brief overview of the Hudson-Jorgenson model sets the stage for the description of two further examples of combining energy and economic models. We have, however, failed to treat other strengths and weaknesses of the system. This successful model, partly because of its modularity, continues to evolve. See, for example, Jorgenson and Fraumeni (1979) for a later version that expands the model to thirty-six sectors and includes an important extension to allow the partially endogenous treatment of technological change.

BESOM The network interpretation of the Hudson-Jorgenson model suggests many possibilities for combining this model with other energy or economic models. The Hudson-Jorgenson model is, in principle, highly modular. We could remove any one of the component models and replace it with other representations of the same process as long as we preserve the definition of the variables connecting with other component models. Hoffman and Jorgenson (1977) reported on one such effort combining the Hudson and Jorgenson model with the Brookhaven Energy System Optimization Model (BESOM).

The BESOM model is a static technological description of substitution possibilities within the energy sector. Taking the inputs of primary energy resources, x, the input prices, P_x, and the demand for energy services, y, as given the BESOM model constructs a grid approximation to characterize the conversion and transformation, z, of primary energy resources, to meet the demands for energy services. The grid approximation results in a linear program:

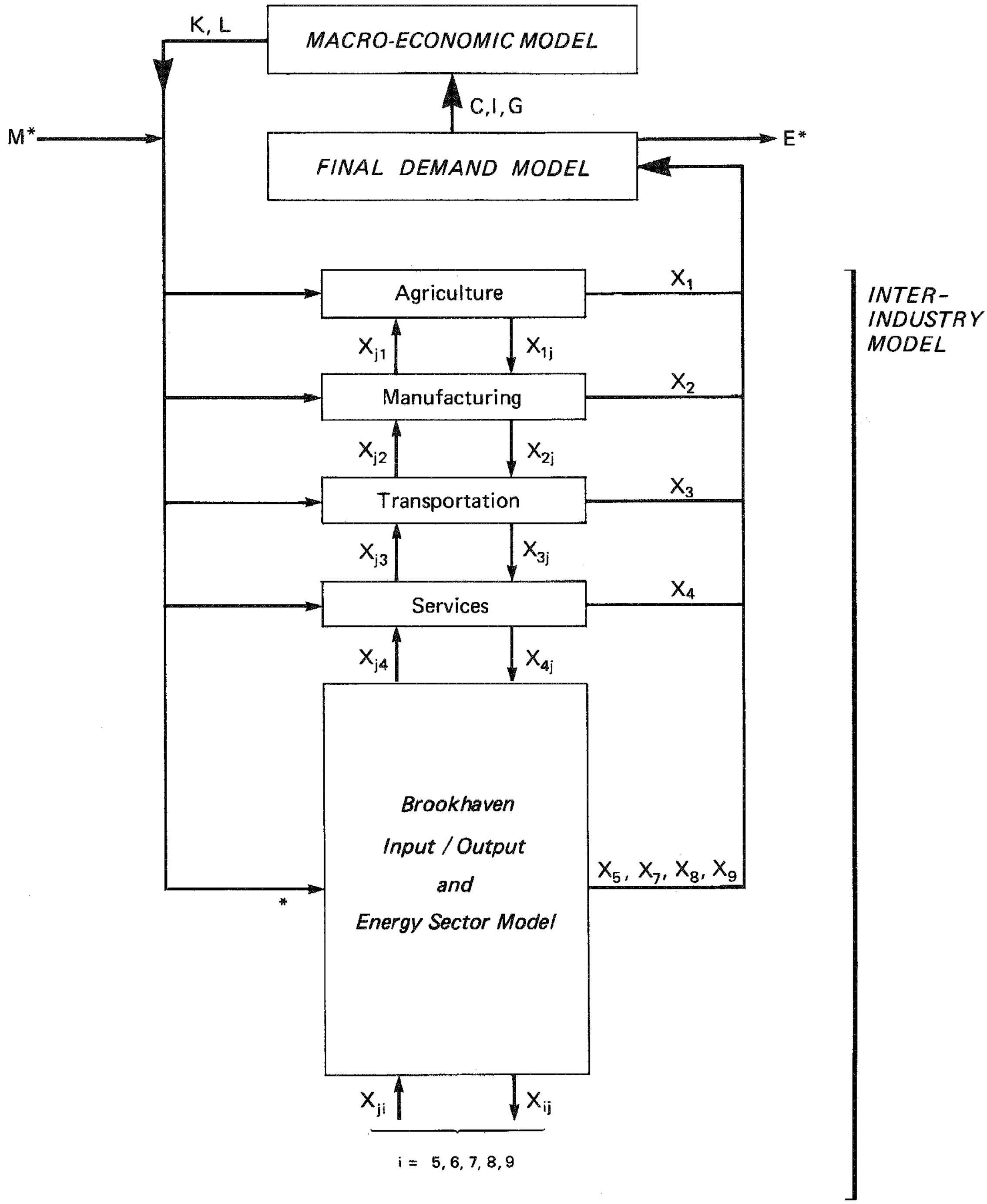
$$\begin{aligned} \min_{x,z} \quad & P_x x + cz \\ & Bx = z, \\ & Dz = y, \\ & z \varepsilon T. \end{aligned}$$

At this level of abstraction, BESOM is identical to the central model in PIES. The only difference is the regional structure in PIES.

This cost minimizing linear program finds the least-cost method of satisfying the energy service demands within the limitations of the available primary energy products. Being a more detailed description of the energy sector substitution possibilities, we might wish to use this model to characterize the energy sector in place of the translog cost functions of the original Hudson-Jorgenson model. Figure 11 illustrates the substitution of the BESOM models for the energy components of the interindustry model. The remainder of the Hudson-Jorgenson model--the other industrial sector and the final demand model--determine the demand for energy products. If we include a description of the energy resource supply within the BESOM model, then the optimization problem can be linked to the remainder of the system to calculate a new equilibrium.

We could search for the full equilibrium between the translog cost functions for the four industrial sectors and the least-cost solution for the energy activities in the BESOM model. Our iterative algorithm moving through the network would be one approach to seeking such an equilibrium. Hoffman and Jorgenson (1977) and Behling et al. (1976) employed a slightly different procedure to maintain compatibility with the solution method embedded in the Hudson-Jorgenson model. The BESOM model and the remainder of the Hudson-Jorgenson system were treated as two separate systems with an input-output model inbetween. When solving the BESOM model the demand for energy products was assumed to be fixed. The BESOM solution generated prices for the energy products and identified a set of technical coefficients describing the inputs and outputs of eleven energy resource sectors, twenty energy conversion processes, and sixteen energy products. These technical coefficients were used to change the corresponding elements of the input-output model and, using the same final demand vector, a new set of energy demand was generated. This process repeated until the technical coefficients in the input-output model and the resulting coefficients in BESOM matched for the consistent level of demand for energy products.

These technical coefficients and product prices were then passed to the Hudson-Jorgenson model and used as an approximate representation of the BESOM model.



KEY:

X_{ij}:	Output of sector i for sector j
X_i :	Output to final demand
C, I, G:	Aggregate Consumption / Investment / Government
K, L:	Capital / Labor Services
M, E:	Imports / Exports
*:	Partial Equilibrium through fixed prices or quantities

Figure 11. SCHEMATIC NETWORK FOR HUDSON-JORGENSEN / BROOKHAVEN COMBINED MODEL

This approximate representation was integrated into the Hudson-Jorgenson model to find a new equilibrium demand for energy products including the full adjustment of the remainder of the interindustry model. This demand for energy products and new set of coefficients for the other industrial sectors were returned to the BESOM model and the cycle repeated. The result is a single combined model with the substitution possibilities of the BESOM model embedded in the larger framework of substitution possibilities of the Hudson-Jorgenson model.

Baughman-Joskow* The combination of the Brookhaven model and the Hudson-Jorgenson model illustrates the possibility for replacing many sectors and achieving an equilibrium for the new model with an expanded description of the substitution possibilities in the energy system. In another combination of the Hudson-Jorgenson model with a different energy sector model, we see a single sector included in a way that reflects the best elements of each model.

The Baughman-Joskow model, described in Joskow and Baughman (1976) and later in Baughman et al. (1979), is a detailed model of the electric utility sector of the energy system. The Baughman-Joskow model includes a regional description of the electric utility industry in four major components:

(i.) a regional demand model which recognizes the substitution possibilities between energy products based on the level of economic activity, the price of electricity, and prices of competing energy products. The prices of the nonelectric energy products are set exogenously by the model user;

(ii.) a capacity planning model which develops an optimal plan for the expansion of electric utility generation capacity;

(iii.) a generation model which determines the plant-type and fuel requirements to meet electricity demand in any time period across the different segments of the load generation curve; and

(iv.) a financial regulatory model which describes the financing requirements of the industry to meet the capacity expansion plan and calculates the electricity prices that follow from the regulatory processes controlling the cost allocation among consumers.

*The outline of the Hudson-Jorgenson-Baughman-Joskow combined model is based on working notes kindly provided by Ed Hudson.

Both the capacity planning and the load generation models seek to minimize the cost of meeting a fixed level of demand. The capacity planning model is based on a forecast of demand, with the forecast updated each period. This leads to a new optimal capacity plan in each period. The generation model minimizes the cost of meeting the demand that actually develops in any period. Both of these models, therefore, fit well into the normative framework of the network of process models.

The financial-regulatory model, by contrast, is an ad hoc representation of the institutional structure of the electric utility industry. This model is purely descriptive and although it fits easily into the network framework, it takes us outside the realm of the normative interpretation of the solution.

The Baughman-Joskow model represents a comprehensive effort to describe the many components of the electric utility industry. This model has been the subject of special attention as an early focus for model assessment efforts; see Kuh and Wood (1979). It is a natural target for combination with a comprehensive model of the remainder of the energy system and the remainder of the economy.

The schematic linkage of the Baughman-Joskow model and the Hudson-Jorgenson model is summarized in Figure 12. Here we replace the electric utilities supply model of the Hudson-Jorgenson interindustry model with the capacity expansion, load generation, and financial-regulatory models of the Baughman-Joskow system. Symmetrically we replace the Baughman-Joskow electricity demand model with the demands for electricity determined by the remainder of the interindustry and final demand models of the Hudson-Jorgenson system. Solving the remainder of the Hudson-Jorgenson model for a number of years at a specified price of electricity would yield a profile of electricity demands for the Baughman-Joskow model. The Baughman-Joskow capacity expansion, generation, and financial models would then yield information on the investments and choices made to generate the electricity to meet these demands. Part of the

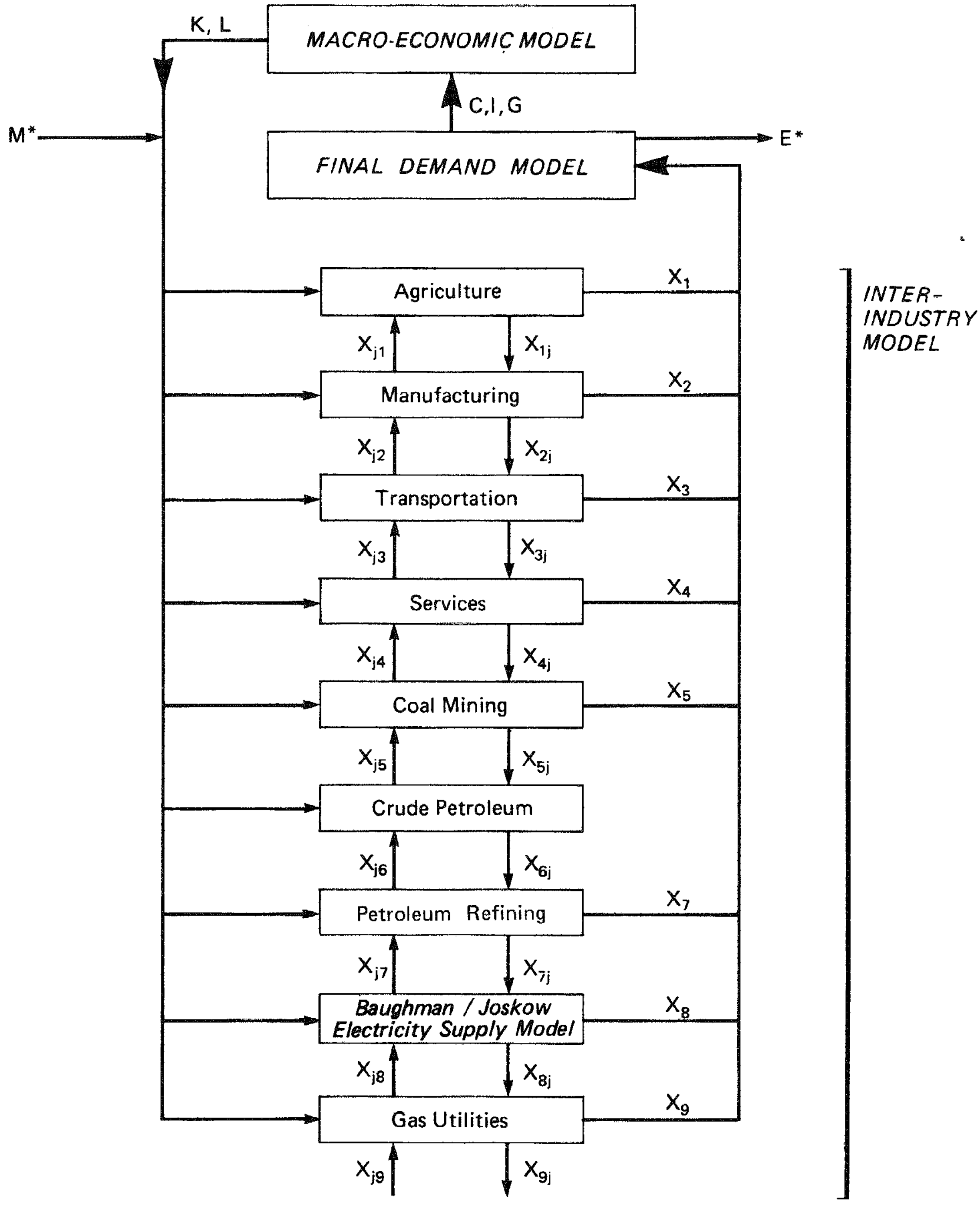


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Figure 12. SCHEMATIC NETWORK FOR HUDSON-JORGENSEN / BAUGHMAN-JOSKOW COMBINED MODEL

result would be the demands for goods and services from other sectors of the interindustry model and for energy materials from the other energy sectors. At the same time, the Baughman-Joskow supply model can compute the price of electricity that would be charged for consumers of the demand generated by the Hudson-Jorgenson model.

In short, we fix the output quantity of electricity and the input prices for materials, capital services, and labor services, and solve the supply models of the Baughman-Joskow system for the output prices and the input quantities. These are passed to the Hudson-Jorgenson model to solve for new prices of materials, capital services, and labor services, and a new level of demand for electricity. In a complete integration of the Baughman-Joskow model the price of capital for the capacity expansion model and price of labor services for operating the electric utility system would be adjusted to maintain an equilibrium with the remainder of the combined energy-economic model. In the first experiments with this combined model, only the integration of energy product prices and demands were completed, but the steps for the full integration were recognized.

This combined energy model captures the strength of both systems. The Hudson-Jorgenson model benefits from a more complete description of the capacity expansion investments in electric utilities, the costs of electricity in any given period, and the impact of regulatory and financial considerations on the price of electricity facing the other sectors of the economy. In return, the users of the Baughman-Joskow model obtain national* electricity demands that give simultaneous attention to the competition of other energy products, the equilibrium level of competing energy product prices, and the equilibrium level and composition of economic activity.

*The Baughman-Joskow model computes the regional energy demands by disaggregating estimates of national demand. The same equations are used to disaggregate the national demands from the Hudson-Jorgenson model.

CONCLUSION

Energy models are powerful tools for studying energy problems. As a practical matter, the only means for constructing a large-scale model is through the careful integration of separate descriptions of components of the overall system. This necessity is a virtue if the combined energy model is developed within the context of a consistent framework that exploits the properties of the overall system structure and makes the model accessible to the users. A careful examination of a range of combined models, both within and without the energy sector, suggests that nearly all of these models can be interpreted as approximations to the pure ideal of a network of process models solved to obtain an optimal solution of the integrated system. This general framework suggests modeling rules for linkages of the component models, properties of the solutions of the component models, and the structure for the systematic exploitation of the particular details of the individual models.

At the same time, the analysis identifies limitations in the existing models in the area of general price level formation and the interpretation of solution in the presence of complicated taxes and regulations. We saw that the optimization problem implied that the solution to the network of process models was determined only by the relative prices, yet most real combined models have constraints on the prices that destroy this homogeneity property. One area for important further research, therefore, is to examine how the determination of the price level, how the introduction of financial variables, and how the introduction of regulations affect the design and solution of combined energy models. We know from some experience that the models are robust in the sense that we can obtain equilibrium solutions, but we do not yet have a consistent principle like optimization for interpreting these descriptive models.

The great flexibility in the alternative approximation techniques for representing combined models offers a wealth of choices for linking models together. Modelers have only begun to tap this reservoir of convenient approximation techniques married to special algorithms for solving combined energy models. The emergence of modeling software languages specifically designed for constructing a network of process models opens the way for a great deal of experimentation in model linkage. For example, there have been very few attempts to use the local equilibrium approximation type algorithm as outlined here.

The modularity, the adaptation to natural data structures, and the decentralized implementation of the combined energy model framework can be tested best in the context of real problems. There are no theoretical innovations here. Rather, the attempt has been to interpret the theory in a manner convenient for the pragmatic and practical development of models that we can use in studying energy policy and planning problems. An immediate research need, therefore, is to combine the ideas, techniques, and methods outlined here and apply these to real decision problems. The experience with real problems offers the hope of sharpening our ability to construct and use large-scale energy-economic models.

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