ELECTRICITY MARKET REFORM:
Market Design and Extended LMP

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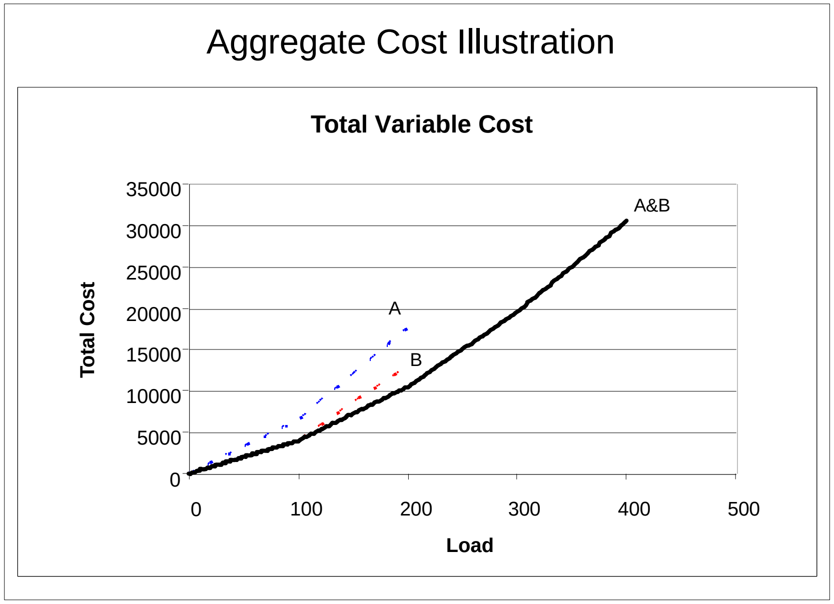
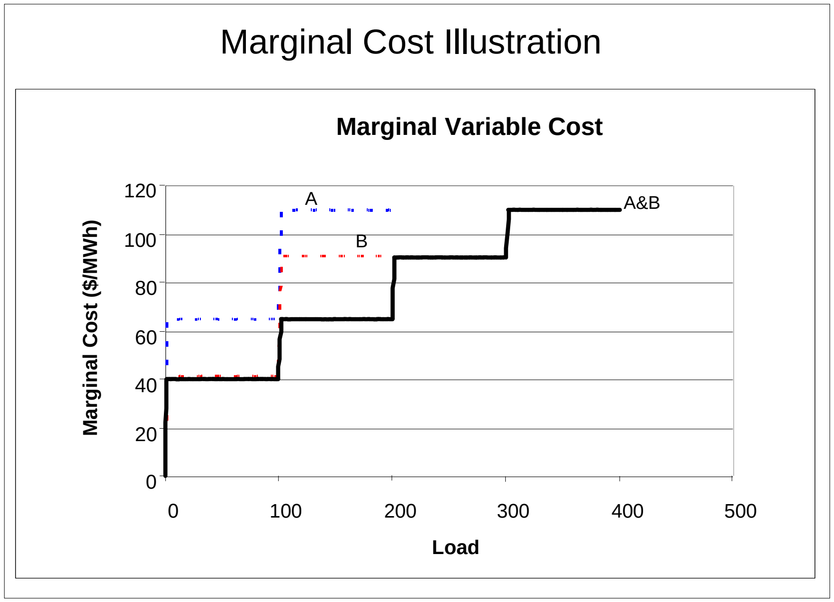
Energy dispatch is continuous but unit commitment requires discrete decisions. Bid-based, security constrained, combined unit commitment and economic dispatch presents a challenge in defining market-clearing prices.

- **Continuous convex economic dispatch**
  - System marginal costs provide locational, market-clearing, linear prices
  - Linear prices support the economic dispatch

- **Discrete, economic, unit commitment and dispatch**
  - Start up and minimum load restrictions enter the model
  - System marginal costs not always well-defined
  - There may be no linear prices that support the commitment and dispatch solution

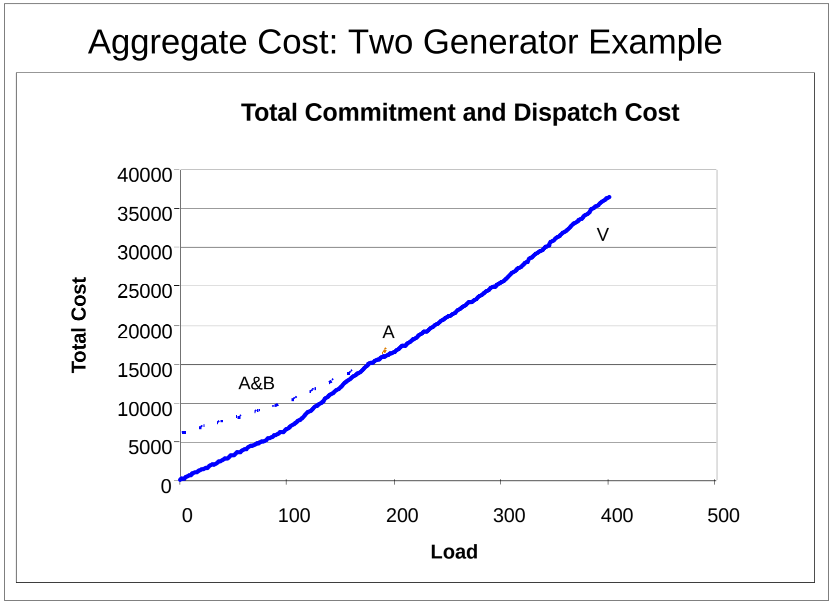
Energy dispatch is continuous, convex and yields linear prices.[1] A simplified example with two generating units illustrates the total and marginal costs.

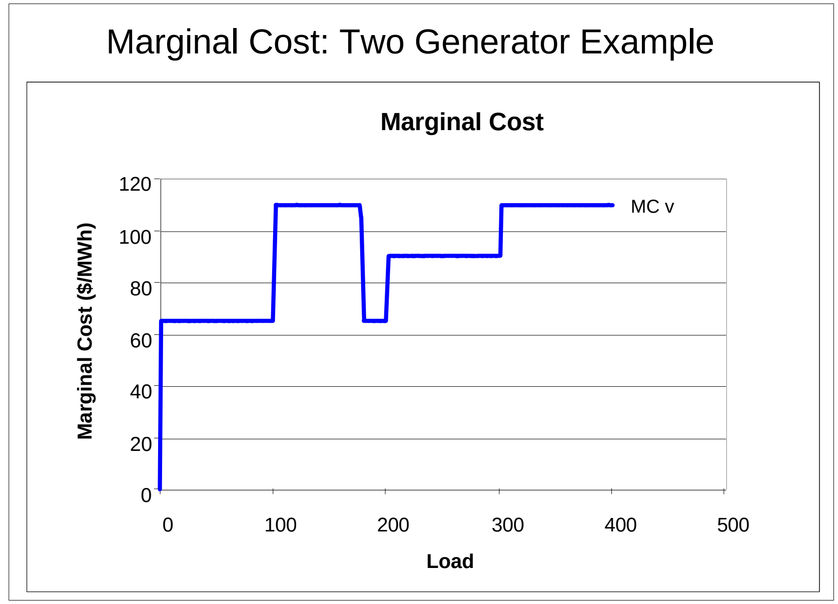




1 Paul R. Gribik, William W. Hogan, and Susan L. Pope, "Market-Clearing Electricity Prices and Energy Uplift," Harvard University, December 31, 2007, available at www.whogan.com.

Unit commitment requires discrete decisions. Now the second unit (B) has a startup cost.





Marginal cost-based linear prices cannot support the commitment and dispatch. The solution has been to make "uplift" payments to assure reliable and economic unit commitment.

Selecting the appropriate approximation model for defining energy and uplift prices involves practical tradeoffs. All involve "uplift" payments to guarantee payments for bid-based cost to participating bidders (generators and loads), to support the economic commitment and dispatch.

Uplift with Given Energy Prices=Optimal Profit – Actual Profit

- **Restricted Model (r)**
    - Fix the unit commitment at the optimal solution.
    - Determine energy prices from the convex economic dispatch.

- **Dispatchable Model (d)**
    - Relax the discrete constraints and treat commitment decisions as continuous.
    - Determine energy prices from the relaxed, continuous, convex model.

- **Extended LMP (ELMP) Model (h)**
    - Equivalent formulations
        - Select the energy prices from the convex hull of the cost function.
        - Select the energy prices from the Lagrangean relaxation (i.e., usual dual problem for pricing the joint constraints).
    - Resulting energy prices minimize the total uplift.

Economic commitment and dispatch is a special case of a general optimization problem.

$$\begin{aligned} v(y) = \underset{x \in X}{Min} \quad & f(x) \\ s.t. \quad & g(x) = y. \end{aligned}$$

From the perspective of a price-taking bidder, uplift is the difference between actual and optimal profits.

Actual profits: $\pi(p,y) = py - v(y)$

Optimal Profits: $\pi^*(p) = \underset{z}{Max}\{pz - v(z)\}$

$$Uplift(p,y) = \pi^*(p) - \pi(p,y)$$

Classical Lagrangean relaxation and pricing creates a familiar dual problem.

$$L(y,x,p) = f(x) + p(y - g(x))$$

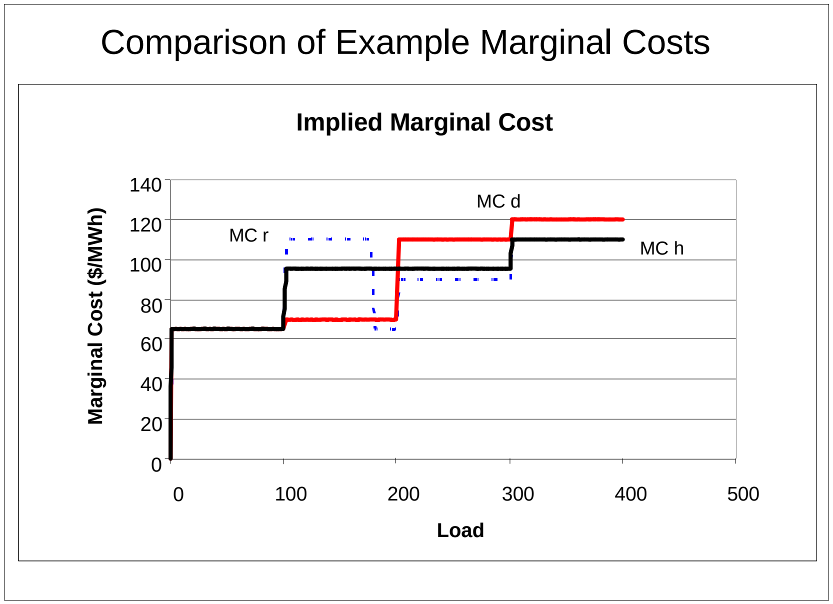
$$\hat{L}(y,p) = \underset{x \in X}{Inf}\{f(x) + p(y - g(x))\}$$

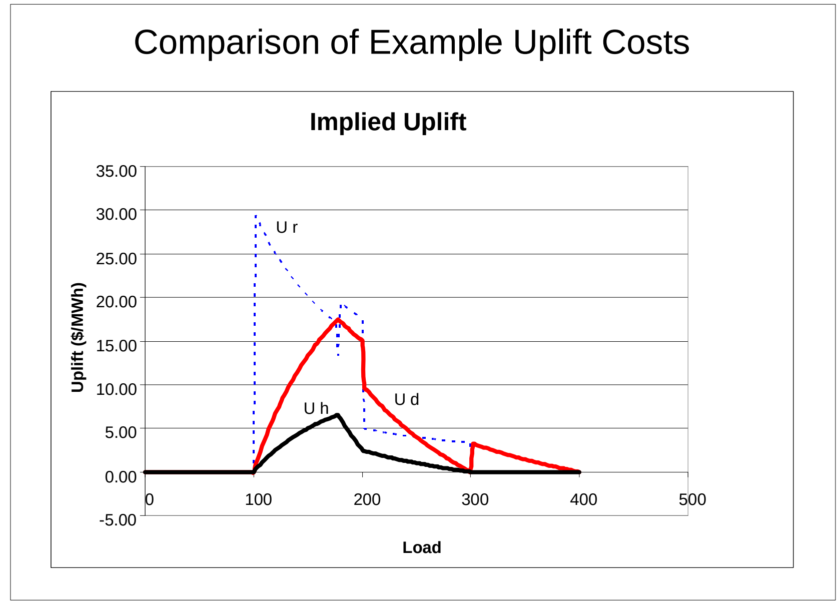
$$L^*(y) = \underset{p}{Sup}\,\hat{L}(y,p) = \underset{p}{Sup}\left\{\underset{x \in X}{Inf}\{f(x) + p(y - g(x))\}\right\}$$

The optimal dual solution minimizes the uplift, and the “duality gap” is equal to the minimum uplift.

$$v(y) - L^*(y) = \underset{p}{Inf}\; Uplift(p,y).$$

Comparing illustrative energy pricing and uplift models.





Both the relaxed and ELMP models produce “standard” implied supply curve. The ELMP model produces the minimum uplift.

Alternative pricing models have different features and raise additional questions.

- **Computational Requirements.** Relaxed model easiest case, ELMP model the hardest. But not likely to be a significant issue. Approximate solutions (e.g., NYISO model) may be workable.

- **Network Application.** All models compatible with network pricing and reduce to standard LMP in the convex case.

- **Operating Reserve Demand.** All models compatible with existing and proposed operating reserve demand curves.

- **Solution Independence.** Restricted model sensitive to actual commitment. Relaxed and ELMP models (largely) independent of actual commitment and dispatch.

- **Financial Transmission Rights.** Transmission revenue collected under the market clearing solution would be sufficient to meet the obligations under the FTRs. However, this may not be true for the revenues under the economic dispatch, which is not a market clearing solution at the ELMP prices, even though the FTRs are simultaneously feasible. The FTR uplift amount included in the decomposition of the total uplift that is minimized by the ELMP prices. This uplift payment would be enough to ensure revenue adequacy of FTRs under ELMP pricing.[2]

- **Day-ahead and real-time interaction.** With uncertainty in real-time and virtual bids, expected real-time price is important, and may be similar under all pricing models.

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[2] Michael Cadwalader , Paul Gribik , William Hogan , Susan Pope, "Extended LMP and Financial Transmission Rights," Working Paper, June 9, 2010, available at www.whogan.com.

A real-time pricing model involves multiple periods and look ahead. Applying and ELMP framework involves choices about what is fixed and what is variable.

- **Real-time quantity anchor.** Conditioning to reflect evolving economic dispatch and commitment. For example, the pricing dispatch would account for ramping limits that constrain the degree that the pricing dispatch could deviate from the actual dispatch to ensure that the price market-clearing dispatch would always be feasible conditioned on the actual dispatch.
- **Real-time price consistency.** Given that the actual conditions equal the forecast conditions, the methodology produces the same set of prices.

For actual commitment and dispatch, past decisions are sunk and real-time quantity anchors apply.

The pricing model could employ more flexibility. The Restricted model can meet both conditions by always ignoring fixed costs. ELMP in general incorporates intertemporal constraints and reflects fixed costs of units not committed.

A stylized version of the unit commitment and dispatch problem for a fixed demand $\mathbf{y}$ is formulated in (Gribik, Hogan, & Pope, 2007) as:

$$v\left(\{\mathbf{y}_t\}\right) = \inf_{\text{g,d,on,start}} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right)$$

subject to

$$\begin{array}{ll}
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\
\mathbf{d}_t = \mathbf{y}_t & \forall t.
\end{array}$$

$\mathbf{y}_t =$ vector of nodal loads in period t

$m_{it} =$ minimum output from unit i in period t if unit is on

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$ramp_{it} =$ maximum ramp from unit i between period t-1 and period t

$StartCost_{it} =$ Cost to start unit i in period t

$NoLoad_{it} =$ No load cost for unit i in period t if unit is on

$\overline{F}_{kt}^{\max} =$ Maximum flow on transmission constraint k in period t.

Variables:

$$start_{it} = \begin{cases} 0 \text{ if unit i is not started in period t} \\ 1 \text{ if unit i is started in period t} \end{cases}$$

$$on_{it} = \begin{cases} 0 \text{ if unit i is off in period t} \\ 1 \text{ if unit i is on in period t} \end{cases}$$

$g_{it} =$ output of unit i in period t

$\mathbf{d}_t =$ vector of nodal demands in period t.

The ELMP is a solution $\mathbf{p}$ for the problem including the loss and transmission limits included as constraints. A "market-clearing" solution is a solution to the inner problem for given prices $\mathbf{p}$.

$$
v^h\left(\{\mathbf{y}_t\}\right) \equiv
$$

$$
\sup_p \left\{
\begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{
\begin{array}{ll}
\inf_{\mathrm{g,d,on,start}} \left(
\begin{array}{l}
\sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\
-\sum_t \mathbf{p}_t^T \mathbf{d}_t
\end{array}
\right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
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start_{it} = 0 \text{ or } 1 & \forall i,t \\
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Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t
\end{array}
\right\}
\end{array}
\right\}
$$

A necessary and sufficient condition for real-time price consistency in ELMP is that all commitment and dispatch variables that are in the economic dispatch or are assigned an uplift payment from the market-clearing solution, must be maintained as variables in the pricing model. This allows for slowly pruning those offers that were not committed in either the economic commitment or the market-clearing commitment and are subsequently excluded from retroactive starts ($Excluded_\tau$ **). Hence, the dual pricing model at time** τ **would take the determined prices for the prior periods** $p_1^*, p_2^*, \cdots p_{\tau-1}^*$ **as fixed and solve the pricing model:**

$$
v^{h\tau}\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l} +\sum_t \mathbf{p}_t^T \mathbf{y}_t \\ + \left\{ \begin{array}{ll} \inf\limits_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\ \text{subject to} & \\ m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\ -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\ start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\ start_{it} = 0 \text{ or } 1 & \forall i,t \\ on_{it} = 0 \text{ or } 1 & \forall i,t \\ \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\ Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\ start_{it} = 0, i \in Excluded_\tau & \end{array} \right\} \\ p_t = p_t^*, t \le \tau - 1 \end{array} \right\}
$$

This model also has an interpretation as the ELMP model for a two stage dualization of the complicating constraints. First we fix the prices for prior periods and price out the constraints to include them as part of the objective function. Then we dualize this reduced model to find the remaining prices. The corresponding statement of the conditional dispatch problem for which we find the ELMP going forward is:

$$
\begin{aligned}
& v^{h\tau}\left(\{\mathbf{y}_t\}\right) \equiv \\
& \inf_{\text{g,d,on,start}} \left(\begin{aligned} & \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ & - \sum_{t \le \tau - 1} \mathbf{p}_t^{*T}\left(\mathbf{d}_t - \mathbf{y}_t\right) \end{aligned} \right) \\
& \text{subject to} \\
& m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
& -ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
& start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
& start_{it} = 0 \text{ or } 1 && \forall i,t \\
& on_{it} = 0 \text{ or } 1 && \forall i,t \\
& \mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
& Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t \\
& start_{it} = 0, i \in Excluded_\tau \\
& \mathbf{d}_t = \mathbf{y}_t && \forall t \ge \tau.
\end{aligned}
$$

Approximations of an ELMP real-time pricing model would include.

- **Block Loaded Units.** Variablize the average cost of units that are all on or off.
- **Fixed Cost Allocation.** The UK Pool solution with on and off peak periods.
- **Relaxation Variants.** Combine relaxed formulation with ad hoc fixed cost allocations.
- **Price Conditioning.** Set different windows and horizons for price consistency objective.

An objective is to obtain a workable pricing model.

- **Integrated With Day-Ahead.** Support a two-settlement system and virtual bids equating day-ahead and expected real-time prices.
- **Stakeholder Testing and Verification.** Simple simulations to understand market impacts.

William W. Hogan is the Raymond Plank Professor of Global Energy Policy, John F. Kennedy School of Government, Harvard University. This paper draws on work for the Harvard Electricity Policy Group and the Harvard-Japan Project on Energy and the Environment. The author is or has been a consultant on electric market reform and transmission issues for Allegheny Electric Global Market, American Electric Power, American National Power, Aquila, Australian Gas Light Company, Avista Energy, Barclays, Brazil Power Exchange Administrator (ASMAE), British National Grid Company, California Independent Energy Producers Association, California Independent System Operator, Calpine Corporation, Canadian Imperial Bank of Commerce, Centerpoint Energy, Central Maine Power Company, Chubu Electric Power Company, Citigroup, Comision Reguladora De Energia (CRE, Mexico), Commonwealth Edison Company, COMPETE Coalition, Conectiv, Constellation Power Source, Coral Power, Credit First Suisse Boston, DC Energy, Detroit Edison Company, Deutsche Bank, Duquesne Light Company, Dynegy, Edison Electric Institute, Edison Mission Energy, Electricity Corporation of New Zealand, Electric Power Supply Association, El Paso Electric, GPU Inc. (and the Supporting Companies of PJM), Exelon, GPU PowerNet Pty Ltd., GWF Energy, Independent Energy Producers Assn, ISO New England, LECG LLC, Luz del Sur, Maine Public Advocate, Maine Public Utilities Commission, Merrill Lynch, Midwest ISO, Mirant Corporation, MIT Grid Study, JP Morgan, Morgan Stanley Capital Group, National Independent Energy Producers, New England Power Company, New York Independent System Operator, New York Power Pool, New York Utilities Collaborative, Niagara Mohawk Corporation, NRG Energy, Inc., Ontario IMO, Pepco, Pinpoint Power, PJM Office of Interconnection, PJM Power Provider (P3) Group, PPL Corporation, Public Service Electric & Gas Company, Public Service New Mexico, PSEG Companies, Reliant Energy, Rhode Island Public Utilities Commission, San Diego Gas & Electric Corporation, Sempra Energy, SPP, Texas Genco, Texas Utilities Co, Tokyo Electric Power Company, Toronto Dominion Bank, Transalta, Transcanada, TransÉnergie, Transpower of New Zealand, Tucson Electric Power, Westbrook Power, Western Power Trading Forum, Williams Energy Group, and Wisconsin Electric Power Company. This work has been done in collaboration with Paul Gribik and Susan Pope. The views presented here are not necessarily attributable to any of those mentioned, and any remaining errors are solely the responsibility of the author. (Related papers can be found on the web at www.whogan.com).