A MODEL FOR A ZONAL OPERATING RESERVE DEMAND CURVE

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The case of electricity restructuring presents examples of fundamental problems that challenge regulation of markets.

- **Marriage of Engineering and Economics.**
    - **Loop Flow.**
    - **Reliability Requirements.**
    - **Incentives and Equilibrium.**

- **Devilish Details.**
    - **Market Power Mitigation.**
    - **Coordination for Competition.**

- **Jurisdictional Disputes.**
    - **US State vs. Federal Regulators.**
    - **European Subsidiarity Principle.**

The short term financial crisis and long term energy policy provide a context with a rapidly changing view of the role of government.

- **Financial Crisis Presents Conflicting Diagnoses**

  "Deregulation, or the failure of regulators to keep up with fast-moving markets, can become unbelievably costly, as we have seen."[1]

- **Going Green Implies a Major Transformation of the Electricity Sector**

  Climate change policy and the expanded focus on renewables present a fast moving array of subsidies, regulations and mandates. Focus on transmission expansion and the smart grid.

- **Electricity Restructuring is not Electricity Deregulation**

  Electricity markets with Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs), the North American Electric Reliability Corporation (NERC), State Public Utility Commissions (PUCs), Public Power Authorities, and the Federal Energy Regulatory Commission (FERC) are highly regulated entities. But "failure of regulators to keep up with fast-moving markets, can become unbelievably costly, as we have seen."

The challenge of "keeping up" emphasizes the dynamic nature of the problems and the importance of understanding the fundamentals of first principles.

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[1] Francis Fukuyama, "The Fall of America, Inc.," Newsweek, October 13, 2008, p. 32.

The Federal Energy Regulatory Commission has responsibility for regulating wholesale electricity markets. The stated framework emphasizes support for competition in wholesale markets as a clear and continuing national policy:

> “While competitive markets face challenges, we should acknowledge that competition in wholesale power markets is national policy. The Energy Policy Act of 2005 embraced wholesale competition as national policy for this country. It represented the third major federal law enacted in the last 25 years to embrace wholesale competition. To my mind, the question before the Commission is not whether competition is the correct national policy. That question has been asked and answered three times by Congress.
>
> If we accept the Commission has a duty to guard the consumer, and that competition is national policy, our duty is clear. It is to make existing wholesale markets more competitive. That is the heart of this review: to not only identify the challenges facing competitive wholesale markets but also identify and assess solutions.”[2]

A task for regulation is to support this policy framework while developing hybrid markets and dealing with both the limits of markets and the failures of market designs.

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[2] Joseph T. Kelliher, “Statement of Chairman Joseph T. Kelliher,” Federal Energy Regulatory Commission, Conference on Competition on Wholesale Power Markets AD07-7-000. February 27, 2007.

The focus on the electricity sector's role in addressing climate change through improved efficiency, development of renewable energy, and use of low carbon fuels creates expanded demands for and of electricity restructuring.

The transformation envisioned is massive, long term, and affects every aspect of electricity production and use.

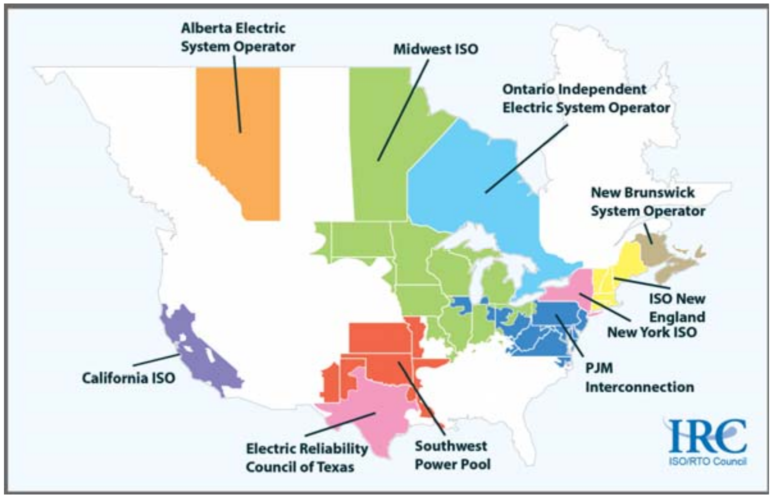
- Uncertain conditions require a broad range of activities to integrate new technology and practices.
- Innovation requires promoting technologies and practices not yet identified or imagined.
- Smart grids can facilitate smart decisions, but only if the electricity structure provides the right information and incentives.
    - Open access to expand entry and innovation.
    - Smart pricing to support the smart grid technologies and information.
    - Internalizing externalities, while exploiting the wisdom of crowds.
        - Price on carbon emissions.
        - Good market design with efficient prices.
        - Compatible infrastructure expansion rules.

Market design in RTOs/ISOs is well advanced but still incomplete.[3]

- **Regional Markets Not Fully Deployed**

- **Reforms of Reforms**

  California MRTU (April 1, 2009) and forthcoming ERCOT Texas Nodal reforms.



- **Market Defect: Scarcity Pricing**

  Smarter pricing to support operations, infrastructure investment and resource adequacy.

- **Market Failure: Transmission Investment**

  - Regulatory mandates for lumpy transmission mixed with market-based investments.
  - Design principles for cost allocation to support a mixed market (i.e., beneficiary pays).

- **Market Challenge: Address Requirements for Climate Change Policy**

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3 William W. Hogan, "Electricity Market Structure and Infrastructure," Conference on Acting in Time on Energy Policy, Harvard University, September 18-19, 2008. (available at www.whogan.com).

The public policy debate over reshaping the electricity industry confronts major challenges in balancing public interests and reliance on markets.

The International Energy Agency (IEA) examined the international experience and produced guidance for electricity restructuring.

- "Governments must ensure a stable and competitive investment framework that sufficiently rewards adequate investments in a timely manner. …
- Governments urgently need to reduce investment risks by giving firmer and more long-term direction on climate change abatement policies. …
- Governments should pursue the benefits of competitive markets to allow for more efficient and more transparent management of investment risks. …
- Governments need to ensure that independent regulators and system operators establish transparent market rules that are clear, coherent and fair. …
- Governments must refrain from price caps and other distorting market interventions. …
- Governments must implement clearer and more efficient procedures for approval of new electricity infrastructure. …"[4]

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[4] International Energy Agency, Tackling Investment Challenges in Power Generation in IEA Countries: Energy Market Experience, Paris, 2007, pp. 15-25.

The International Energy Agency identified the centerpiece of successful market design.

"Locational marginal pricing (LMP) is the electricity spot pricing model that serves as the benchmark for market design – the textbook ideal that should be the target for policy makers. A trading arrangement based on LMP takes all relevant generation and transmission costs appropriately into account and hence supports optimal investments."[5]

[5] International Energy Agency, Tackling Investment Challenges in Power Generation in IEA Countries: Energy Market Experience, Paris, 2007, p. 16.

Application of the broad goals identified by the IEA would be compatible with recommendations by Paul Joskow for a new Federal Power Act.

"What provisions might a Federal Power Act of 2009 contain?

- [Federalize transmission] …
- [Mandate Regional Transmission Organizations] The key provisions of FERC Order 2000 should be put into law. This would require the creation of RTOs that manage the operation of large regional transmission networks, implement FERC's transmission access, pricing, and planning regulations, and operate voluntary wholesale markets for electric energy, ancillary services, capacity and transmission rights. There is abundant evidence (a) that RTOs are needed to support efficient competitive markets, (b) that expanding the geographic expanse of RTOs and improving the market designs for energy, ancillary services and capacity lead to efficiency improvements, (c) and that wholesale market designs built around what is generally referred to as the "standard market design," augmented by capacity obligations and capacity markets, promote economic efficiency.
- [Unbundle generation and distribution] …
- [States determine retail access] …
- [Limit generation subsidies to merchant investments] …
- [Allocate any free CO2 allowances to electricity consumers] …
- [State regulatory jurisdiction continue over distribution facilities] …" [6]

[6] Paul Joskow, "Challenges For Creating A Comprehensive National Electricity Policy," Technology Policy Institute Keynote Speech, Washington DC, September 26, 2008. (available at http://www.hks.harvard.edu/hepg/).

The US experience illustrates successful market design and remaining challenges for both theory and implementation.

- **Design Principle: Integrate Market Design and System Operations**

  Provide good short-run operating incentives.
  Support forward markets and long-run investments.

- **Design Framework: Bid-Based, Security Constrained Economic Dispatch**

  Locational Marginal Prices (LMP) with granularity to match system operations.
  Financial Transmission Rights (FTRs).

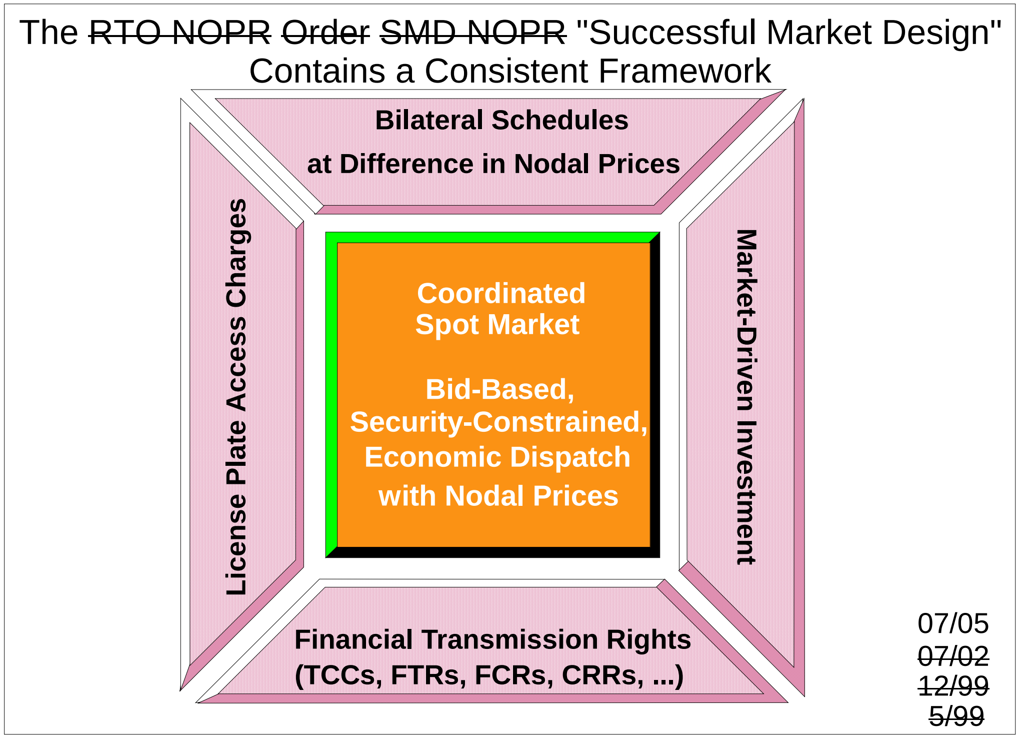
- **Design Implementation: Pricing Evolution**

  Better scarcity pricing to support resource adequacy.
  Unit commitment and lumpy decisions with coordination, bid guarantees and uplift payments.

- **Design Challenge: Infrastructure Investment**

  Hybrid models to accommodate both market-based and regulated investments.
  Applying beneficiary-pays principle to support integration with rest of the market design.

The example of successful central coordination, ~~CRT, Regional Transmission Organization (RTO) Millennium Order (Order 2000)~~ ~~Standard Market Design (SMD) Notice of Proposed Rulemaking (NOPR),~~ "Successful Market Design" provides a workable market framework that is working in places like New York, PJM in the Mid-Atlantic Region, New England, the Midwest, and California.

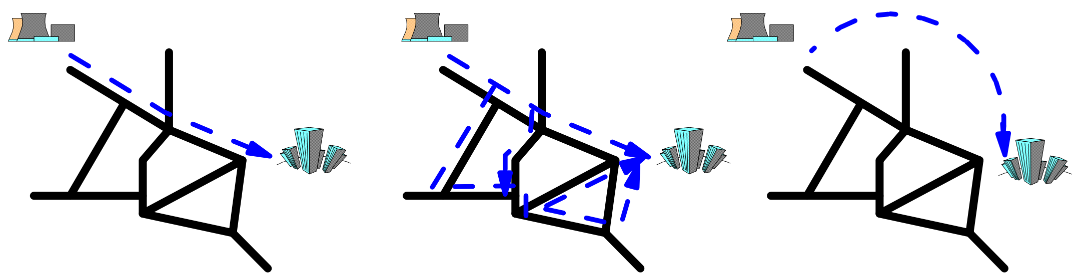


Poolco…OPCO…ISO…IMO…Transco…RTO… ITP…WMP…: "A rose by any other name …"

Defining and managing transmission usage is a principal challenge in electricity markets.

Transmission Capacity Definitions

Contract Path → Flow-Based Paths → Point-to-Point



Contract Path Fiction

OASIS Schedules
and TLR

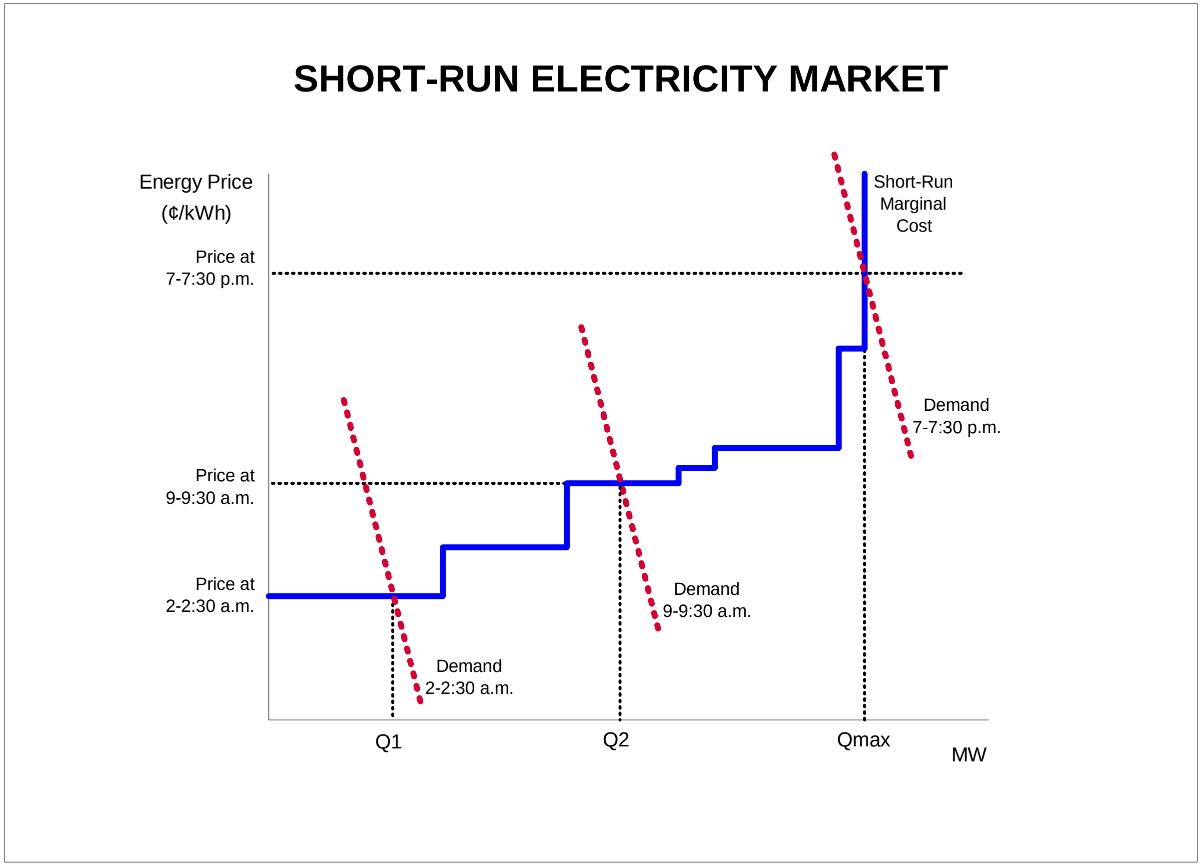
Parallel Flows

Flowgate Rights
FGRs

Flows Implicit

Financial Transmission
Rights
FTRs

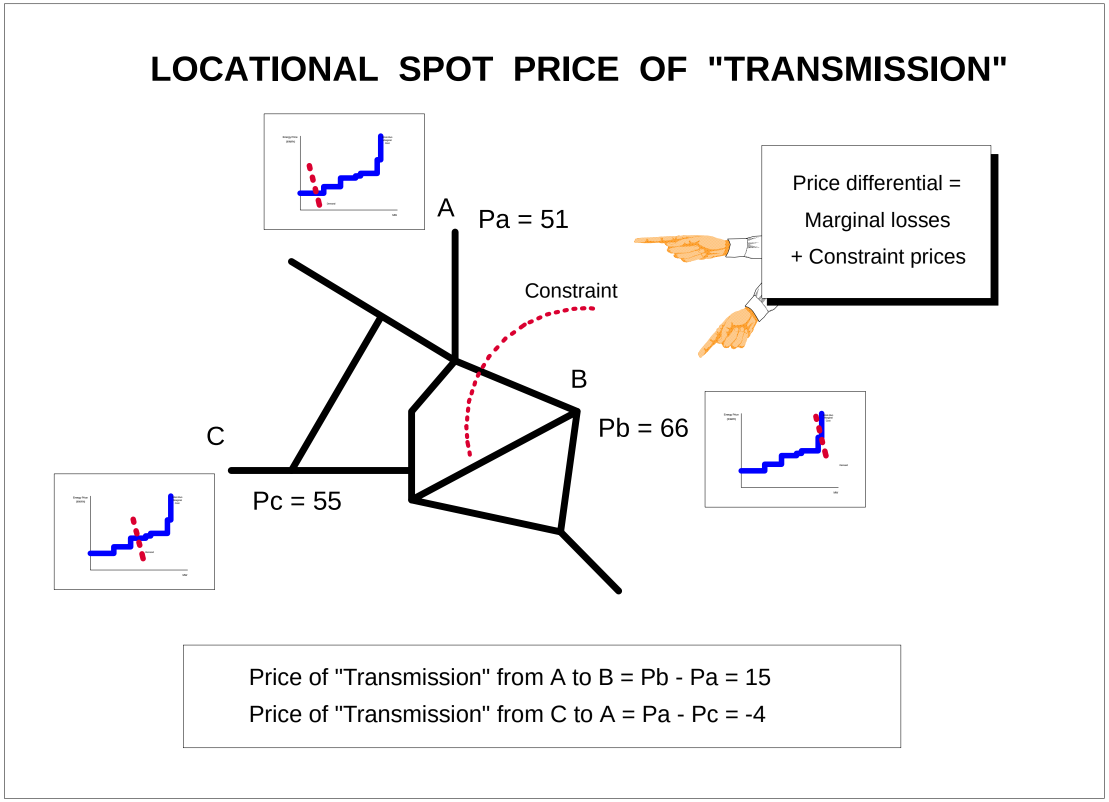
An efficient short-run electricity market determines a market clearing price based on conditions of supply and demand. Everyone pays or is paid the same price.



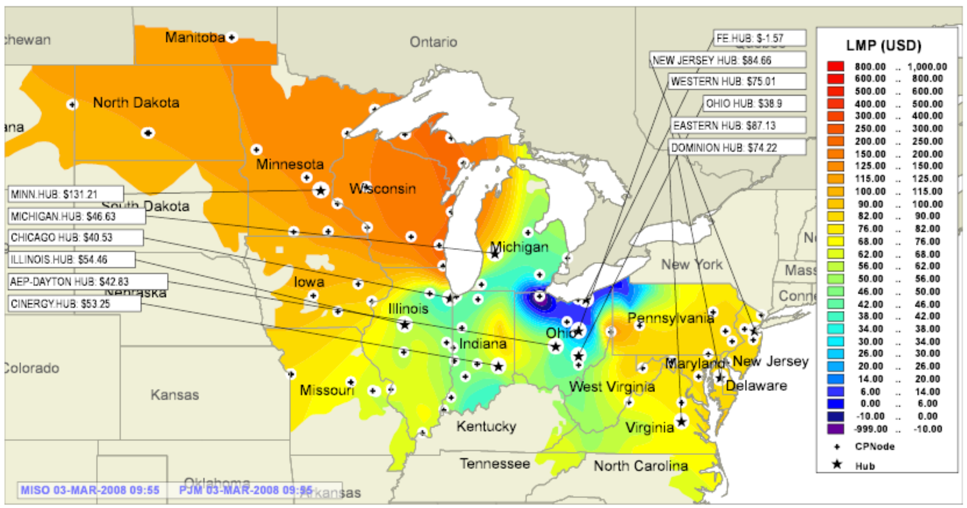
The natural extension of a single price electricity market is to operate a market with locational spot prices.

- It is a straightforward matter to compute "Schweppe" spot prices based on marginal costs at each location.

- Transmission spot prices arise as the difference in the locational prices.



RTOs operate spot markets with locational prices. For example, PJM updates prices and dispatch every five minutes for over 8,000 locations. Locational spot prices for electricity exhibit substantial dynamic variability and persistent long-term average differences.



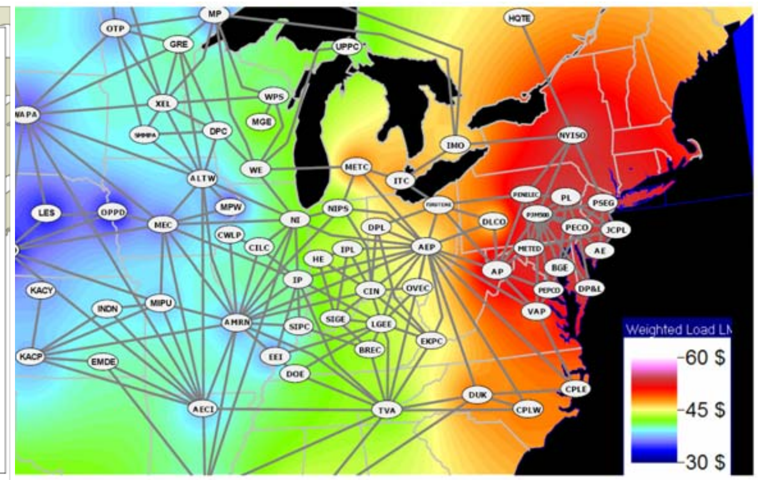


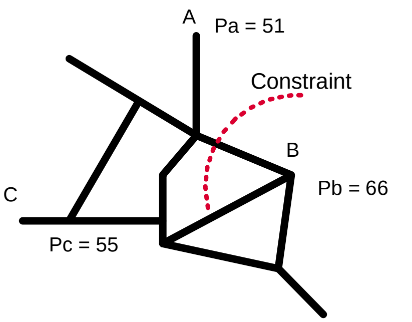
Figure 2.2-3 Contour Map of Annual Load Weighted LMP

Minnesota Hub: $131.21/MWh. First Energy Hub: $-1.57/MWh.

From MISO-PJM Joint and Common Market, http://www.jointandcommon.com/ for March 3, 2008, 9:55am. Projected 2011 annual average from 2006 Midwest ISO-PJM Coordinated System Plan.

A mechanism for hedging volatile transmission prices can be established by defining financial transmission rights to collect the congestion rents inherent in efficient, short-run spot prices.

NETWORK TRANSMISSION FINANCIAL RIGHTS



Price of "Transmission" from A to B = Pb - Pa = 15
Price of "Transmission" from A to C = Pc - Pa = -4

- DEFINE TRANSMISSION CONGESTION CONTRACTS BETWEEN LOCATIONS.
- FOR SIMPLICITY, TREAT LOSSES AS OPERATING COSTS.
- RECEIVE CONGESTION PAYMENTS FROM ACTUAL USERS; MAKE CONGESTION PAYMENTS TO HOLDERS OF CONGESTION CONTRACTS.
- TRANSMISSION CONGESTION CONTRACTS PROVIDE PROTECTION AGAINST CHANGING LOCATIONAL DIFFERENCES.

Financial Transmission Rights (FTRs), including Transmission Congestion Contracts (TCCs) and Congestion Revenue Rights (CRRs), present a variety of issues.

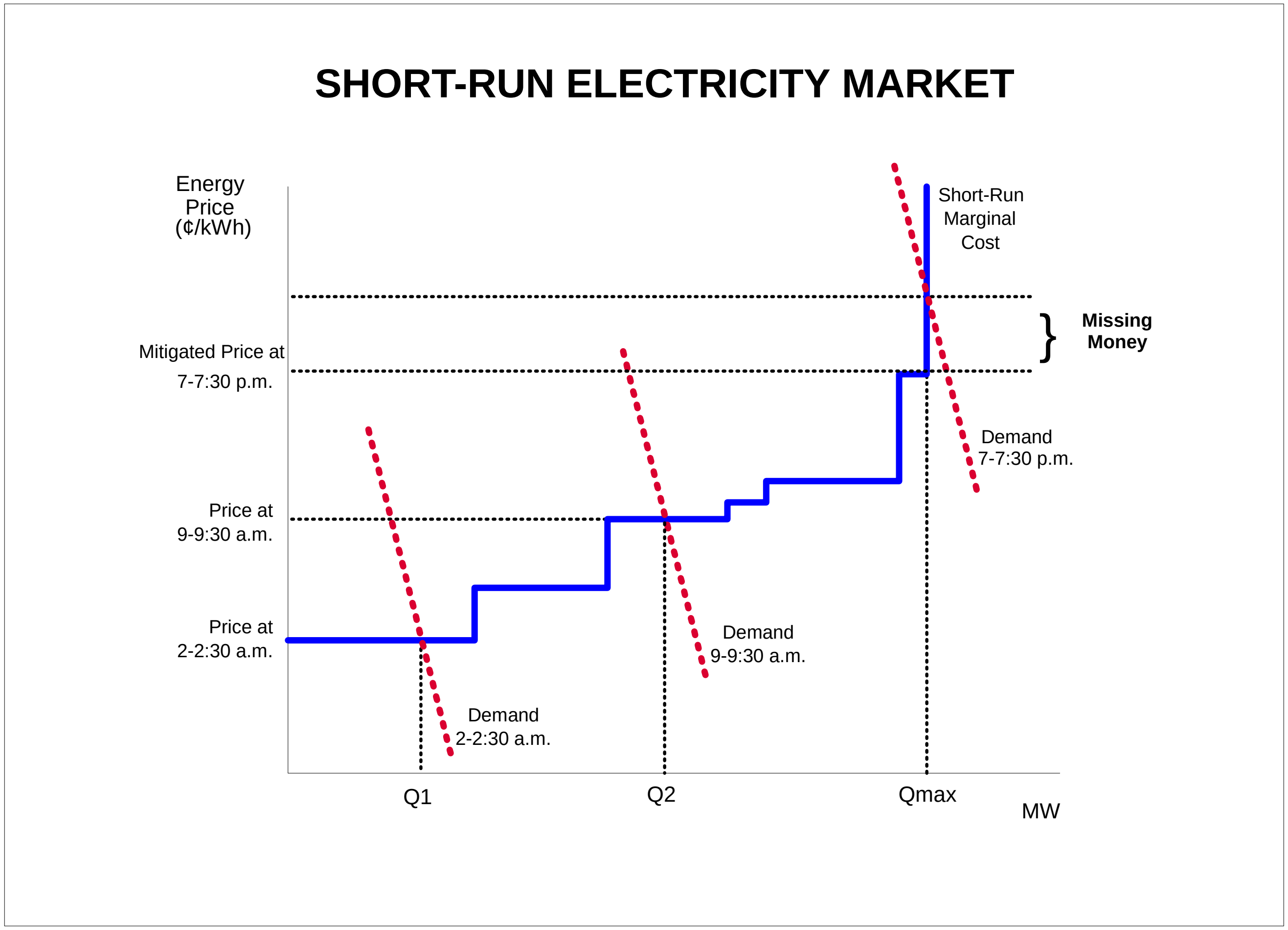
- **Definitions.**
  - Duration.
  - Obligations vs. Options.
  - Auction Revenue Rights.
  - Sequential Markets.
  - Expansion Rules.

- **Revenue Adequacy.**
  - Theory: Simultaneous Feasibility Ensures Full Funding with Same Grid.
  - Practice: Carve Outs, Outages and Loop Flow Forecasts can Affect Feasibility.

- **Market Performance.**
  - Arbitrage and FTR Prices.
  - Gaming and Credit Risks.
  - Market Power Interactions.

- **Investment and Trading.**
  - Grid Expansion.
  - Continuous Trading: Nodal Exchange.
    (http://www.nodalexchange.com/about_nodal/overview.php)

Early market designs presumed a significant demand response. Absent this demand participation most markets implemented inadequate pricing rules equating prices to marginal costs even when capacity is constrained. This produces a "missing money" problem.



Scarcity pricing presents an important challenge for Regional Transmission Organizations (RTOs) and electricity market design. Simple in principle, but more complicated in practice, inadequate scarcity pricing is implicated in several problems associated with electricity markets.

- **Investment Incentives.** Inadequate scarcity pricing contributes to the "missing money" needed to support new generation investment. The policy response has been to create capacity markets. Better scarcity pricing would reduce the challenges of operating good capacity markets.

- **Demand Response.** Higher prices during critical periods would facilitate demand response and distributed generation when it is most needed. The practice of socializing payments for capacity investments compromises the incentives for demand response and distributed generation.

- **Renewable Energy.** Intermittent energy sources such as solar and wind present complications in providing a level playing field in pricing. Better scarcity pricing would reduce the size and importance of capacity payments and improve incentives for renewable energy.

- **Transmission Pricing.** Scarcity pricing interacts with transmission congestion. Better scarcity pricing would provide better signals for transmission investment.

Smarter scarcity pricing would mitigate or substantially remove the problems in all these areas. While long-recognized, the need for smarter prices for a smarter grid promotes interest in better theory and practice of scarcity pricing.[7]

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[7] FERC, Order 719, October 17, 2008.

The theory and practice of scarcity pricing intersect important elements of electricity systems and economic dispatch.

- **Reliability.** By definition, scarcity conditions arise when the system is constrained and dispatch is modified to respect reliability constraints.
- **Dispatch.** Simultaneous optimization of energy and reserves means that scarcity in either affects prices for both.
- **Resource Adequacy.** The standards for resource adequacy and operating security are not fully integrated or compatible.

A critical connection is the treatment of operating reserves and construction of operating reserve demand curves. The basic idea of applying operating reserve demand curves is well tested and found in operation in important RTOs.

- **NYISO.** See NYISO Ancillary Service Manual, Volume 3.11, Draft, April 14, 2008, pp, 6-19-6-22.
- **ISONE.** FERC Electric Tariff No. 3, Market Rule I, Section III.2.7, February 6, 2006.
- **MISO.** FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009.[8]

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[8] "For each cleared Operating Reserve level less than the Market-Wide Operating Reserve Requirement, the Market-Wide Operating Reserve Demand Curve price shall be equal to the product of (i) the Value of Lost Load ("VOLL") and (ii) the estimated conditional probability of a loss of load given that a single forced Resource outage of 100 MW or greater will occur at the cleared Market-Wide Operating Reserve level for which the price is being determined. … The VOLL shall be equal to $3,500 per MWh." MISO, FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009, Sheet 2226.

The underlying models of operating reserve demand curves differ across RTOs. One need is for a framework that develops operating reserve demand curves from first principles to provide a benchmark for the comparison of different implementations.

- **Operating Reserve Demand Curve Components.** The inputs to the operating reserve demand curve construction can differ and a more general model would help specify the result.
- **Locational Differences and Interactions.** The design of locational operating reserve demand curves presents added complications in accounting for transmission constraints.
- **Economic Dispatch.** The derivation of the locational operating demand curves has implications for the integration with economic dispatch models for simultaneous optimization of energy and reserves.

Begin with an expected value formulation of the lossless economic dispatch that might appeal in principle. Given benefit (*B*) and cost (*C*) functions, demand (*d*), generation (*g*), plant capacity (*Cap*), reserves (*r*), commitment decisions (*u*), transmission constraints (*H*), and state probabilities (*p*):

$$\underset{y^i, d^i, g^i, r, u \in \{0,1\}}{Max} \; p_0\left(B^0\left(d^0\right) - C^0\left(g^0, r, u\right)\right) + \sum_{i=1}^{N} p_i\left(B^i\left(d^i, d^0\right) - C^i\left(g^i, g^0, r, u\right)\right)$$

$$s.t.$$

$$y^i = d^i - g^i, \quad i = 0,1,2,\cdots,N,$$

$$\iota^t y^i = 0, \quad i = 0,1,2,\cdots,N,$$

$$H^i y^i \le b^i, \quad i = 0,1,2,\cdots,N,$$

$$g^0 + r \le u \bullet Cap^0,$$

$$g^i \le g^0 + r, \quad i = 1,2,\cdots,N,$$

$$g^i \le u \bullet Cap^i, \quad i = 0,1,2,\cdots,N.$$

Suppose there are *K* possible contingencies. The interesting cases have $K \gg 10^3$. The number of possible system states is $N = 2^K$, or more than the stars in the Milky Way. Some approximation will be in order.[9]

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[9] Shams N. Siddiqi and Martin L. Baughman, "Reliability Differentiated Pricing of Spinning Reserve," IEEE Transactions on Power Systems, Vol. 10, No. 3, August 1995, pp.1211-1218. José M. Arroyo and Francisco D. Galiana, "Energy and Reserve Pricing in Security and Network-Constrained Electricity Markets," IEEE Transactions On Power Systems, Vol. 20, No. 2, May 2005, pp. 634-643. François Bouffard, Francisco D. Galiana, and Antonio J. Conejo, "Market-Clearing With Stochastic Security—Part I: Formulation," IEEE Transactions On Power Systems, Vol. 20, No. 4, November 2005, pp. 1818-1826; "Part II: Case Studies," pp. 1827-1835.

Introduce random changes in load ε^i and possible lost load l^i in at least some conditions.

$$\underset{y^i, g^i, l^i, r, u\in\{0,1\}}{Max}\; p_0\left(B^0\left(d^0\right) - C^0\left(g^0, r, u\right)\right) + \sum_{i=1}^{N} p_i\left(B^i\left(d^o + \varepsilon^i - l^i, d^0\right) - C^i\left(g^i, g^0, r, u\right)\right)$$

$$\begin{aligned}
&s.t.\\
&\quad y^0 = d^0 - g^0,\\
&\quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N,\\
&\quad \iota^t y^i = 0, \quad i = 0, 1, 2, \cdots, N,\\
&\quad H^i y^i \le b^i, \quad i = 0, 1, 2, \cdots, N,\\
&\quad g^0 + r \le u \bullet Cap^0,\\
&\quad g^i \le g^0 + r, \quad i = 1, 2, \cdots, N,\\
&\quad g^i \le u \bullet Cap^i, \quad i = 0, 1, 2, \cdots, N.
\end{aligned}$$

Simplify the benefit and cost functions:

$$B^i\left(d^o + \varepsilon^i - l^i, d^0\right) \approx B^0\left(d^0\right) + k_d^i - v^t l^i \quad , \qquad C^i\left(g^i, g^0, r, u\right) \approx C^0\left(g^0, r, u\right) + k_g^i .$$

This produces an approximate objective function:

$$p_0\left(B^0\left(d^0\right) - C^0\left(g^0, r, u\right)\right) + \sum_{i=1}^{N} p_i\left(B^i\left(d^o - l^i, d^0\right) - C^i\left(g^i, g^0, r, u\right)\right) = B^0\left(d^0\right) - C^0\left(g^0, r, u\right) + \sum_{i=1}^{N} p_i\left(k_d^i - k_g^i\right) - v^t \sum_{i=1}^{N} p_i l^i .$$

The revised formulation highlights the pre-contingency objective function and the role of the value of the expected undeserved energy.

$$\begin{aligned}
&\underset{y^i, g^i, l^i, r, u \in \{0,1\}}{Max} \; B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - v^t \sum_{i=1}^{N} p_i l^i \\
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N, \\
&\quad \iota^t y^i = 0, \quad i = 0, 1, 2, \cdots, N, \\
&\quad H^i y^i \le b^i, \quad i = 0, 1, 2, \cdots, N, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad g^i \le g^0 + r, \quad i = 1, 2, \cdots, N, \\
&\quad g^i \le u \bullet Cap^i, \quad i = 0, 1, 2, \cdots, N.
\end{aligned}$$

There are still too many system states.

Define the optimal value of expected unserved energy (VEUE) as the result of all the possible optimal post-contingency responses given the pre-contingency commitment and scheduling decisions.

$$VEUE\left(d^0, g^0, r, u\right) = \underset{y^i, g^i, l^i}{Min}\, v^t \sum_{i=1}^{N} p_i l^i$$

$$s.t.$$

$$y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N,$$

$$\iota^t y^i = 0, \quad i = 1, 2, \cdots, N,$$

$$H^i y^i \le b^i, \quad i = 1, 2, \cdots, N,$$

$$g^i \le g^0 + r, \quad i = 1, 2, \cdots, N,$$

$$g^i \le u \bullet Cap^i, \quad i = 1, 2, \cdots, N.$$

This second stage problem subsumes all the redispatch and curtailment decisions over the operating period after the commitment and scheduling decisions.

The expected value formulation reduces to a much more manageable scale with the introduction of the implicit VEUE function.

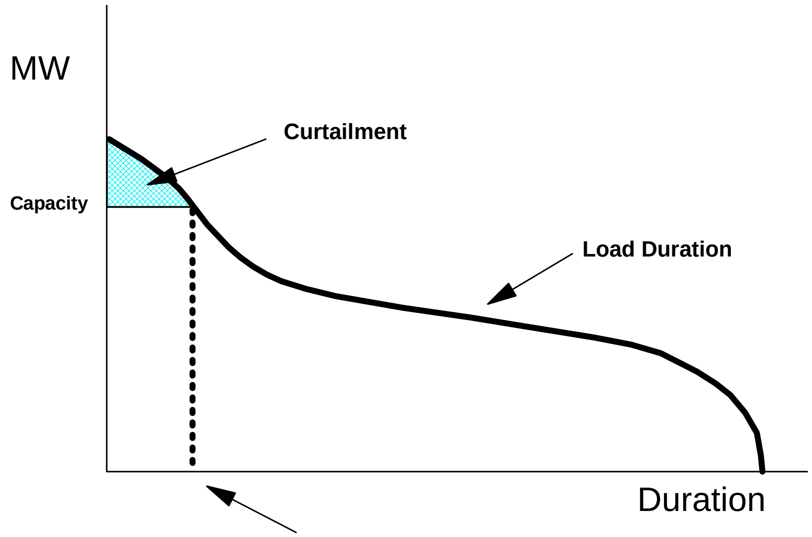
$$\underset{y^0, d^0, g^0, r, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - VEUE\left(d^0, g^0, r, u\right)$$

$$\begin{aligned} &s.t. \\ &\quad y^0 = d^0 - g^0, \\ &\quad H^0 y^0 \le b^0, \\ &\quad g^0 + r \le u \bullet Cap^0, \\ &\quad \iota^t y^0 = 0, \\ &\quad g^0 \le u \bullet Cap^0. \end{aligned}$$

The optimal value of expected unserved energy defines the demand for operating reserves. This formulation of the problem follows the outline of existing operating models except for the exclusion of contingency constraints.

There is a simple stylized connection between reliability standards and resource economics. Defining expected load shedding duration, choosing installed capacity, or estimating value of lost load address different facets of the same problem.

A Simple Reliability Model

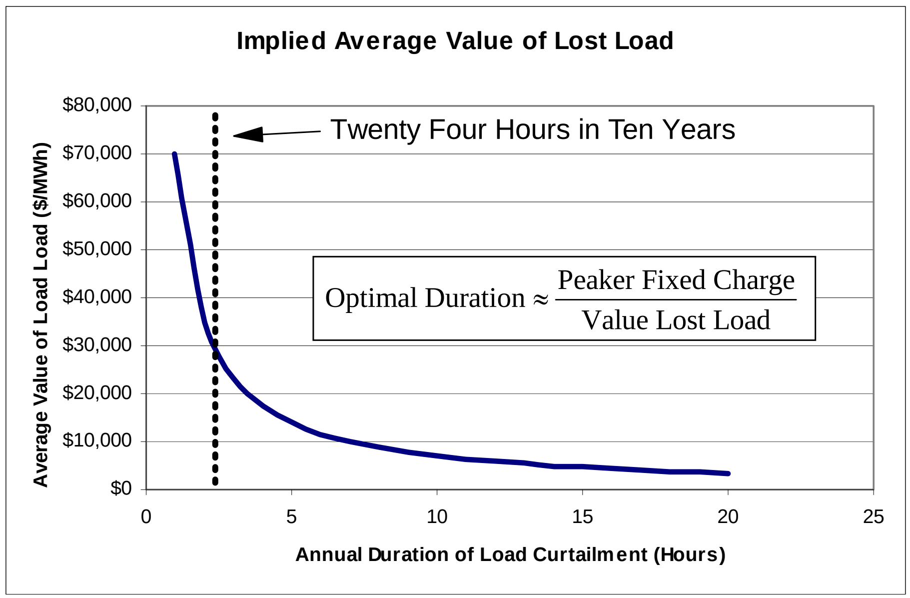


$$\text{Optimal Duration} \approx \frac{\text{Peaker Fixed Charge}}{\text{Value Lost Load}}$$

(Steven Stoft, Power System Economics, IEE Press, Wiley Interscience, 2002, p. 138)

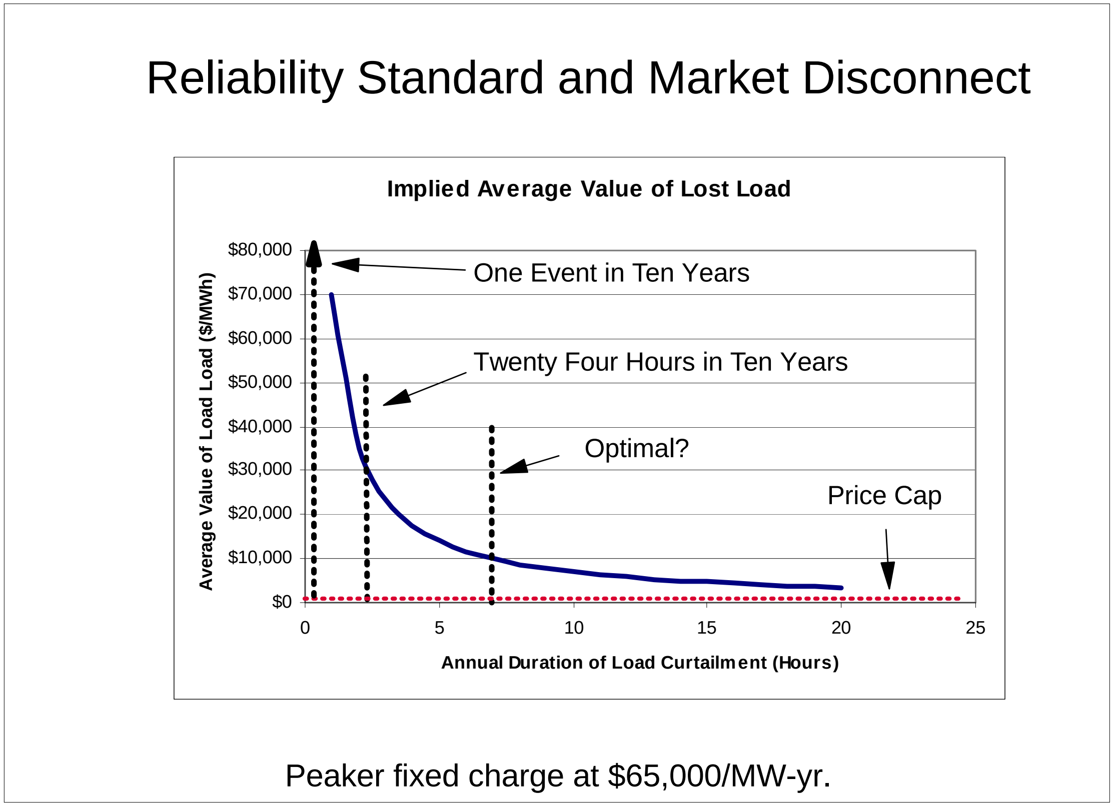
The simple connection between reliability planning standards and resource economics illustrates a major disconnect between market pricing and the implied value of lost load.

Reliability Planning Standard and Value of Lost Load



Peaker fixed charge at $65,000/MW-yr.

There is a large disconnect between long-term planning standards and market design. The installed capacity market analyses illustrate the gap between prices and implied values. The larger disconnect is between the operating reserve market design and the implied reliability standard.



Implied prices differ by orders of magnitude. $\left(\text{Price Cap} \approx \$10^3;\ \text{VOLL} \approx \$10^4;\ \text{Reliability Standard} \approx \$10^5\right)$

Ignore the network features for the first illustration. Assume all the load and generations is at a single location. Focus on the deviations form the base dispatch. Unserved energy demand is a random variable with a distribution for the probability that load exceeds available capacity.

$$Unserved\ Energy = Max\left(0, Load - Available\ Capacity\right)$$

Hence

$$\begin{aligned} Unserved\ Energy &= Max\left(0, E\left(Load\right) + \Delta\ Load - \left(Committed\ Capacity - \Delta Capacity\right)\right) \\ &= Max\left(0, \Delta\ Load + Outage + \left(E\left(Load\right) - Committed\ Capacity\right)\right) \\ &= Max\left(0, \Delta\ Load + Outage - Operating\ Reserve\right). \end{aligned}$$

This produces the familiar loss of load probability (*LOLP*) calculation, for which there is a long history of analysis and many techniques. With operating reserves (*r*),

$$LOLP = \Pr\left(\Delta\ Load + Outage \geq r\right) = \overline{F}_{LOL}\left(r\right).$$

A common characterization of a reliability constraint is that there is a limit on the *LOLP.* This imposes a constraint on the required reserves (*r*).

$$\overline{F}_{LOL}\left(r\right) \leq LOLP_{Max}.$$

This constraint formulation implies an infinite cost for unserved energy above the constraint limit, and zero value for unserved energy that results within the constraint.

An alternative approach is to consider the expected unserved energy (*EUE*) and the Value of Lost Load (*VOLL*).

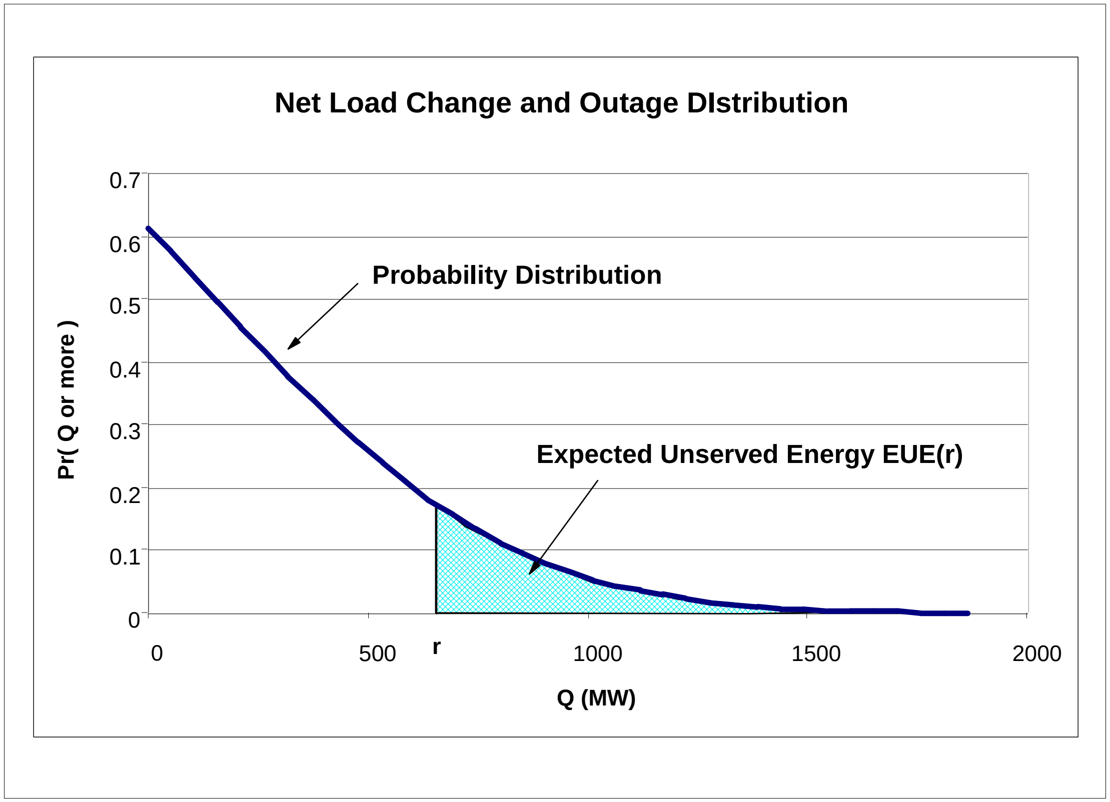
Suppose the *VOLL* per MWh is v. Then we can obtain the *EUE* and its total value (*VEUE*) as:

$$EUE(r) = \int_r^{\infty} \overline{F}_{LOL}(x)\, dx.$$

$$VEUE(r) = v\int_r^{\infty} \overline{F}_{LOL}(x)\, dx.$$

There is a chance that no outage occurs and that net load is less than expected, or $\overline{F}_{LOL}(0) < 1.$

The real changes may not be continuous, but it is common to apply continuous approximations. Total value of expected unserved energy is of same magnitude as the cost of meeting load.

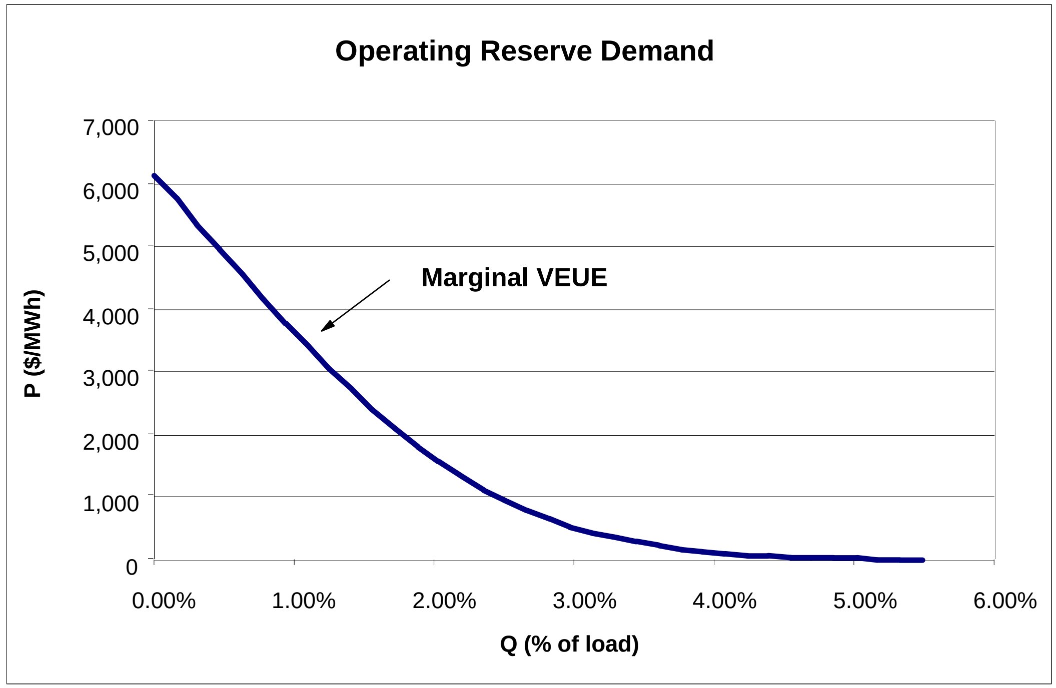


Operating reserve demand is a complement to energy demand for electricity. The probabilistic demand for operating reserves reflects the cost and probability of lost load.[10]

Example Assumptions

Expected Load (MW)	34000
Std Dev %	1.50%
Expected Outage %	0.45%
Std Dev %	0.45%
Expected Total (MW)	153
Std Dev (MW)	532.46
VOLL ($/MWh)	10000

Under the simplifying assumptions, if the dispersion of the LOLP distribution is proportional to the expected load, the operating reserve demand is proportional to the expected load.



[10] "For each cleared Operating Reserve level less than the Market-Wide Operating Reserve Requirement, the Market-Wide Operating Reserve Demand Curve price shall be equal to the product of (i) the Value of Lost Load ("VOLL") and (ii) the estimated conditional probability of a loss of load given that a single forced Resource outage of 100 MW or greater will occur at the cleared Market-Wide Operating Reserve level for which the price is being determined. … The VOLL shall be equal to $3,500 per MWh." MISO, FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009, Sheet 2226.

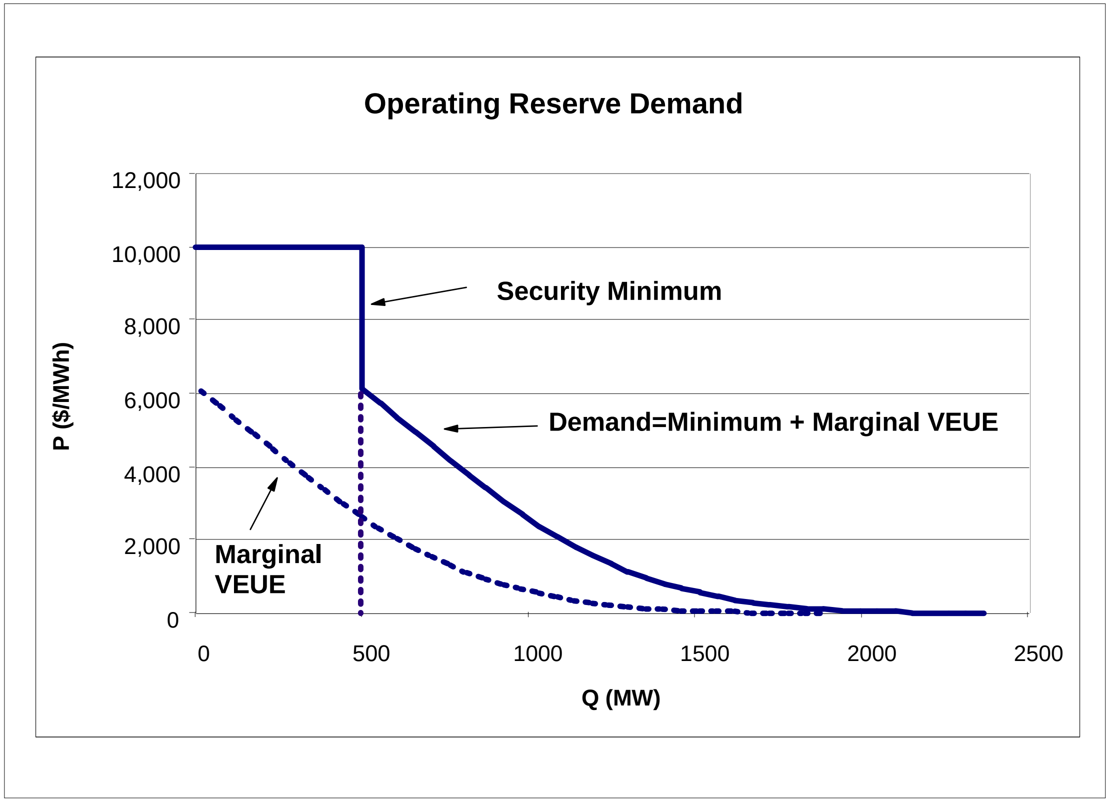
The deterministic approach to security constrained economic dispatch includes lower bounds on the required reserve to ensure that for a set of monitored contingencies (e.g., an n-1 standard) there is sufficient operating reserve to maintain the system for an emergency period.

Suppose that the maximum generation outage contingency quantity is $r_{Min}\left(d^0, g^0, u\right)$. Then we would have the constraint:

$$r \geq r_{Min}\left(d^0, g^0, u\right).$$

In effect, the contingency constraint provides a vertical demand curve that adds horizontally to the probabilistic operating reserve demand curve.

If the security minimum will always be maintained over the monitored period, the VEUE price at r=0 applies. If the outage shocks allow excursions below the security minimum during the period, the VEUE starts at the security minimum.



A difficulty with defining a locational operating reserve demand curve is the complexity of the interactions among locations plus interactions with the transmission grid. A similar problem appears in the evaluation of planned transmission and generation investment.

- **Expected Values.** The basic formulation of the real-time economic dispatch problem is built on a particular configuration of the transmission grid and the usual application of Kirchoff's laws. The operating reserve and long-term planning problem share a focus on the expected values of outcomes across different conditions. The expected value in principle applies probabilities across many configurations and the expected value need not follow the individual dictates of Kirchoff's laws.

- **Zonal Model.** The expected value formulation rationalizes approximation in a zonal model. The zonal application across a wide range of conditions is a regular feature of RTO transmission planning and resource adequacy calculations.

  - **Zones with Closed Interfaces.** Areas with limited transmission are defined and treated as having a close interface with a capacity limit for emergency transfers from the rest of the system.

  - **Capacity Emergency Transfer Limit (CETL).** Conservative transmission standards (e.g., 1 day in 25 years) apply to simulations that determine the transfer limit.[11]

- **Interface Limits.** Although the exact CETL calculations might not be appropriate for short-term reserve management, the analogy could be applied to determine closed interface limits.

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11 PJM , *2008 PJM Reserve Requirement Study*, October 8, 2008, Appendix H.

The expected value formulation with zonal approximation of the value of expected unserved energy provides a model for integrating zonal locational operating reserve requirements.

$$\begin{aligned}
&\underset{y^0, d^0, g^0, r, \bar{r}, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - ZVEUE\left(d^0, g^0, r, \bar{r}, u\right) \\
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad H^0 y^0 \leq b^0, \\
&\quad g^0 + r \leq u \bullet Cap^0, \\
&\quad A^0 y^0 + \bar{r} \leq \bar{r}_{int}, \\
&\quad r \leq r_{cap}, \\
&\quad \iota^t y^0 = 0, \\
&\quad g^0 \leq u \bullet Cap^0.
\end{aligned}$$

The operating reserves trade off against energy dispatch ($g^0 + r \leq u \bullet Cap^0$), allocation of zonal interface capacity trades off with power flow dispatch across the interface ($A^0 y^0 + \bar{r} \leq \bar{r}_{int}$), and there are limits on use of individual reserves ($r \leq r_{cap}$).

With sufficient regularity assumptions and a given unit commitment ($\tilde{u}$), if ($\tilde{d}^0, \tilde{g}^0, \tilde{r}, \tilde{\bar{r}}$) is a solution of the optimal dispatch, then it is also a solution of the approximation problem using "demand curves" to characterize the zonal value of expected unserved energy.

$$\underset{y^0,d^0,g^0,r,\bar{r}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, \tilde{u}\right) - \nabla ZVEUE\left(\tilde{d}^0, \tilde{g}^0, \tilde{r}, \tilde{\bar{r}}, \tilde{u}\right)^t \left(d^0, g^0, r, \bar{r}\right)$$

$$s.t.$$

$$y^0 = d^0 - g^0,$$

$$H^0 y^0 \le b^0,$$

$$g^0 + r \le \tilde{u} \bullet Cap^0,$$

$$A^0 y^0 + \bar{r} \le \bar{r}_{int},$$

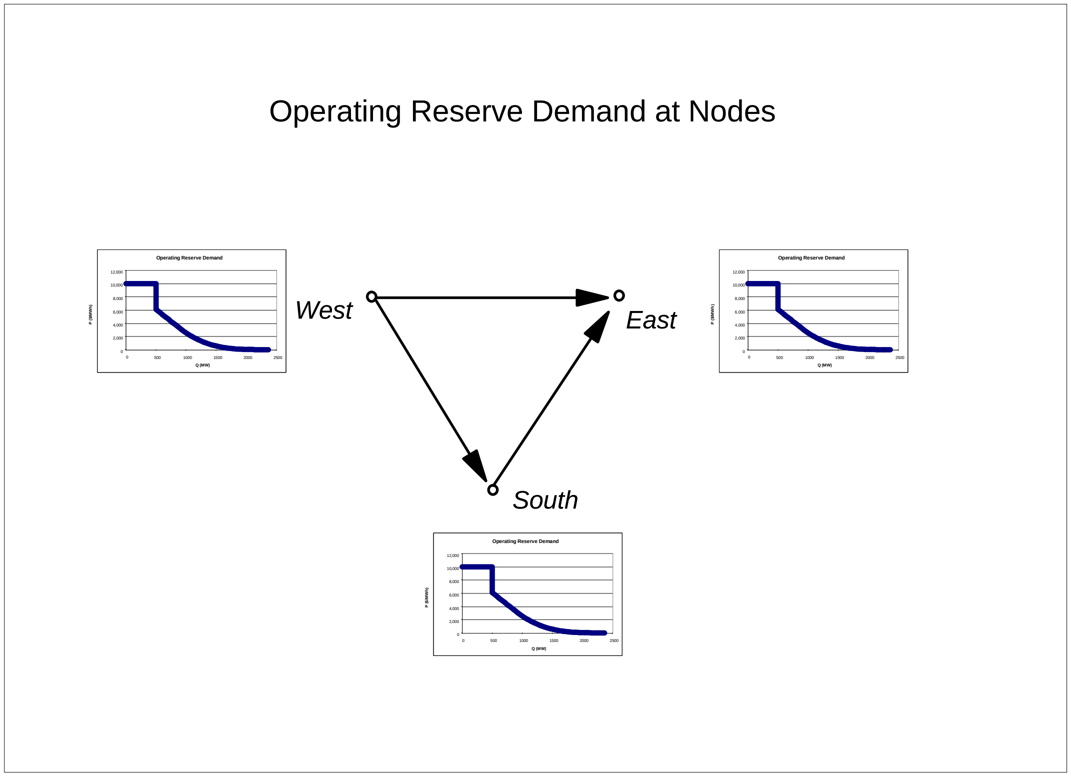
$$r \le r_{cap},$$

$$\iota^t y^0 = 0,$$

$$g^0 \le \tilde{u} \bullet Cap^0.$$

Hence if we could characterize the gradient of $ZVEUE$, this would open the way to an iterative solution of the dispatch problem with a demand curve for operating reserves (e.g, PIES method with diagonal demand). The gradient also allows estimation of $ZVEUE$ needed for full unit commitment problem.

Suppose that the *LOLP* distribution at each node could be calculated.[12] This would give rise to an operating reserve demand curve at each node.



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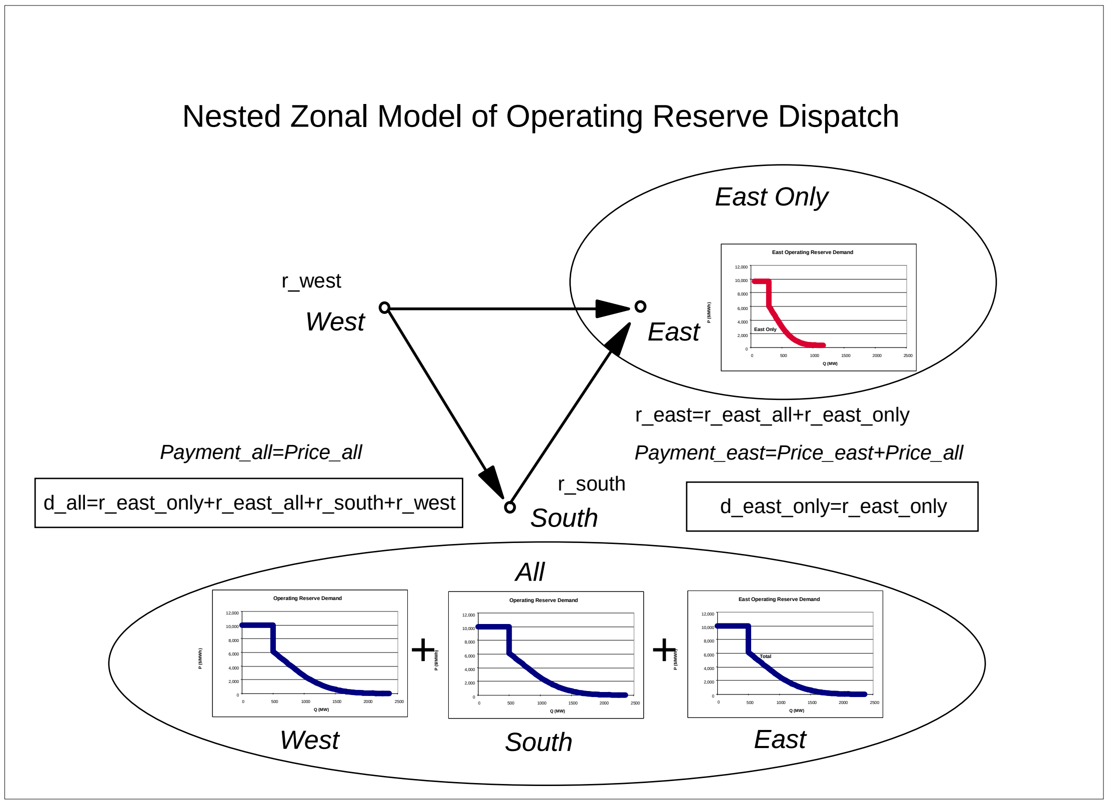
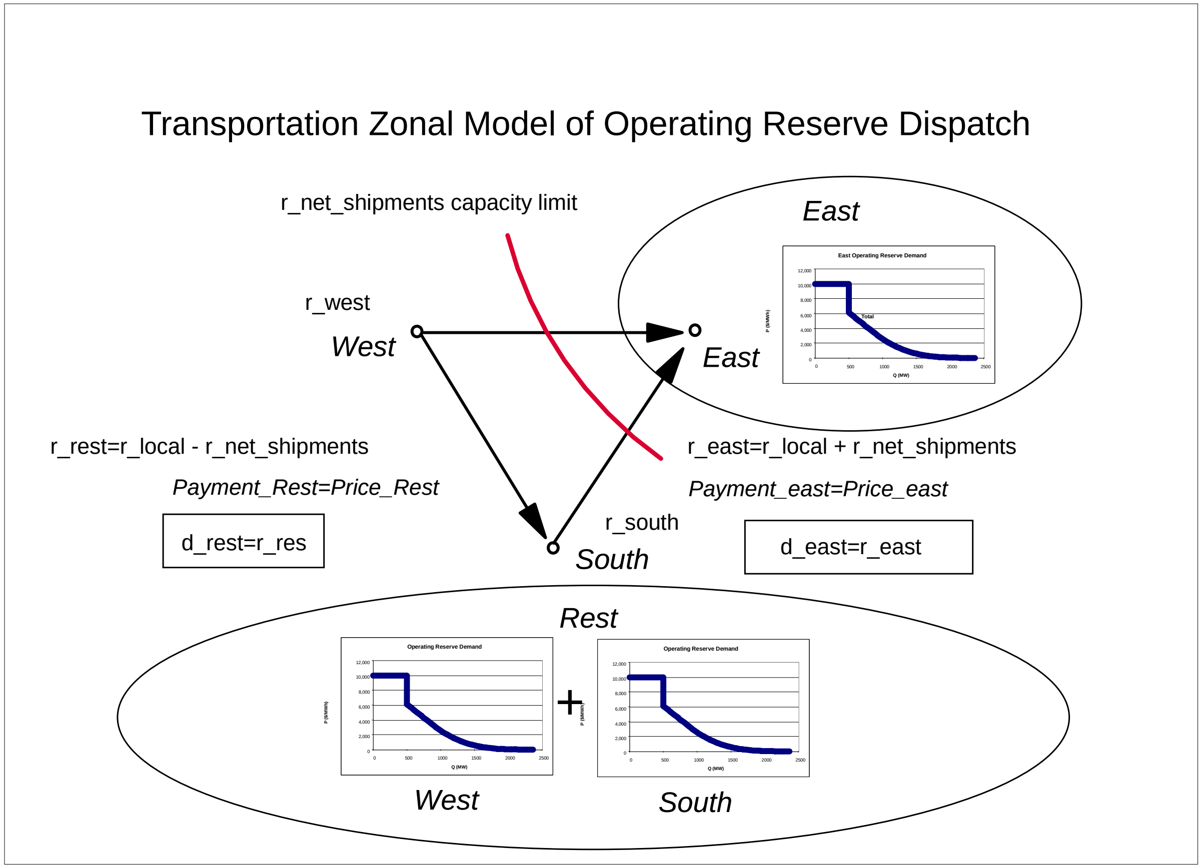
12 Eugene G. Preston, W. Mack Grady, Martin L. Baughman, "A New Planning Model for Assessing the Effects of Transmission Capacity Constraints on the Reliability of Generation Supply for Large Nonequivalenced Electric Networks," IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997, pp. 1367-1373. J. Choi, R. Billinton, and M. Futuhi-Firuzabed, "Development of a Nodal Effective Load Model Considering Transmission System Element Unavailabilities," IEE Proceedings - Generation, Transmission and Distribution, Vol. 152, No. 1, January 2005, pp. 79-89.

The next piece is a model of simultaneous dispatch of operating reserves and energy. One approach for the operating reserve piece is a cascading zonal model (e.g., NYISO reserve pricing).



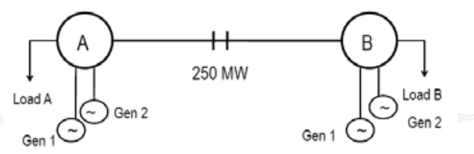
The result is that the input operating reserve price functions are additive premiums that give rise to an implicit operating reserve demand curves with higher prices.

An alternative approach would be to overlay a transportation model with interface transfer limits on operating reserve "shipments." The resulting prices are on the demand curves, but the model requires estimation of the (dynamic) transfer capacities. This is similar to the PJM installed capacity deliverability model, but specified an hour ahead rather than years ahead.



A PJM example illustrates a cascading model of operating reserve demand curves of the type in use in RTOs. There is a reserve demand for Zone B, and a reserve demand for the total including Zone A and Zone B. Transmission capacity can be used for energy or reserved for operating reserves. The reserves have individual limits (e.g., ramping) and joint limits with energy.[13]

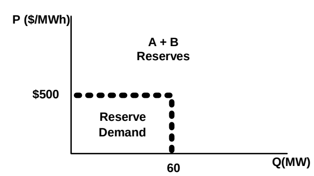
Operating Reserve Demand Curve Example

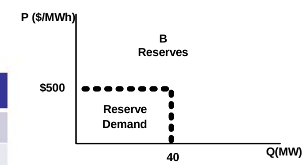


Generator	Energy Offer ($/MWh)	(Eco Min) MW	Total Capacity (Eco Max) (MW)	10 minute Reserve Capability (MW)
Zone A				
1	$20	0	600	60
2	$20 + ½ P_2^A	0	400	40
Zone B				
1	$60	0	400	40
2	$100 + P_2^B	0	300	30

10 Minute Reserve Requirement and Penalty Factors

Zone	Requirement (MW)	Penalty Factor ($/MW)
A+B (RTO)	60	$500
B	40	$500





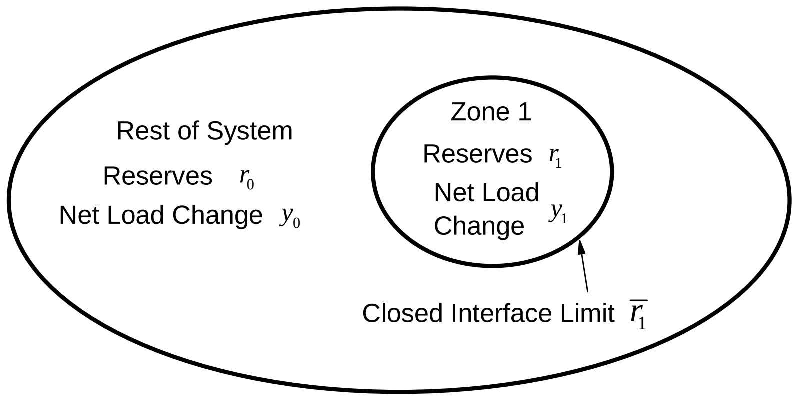
13 PJM, "ORDC/RCPF Example to Show Locational Price Impacts-Part 1," Scarcity Pricing Working Group, September 3, 2009.

Different variants of operating reserve demand curves can be and have been integrated with energy dispatch. A challenge for any locational operating reserve demand curve is to define a framework for deriving the form of the demand curve.

- **Generalize Loss of Load Probability (LOLP) and expected unserved energy from the aggregate system.** The simple model of loss of load from random changes in demand and generation provides a starting point but does not address locational interactions.
- **Integrate reservation of interface capacity.** A zonal model of interface capacity would include tradeoffs between normal energy dispatch and reservation of interface capacity to allow transfer of operating reserves.
- **Derive interaction between reserves in different locations.** Under some conditions, reserves in one location can support outages in another location.

The task is to define a locational operating reserve model that approximates and prices the dispatch decisions made by operators. To illustrate, consider the simplest case with one constrained zone and the rest of the system. The reserves are defined separately and there is a known transfer limit for the closed interface between the constrained zone and the rest of the system.

Zonal Interface Limit on Emergency Transfers



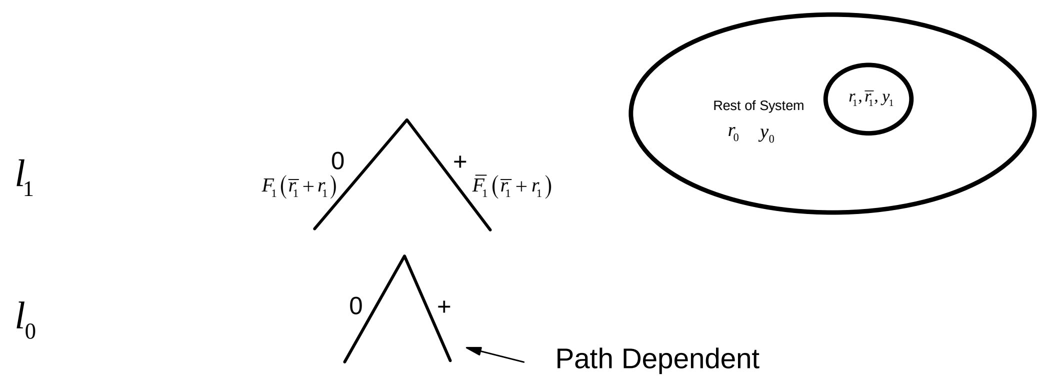
$$y_0 \sim f_0, y_1 \sim f_1, F_o(y_0) = \int_{-\infty}^{y_0} f_0(x_0)\,dx_0, F_1(y_1) = \int_{-\infty}^{y_1} f_1(x_1)\,dx_1$$

The zonal value of expected unserved energy (ZVEUE) would be an added component of the objective function in economic dispatch. The basic problem determines the configuration of lost load. The derivatives of ZVEUE define the demand curves for operating reserves.

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0l_0+v_1l_1\middle|y_0+y_1-l_0-l_1\leq r_0+r_1,\,y_1-l_1\leq\overline{r}_1+r_1\right\}\right]$$

Loss of Load Probability Structure

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0l_0+v_1l_1\middle|y_0+y_1-l_0-l_1\leq r_0+r_1,\,y_1-l_1\leq\overline{r}_1+r_1\right\}\right]$$



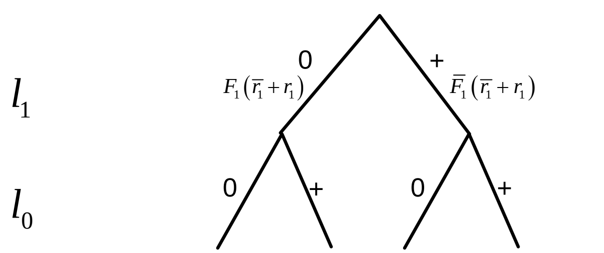
$$y_i \sim f_i,\; F_i\left(y_i\right)=\int_{-\infty}^{y_i} f_i\left(x_i\right)dx_i \qquad VOLL_0=v_0\leq VOLL_1=v_1.$$

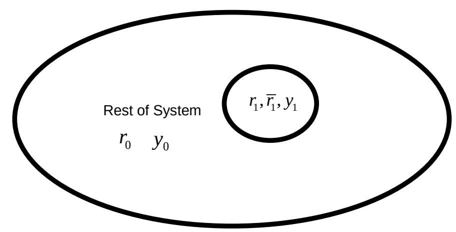
The full ZVEUE is difficult to characterize and calculate. However, inspection of the possible configurations of outages reveals the marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves

Loss of Load Probabilities

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \,\middle|\, y_0+y_1-l_0-l_1\leq r_0+r_1,\ y_1-l_1\leq\overline{r}_1+r_1\right\}\right]$$

Reserve Incremental Values



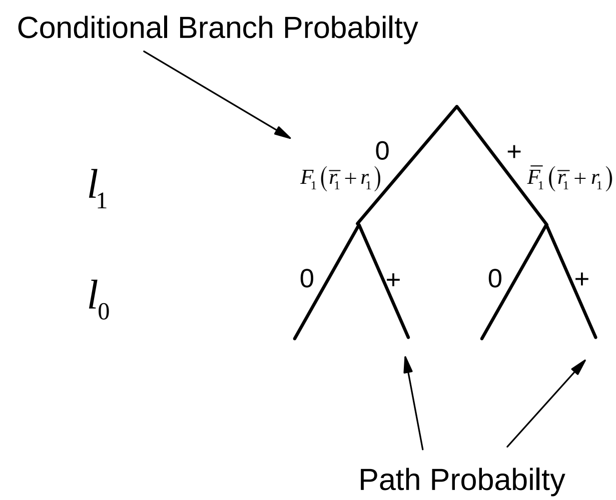


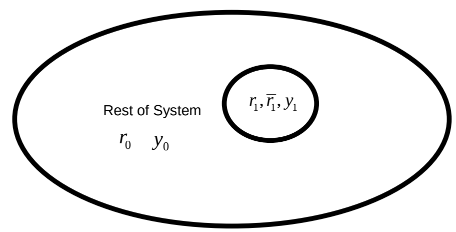
r_1	0	v_0	v_1	v_1
$\overline{r}_1$	0	0	v_1	v_1-v_0
r_0	0	v_0	0	v_0

The full ZVEUE is difficult to characterize and calculate. However, inspection of the possible configurations of outages reveals the probabilities for the possible marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves.

Loss of Load Probabilities

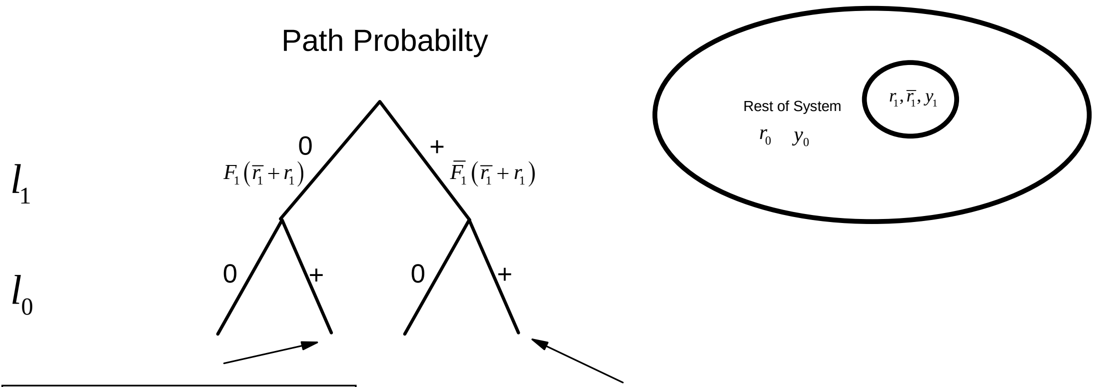
$$ZVEUE\left(r_0, \overline{r}_1, r_1\right) = E_y\left[\underset{l_i \geq 0}{Min} \quad \left\{ v_0 l_0 + v_1 l_1 \middle| y_0 + y_1 - l_0 - l_1 \leq r_0 + r_1, y_1 - l_1 \leq \overline{r}_1 + r_1 \right\}\right]$$





Assuming locational independence of outages, it is straightforward to calculate the probabilities on each path. The loss of load probabilities times the locational VOLL yields the operating reserve demand as a function of all the locational reserves and interface capacities.

Demand Curve Elements



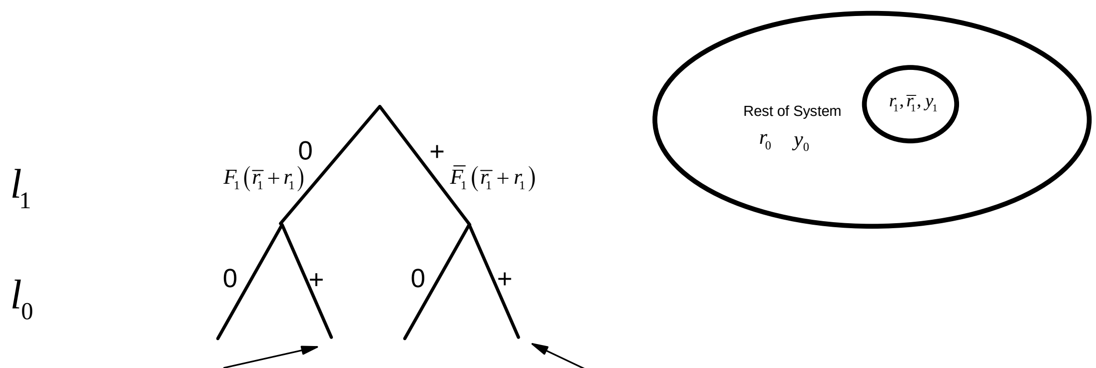
$$\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1-x_1}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i = \int_{-\infty}^{\bar{r}_1+r_1}\bar{F}_0(r_0+r_1-x_1)\,f_1(x_1)\,dx_1$$

$$\int_{\bar{r}_1+r_1}^{\infty}\int_{r_0-\bar{r}_1}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i = \bar{F}_1(\bar{r}_1+r_1)\,\bar{F}_0(r_0-\bar{r}_1)$$

Assuming locational independence of outages, it is straightforward to calculate the probabilities on each path. The loss of load probabilities times the locational VOLL yields the operating reserve demand as a function of all the locational reserves and interface capacities.

Demand Curve Elements: Rest of System

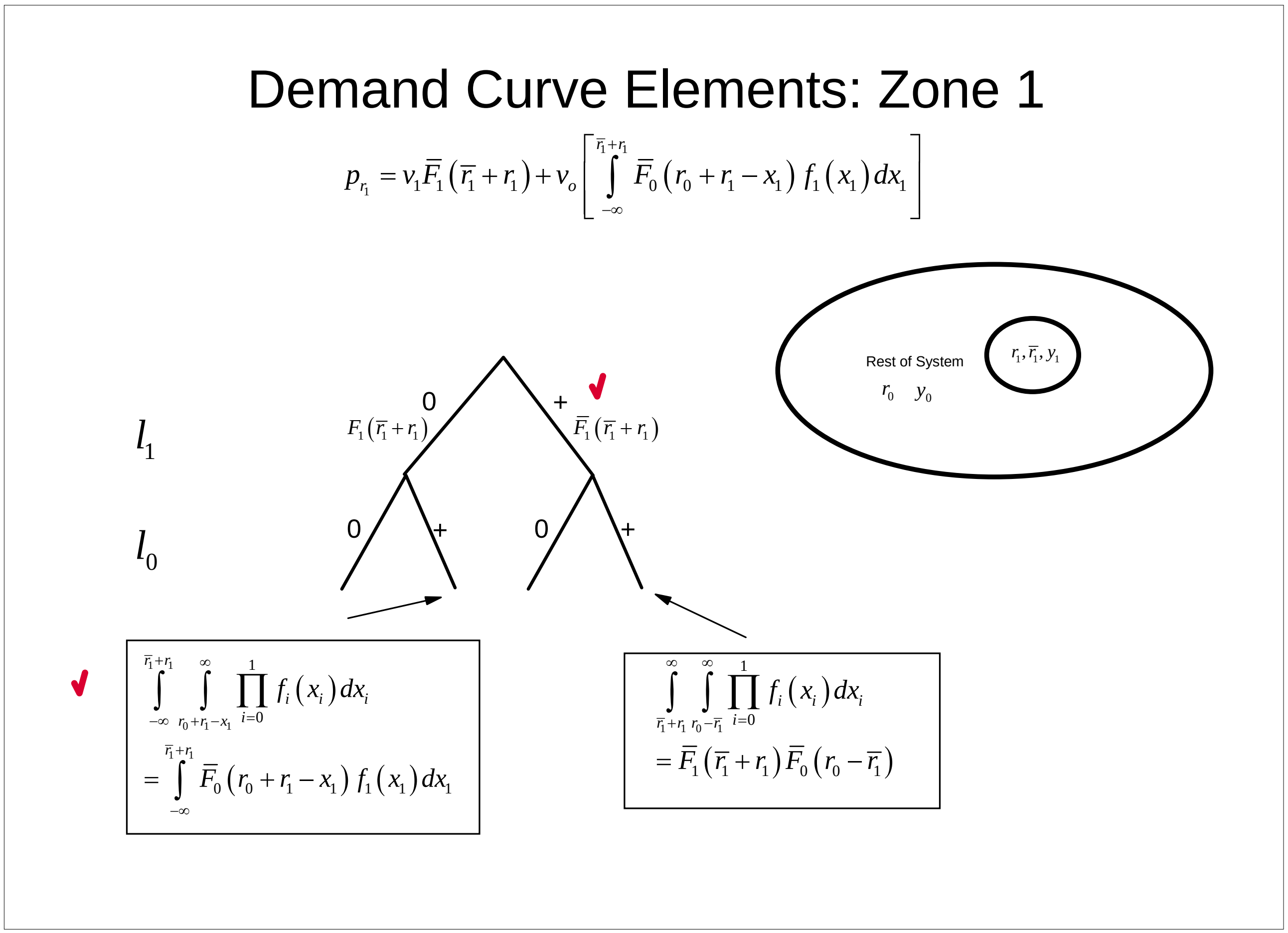
$$p_{r_0} = v_o\left[\int_{-\infty}^{\overline{r}_1 + r_1} \overline{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1 + \overline{F}_1\left(\overline{r}_1 + r_1\right)\overline{F}_0\left(r_0 - \overline{r}_1\right)\right]$$



$$\int_{-\infty}^{\overline{r}_1+r_1}\int_{r_0+r_1-x_1}^{\infty}\prod_{i=0}^{1} f_i\left(x_i\right)dx_i = \int_{-\infty}^{\overline{r}_1+r_1}\overline{F}_0\left(r_0+r_1-x_1\right) f_1\left(x_1\right)dx_1$$

$$\int_{\overline{r}_1+r_1}^{\infty}\int_{r_0-\overline{r}_1}^{\infty}\prod_{i=0}^{1} f_i\left(x_i\right)dx_i = \overline{F}_1\left(\overline{r}_1+r_1\right)\overline{F}_0\left(r_0-\overline{r}_1\right)$$

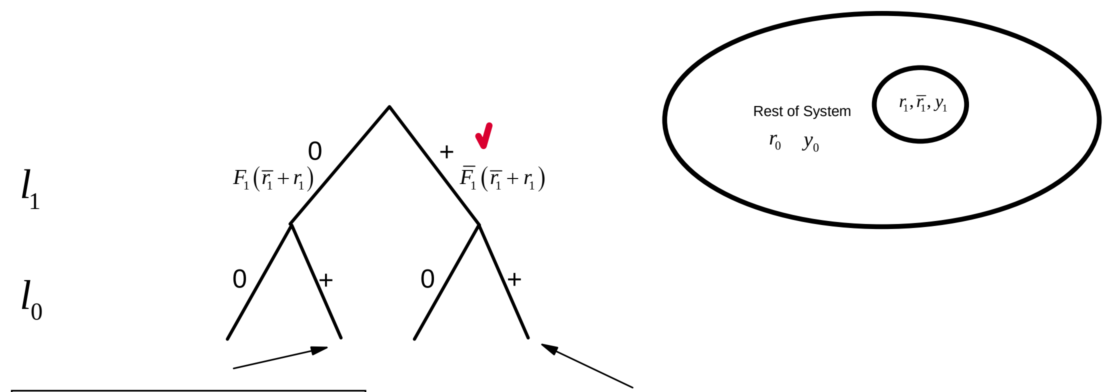
A similar inspection of the possible paths in the trees identifies the probability that an increment of operating reserve would change the unserved energy. The possible configurations of outages reveals the marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves.



A similar calculation provides the demand for interface capacity as a function of the level of locational operating reserves and interface capacity.

Demand Curve Elements: Interface

$$p_{\bar{r}_1} = v_1 \bar{F}_1(\bar{r}_1 + r_1) - v_0 \left[\bar{F}_1(\bar{r}_1 + r_1) \bar{F}_0(r_0 - \bar{r}_1) \right]$$



$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i = \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0(r_0 + r_1 - x_1) f_1(x_1)\, dx_1$$

$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i = \bar{F}_1(\bar{r}_1 + r_1) \bar{F}_0(r_0 - \bar{r}_1)$$

The probability trees provide a workable means for beginning with the locational probability distributions of load and outages and calculating the resulting demand curves. The appendix outlines the extensions to multiple nested and parallel zones.

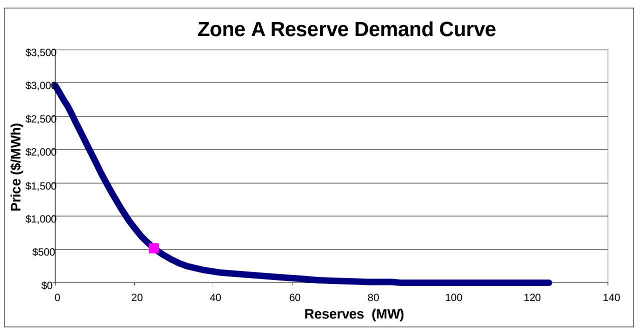
The implied demand curves illustrate critical properties.

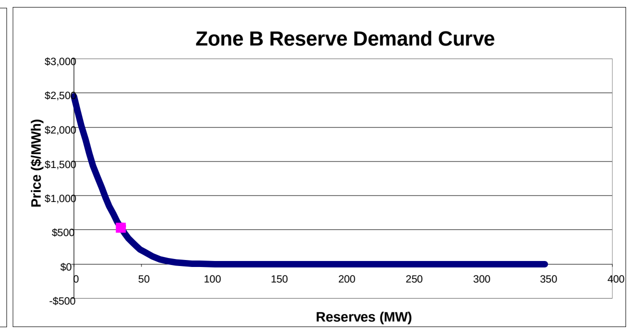
- **Interaction.** The demand curves are interdependent, but the dependence is not in the form of the nested or cascading model often assumed.
- **Maximum Value.** The value of loss load in the zone is an upper bound for the reserve price in the zone.
- **Convergence.** As the interface capacity increases, the implied demand curves in the constrained zone and for the rest of the system converge to the same prices.
- **Interface Demand.** In addition to the demand for operating reserves, there is an implied demand curve for the interface transfer limit.
- **No Thresholds.** The implied demand curve scarcity prices are positive at all levels. At higher reserves the prices are lower, but there is no threshold where the scarcity price falls to zero.

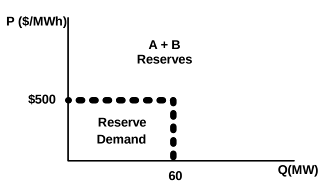
To illustrate application of the interdependent zonal model and the cascading zonal model in the PJM example, requires the underlying outage distribution. The benchmark choice of parameters approximates the assumptions of the PJM cascade model with infinite interface capacity.

Benchmarking Operating Reserve Demand

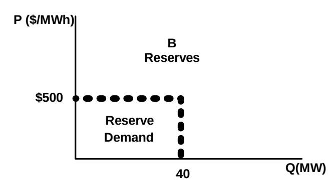
Load in: Zone A =500MW, Zone B=700MW







Outages	Zone A	Zone B
Expected Total (MW)	9.60	14.40
Std Dev (MW)	16.64	24.96
VOLL ($/MWh)	6000	10000



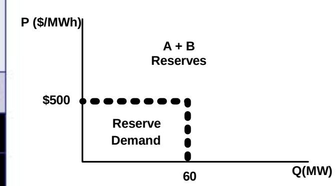
At the benchmark load of Zone A at 500 MW and Zone B at 700 MW, economic dispatch with the cascade model produces energy and reserve dispatch with associated prices. Reserve prices are positive because of the energy redispatch required to maintain reserve levels.

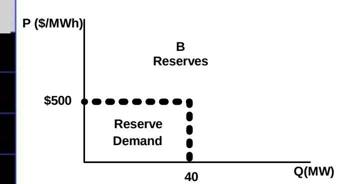
Dispatch and Prices at Benchmark Load

pjm

No Reserve Shortage, Congestion
Load A = 500, Load B = 700 MW

Generator	Running Cost ($/MWh)	Output (MW)	Reserve (MW)	Energy Price ($/MWh)	Reserve Price ($/MWh)
Zone A				$90	$0
1	$20	600	0	[redacted]	[redacted]
2	$20 + ½ P_2^A = $90	140	40	[redacted]	[redacted]
Zone Total		740	40	[redacted]	[redacted]
Zone B				$160	$70
1	$60	400	0	[redacted]	[redacted]
2	$100 + P_2^B = $160	60	30	[redacted]	[redacted]
Flow A to B		240	10	[redacted]	[redacted]
Deliverable		700	40	[redacted]	[redacted]
RTO Total		1200	70		

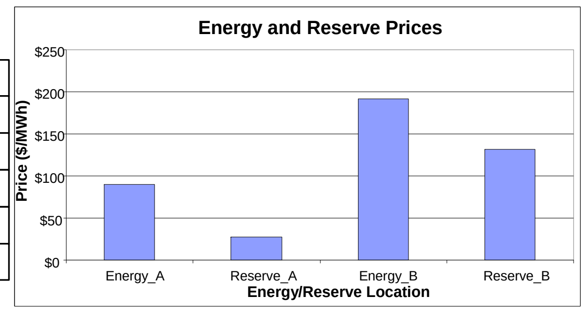


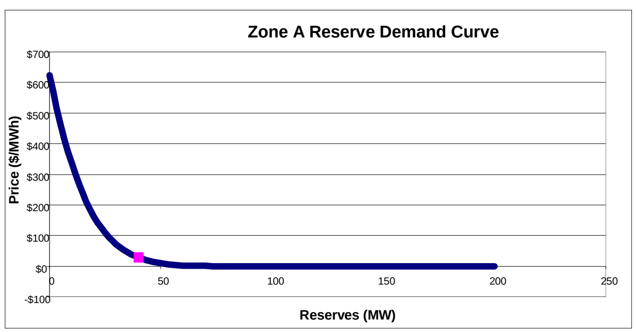


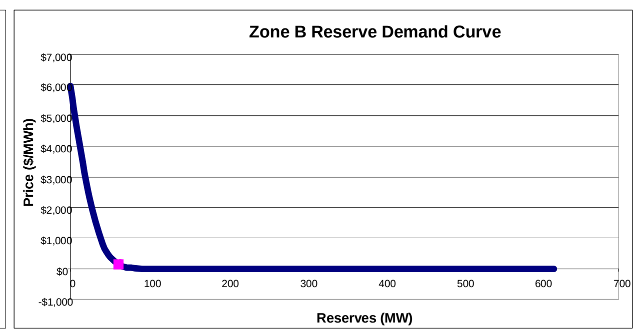
At the benchmark loads, the interdependent locational demand curves yield similar dispatch and locational prices. However, both reserve prices are positive, reflecting the continuous nature of the alternative operating reserve demand curve. The figure indicates the local projection of the operating reserve demand curves at the economic dispatch solution.

Dispatch and Prices at Benchmark Load

Load A	500	Load B	700
	Energy	Reserves	
GA_1	600	0	
GA_2	140	40	
GB_1	368	32	
GB_2	92	30	





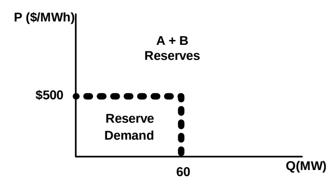


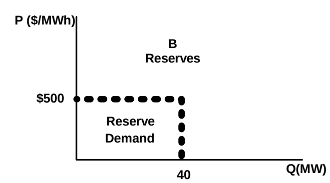
Increasing the load in Zone B fully triggers reserve shortages and the assumed operating reserve penalty factor of $500/MWh.

Dispatch and Prices at High Load

Reserve Shortage in Zone B, Congestion
Load A = 500, Load B = 925 MW

Generator	Running Cost ($/MWh)	Output (MW)	Reserve (MW)	Energy Price ($/MWh)	Reserve Price ($/MWh)
Zone A				$95	$0
1	$20	600	0		
2	$20 + ½ P_2^A = $95	150	40		
Zone Total		750	40		
Zone B				$875	$500
1	$60	400	0		
2	$100 + P_2^B = $375	275	25		
Flow A to B		250	0		
Deliverable		925	25		
RTO Total		1425	65		

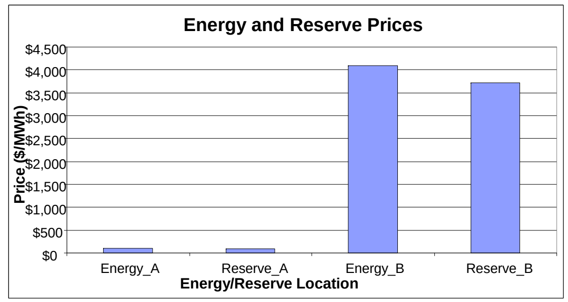


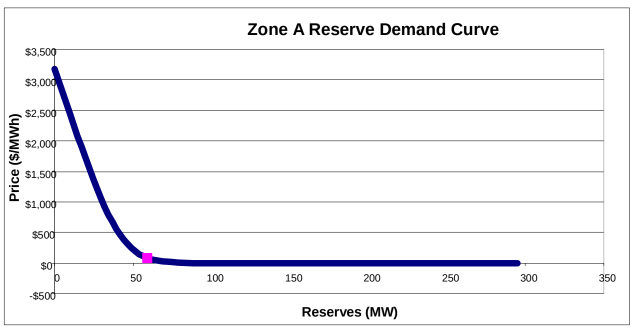


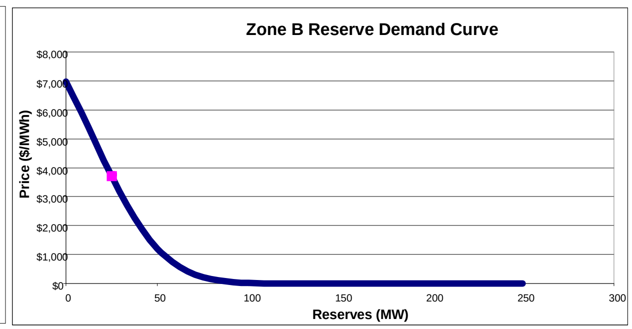
At high load, with the implied shortage of operating reserve, the demand prices for reserves and energy increase substantially in the constrained Zone B.

Dispatch and Prices at High Load

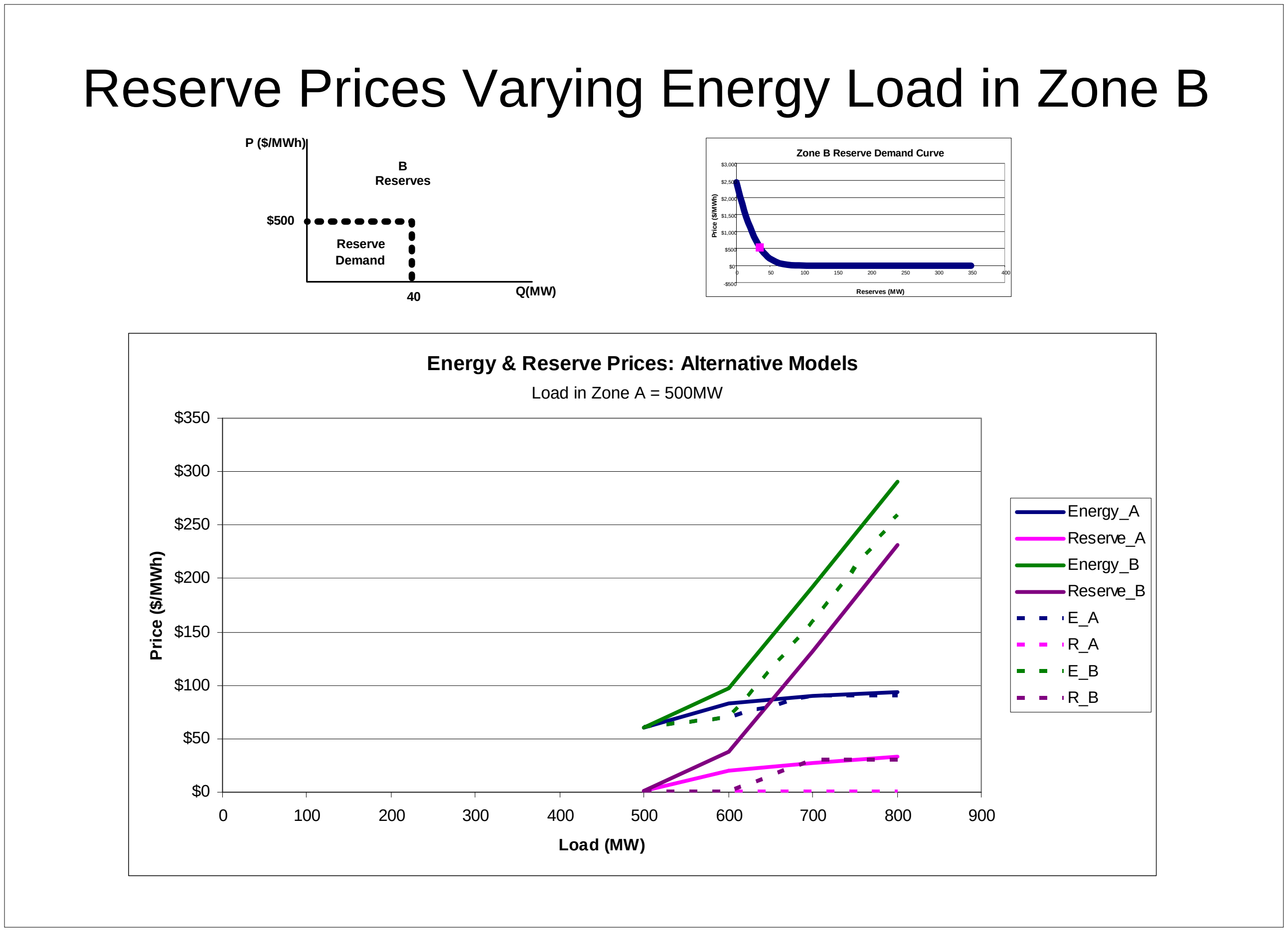
Load A	500	Load B	925
	Energy	Reserves	
GA_1	581	19	
GA_2	169	40	
GB_1	400	0	
GB_2	275	25	



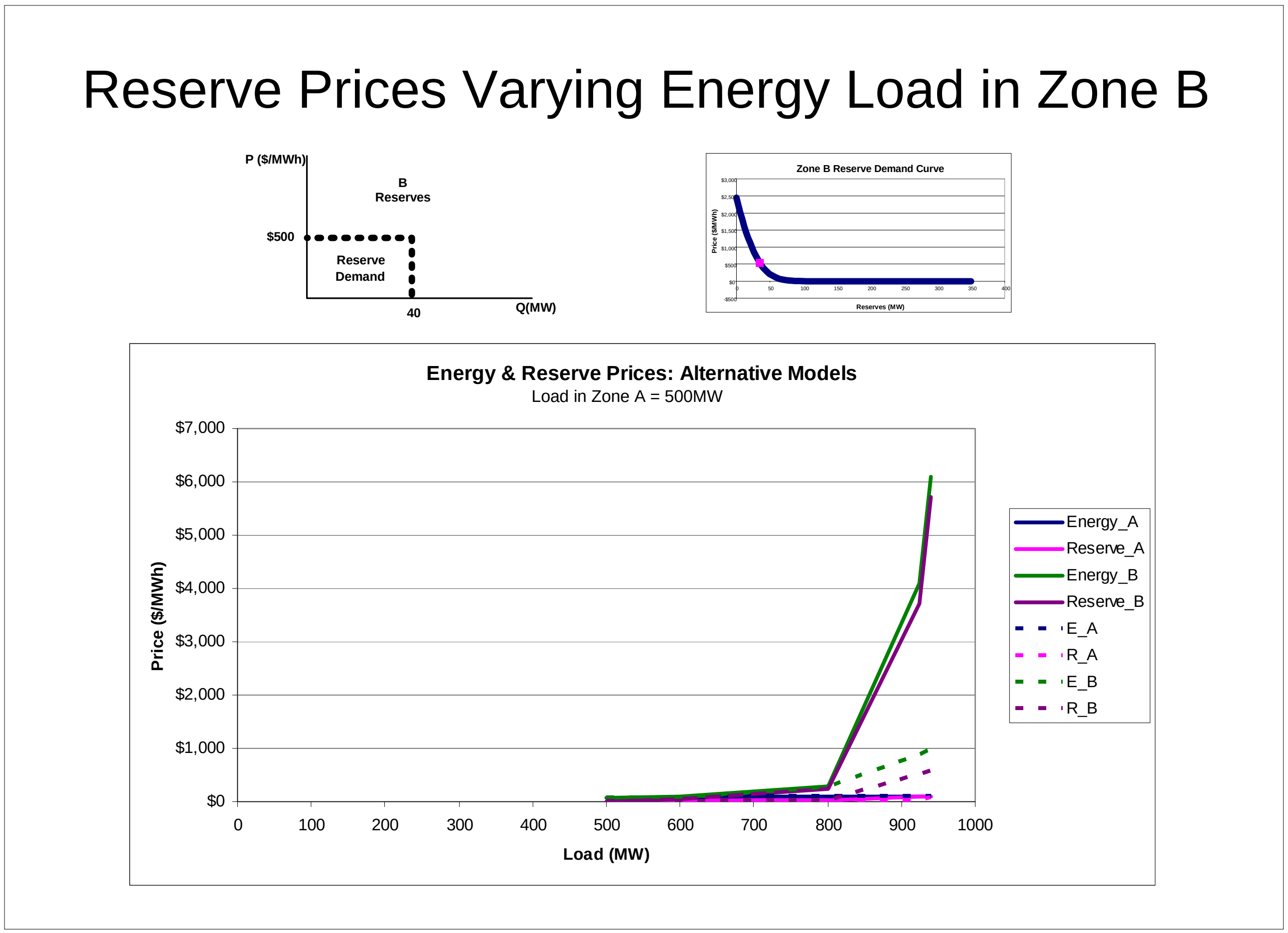




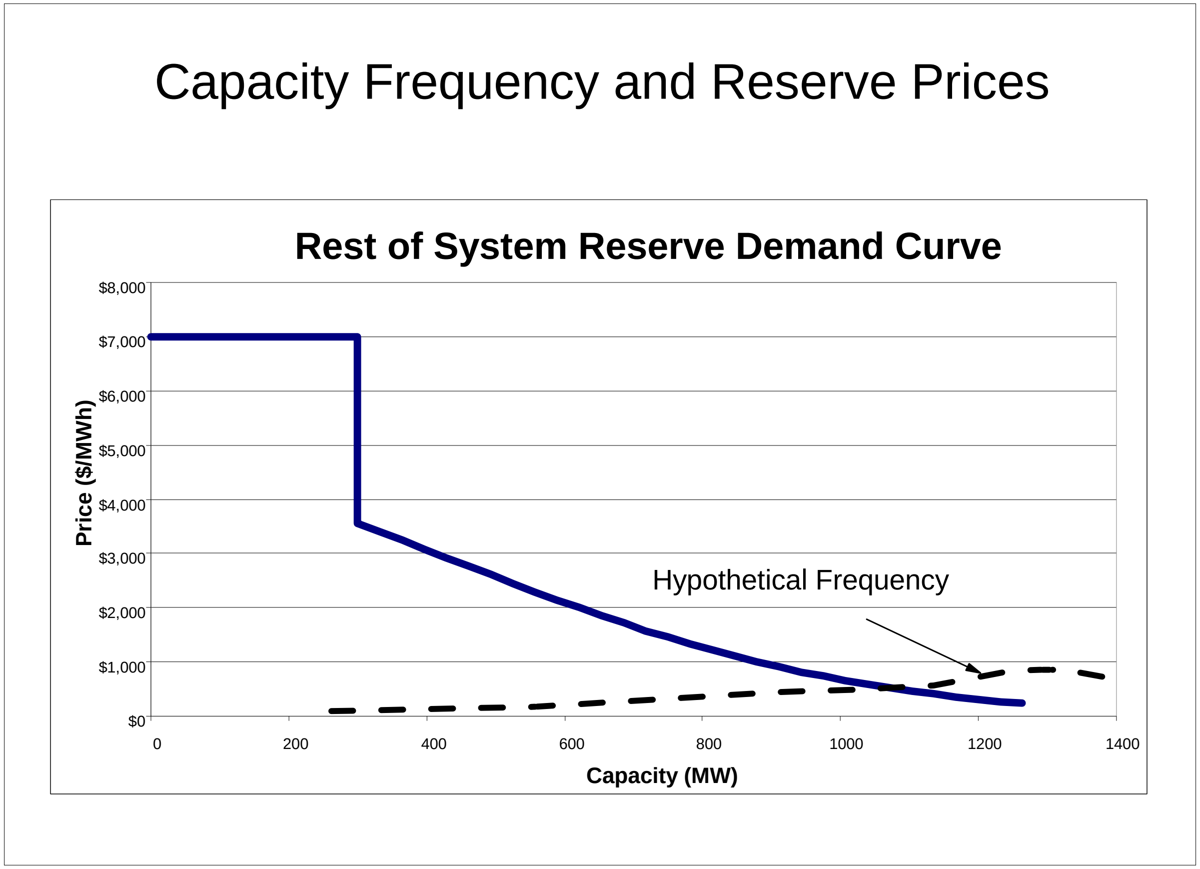
Varying the load at Zone B illustrates the differences in energy and operating reserve locational prices for the PJM cascade assumptions and the alternative interdependent demand curves. Operating reserve prices are generally higher for the interdependent demand curves.



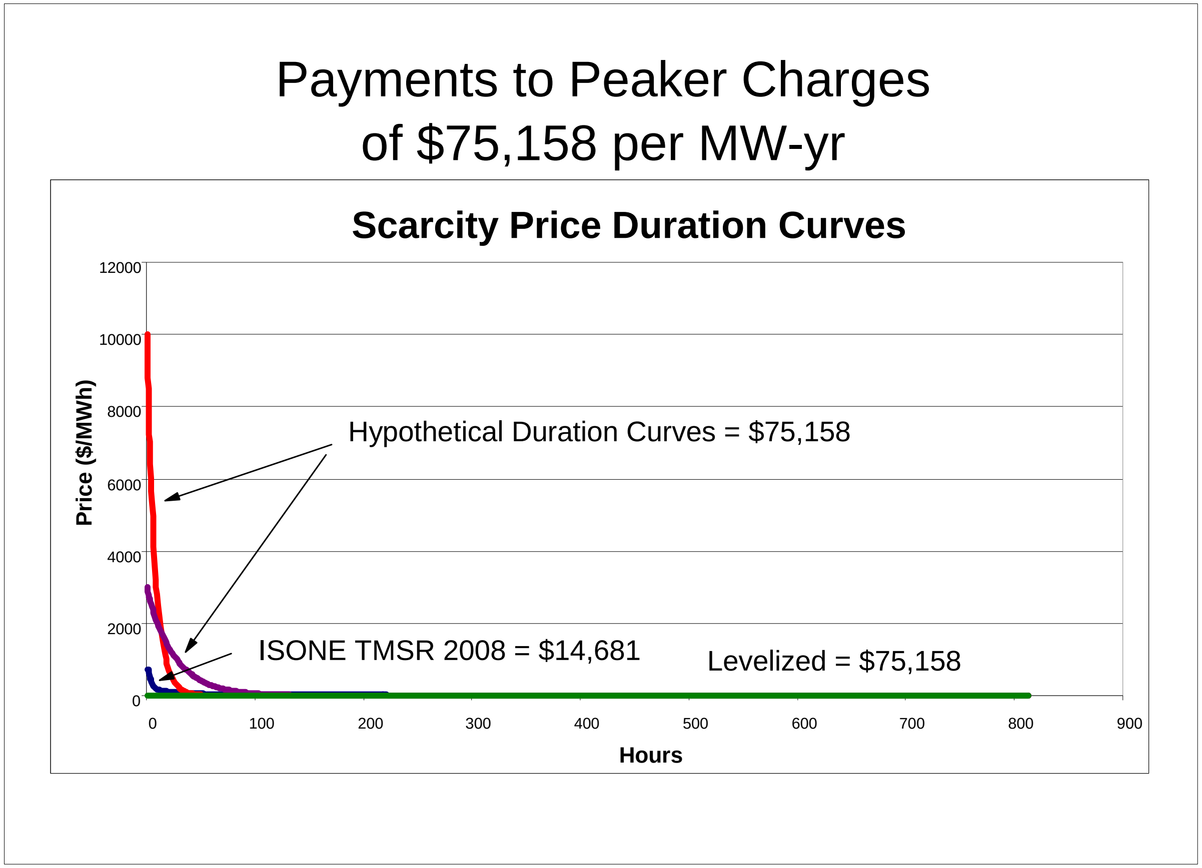
At very high loads in Zone B, the difference in scarcity prices between the alternative models is more pronounced.



An interesting question is the frequency of different reserve levels and the interaction with the operating reserve demand curve. This will determine the scarcity price duration curve.



Different scarcity pricing duration curves will determine the contribution of scarcity prices to total payments for energy and reserves. For example, consider the PJM estimate of a fixed charge for a peaker at $75,158 per MW-yr. The hypotheticals illustrate consistent alternative duration curves. These are compared with the actual 2008 price duration curve in ISONE for ten minute spinning reserves (TMSR) for location ID 7000.



Improved pricing through an explicit operating reserve demand curve raises a number of issues.

Demand Response: Better pricing implemented through the operating reserve demand curve would provide an important signal and incentive for flexible demand participation in spot markets.

Price Spikes: A higher price would be part of the solution. Furthermore, the contribution to the "missing money" from better pricing would involve many more hours and smaller price increases.

Practical Implementation: The NYISO, ISONE and MISO implementations dispose of any argument that it would be impractical to implement an operating reserve demand curve. The only issues are the level of the appropriate price and the preferred model of locational reserves.

Operating Procedures: Implementing an operating reserve demand curve does not require changing the practices of system operators. Reserve and energy prices would be determined simultaneously treating decisions by the operators as being consistent with the adopted operating reserve demand curve.

Multiple Reserves: The demand curve would include different kinds of operating reserves, from spinning reserves to standby reserves.

Reliability: Market operating incentives would be better aligned with reliability requirements.

Market Power: Better pricing would remove ambiguity from analyses of high prices and distinguish (inefficient) economic withholding through high offers from (efficient) scarcity pricing derived from the operating reserve demand curve.

Hedging: The Basic Generation Service auction in New Jersey provides a prominent example that would yield an easy means for hedging small customers with better pricing.

Increased Costs: The higher average energy costs from use of an operating reserve demand curve do not automatically translate into higher costs for customers. In the aggregate, there is an argument that costs would be lower.

Compared to a perfect model, there are many simplifying assumptions needed to specify and operating reserve demand curve. The sketch of the operating reserve demand curve(s) in a network could be extended.

- **Empirical Estimation.** Use existing LOLP models or LOLP extensions with networks to estimate approximate LOLP distributions at nodes.
- **Value of Lost Load.** There are different estimates of lost load. For demand curve estimation the relevant value is the marginal of the average VOLL across the group that would first be curtailed in the event of an outage greater than the available reserves.
- **Multiple Periods.** Incorporate multiple periods of commitment and response time. Handled through the usual supply limits on ramping.
- **Operating Rules.** Incorporate up and down ramp rates, deratings, emergency procedures, etc.
- **Pricing incidence.** Charging participants for impact on operating reserve costs, with any balance included in uplift.[14]
- **Minimum Uplift Pricing.** Dispatch-based pricing that resolves inconsistencies by minimizing the total value of the price discrepancies.
- **…**

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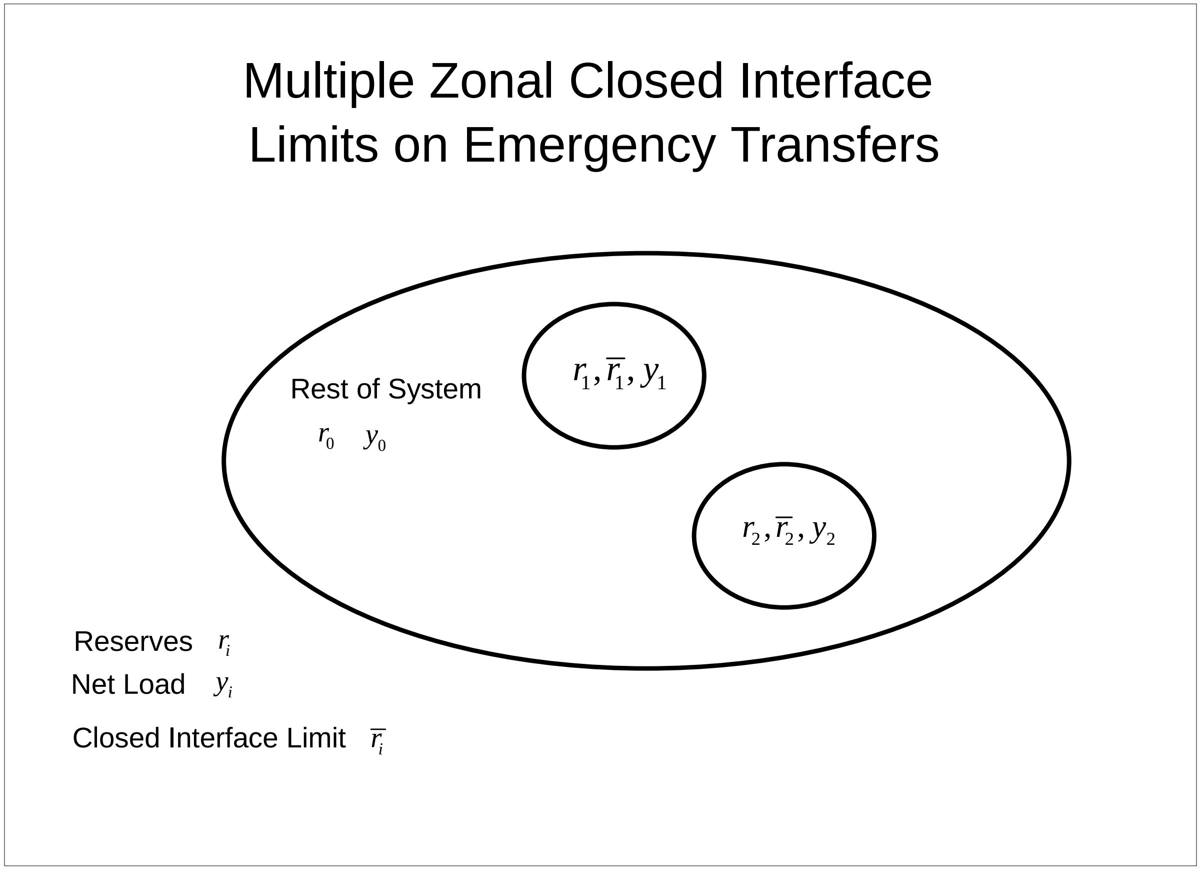
14 Brendan Kirby and Eric Hirst, "Allocating the Cost of Contingency Reserves," *The Electricity Journal*, December 2003, 99. 39-47.

Supplemental material

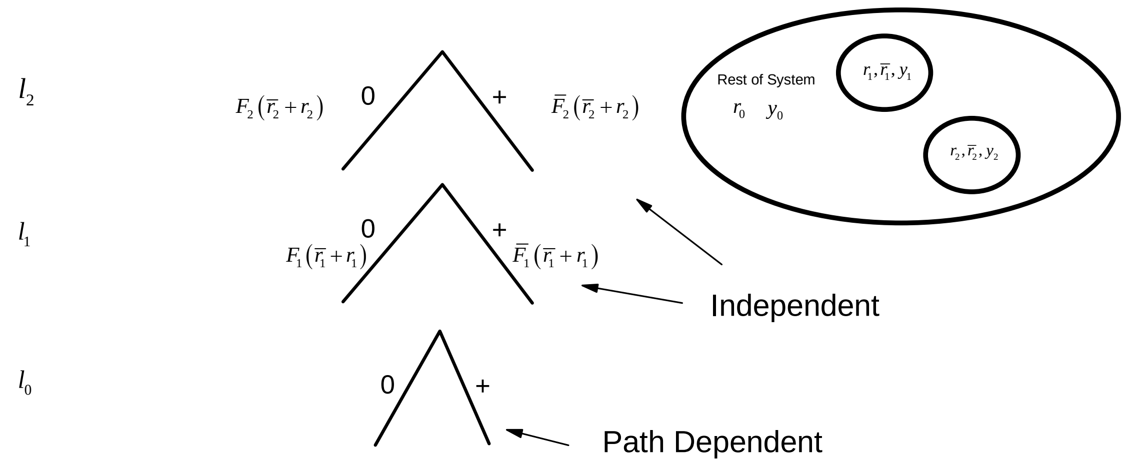
- **On design of operating reserve demand curve.**

The case of multiple constrained zones is a natural extension of the case for a single constrained zone.



The probability of losses depends on the path of binding interface constraints.

Loss of Load Probability Structure

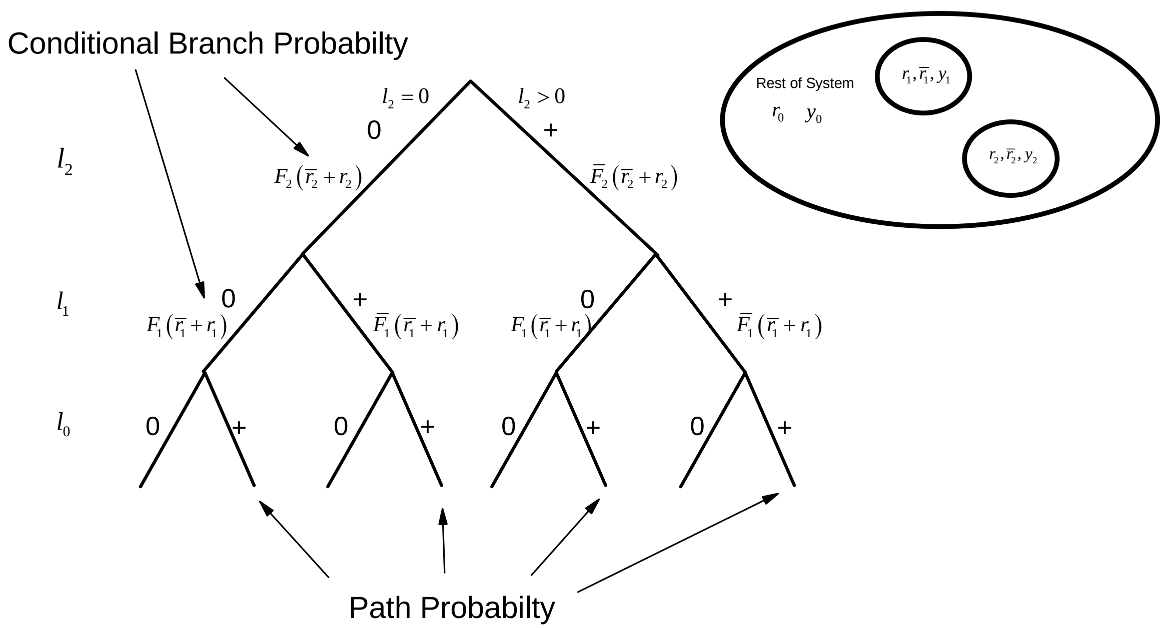


$$y_i \sim f_i,\ F_i(y_i) = \int_{-\infty}^{y_i} f_i(x_i)\,dx_i$$

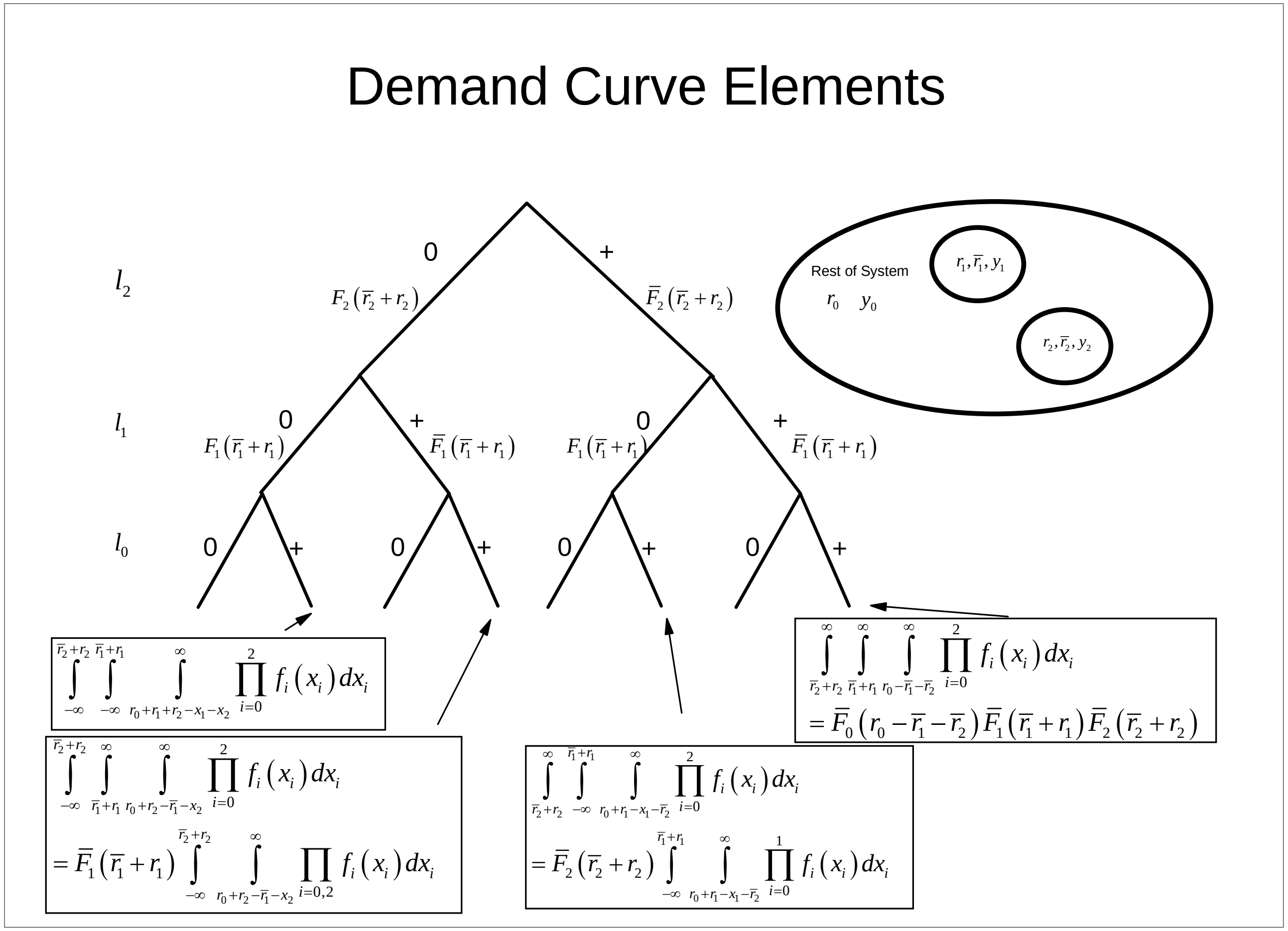
The probability tree captures the dependencies of loss of load.

Loss of Load Probabilities

$$ZVEUE\left(r_0,\bar{r}_1,r_1,\bar{r}_2,r_2\right)=E_y\left[\underset{l_i\ge 0}{Min}\ \left\{v_0 l_0+v_1 l_1+v_2 l_2 \middle| y_0+y_1+y_2-l_0-l_1-l_2\le r_0+r_1+r_2, y_1-l_1\le\bar{r}_1+r_1, y_2-l_2\le\bar{r}_2+r_2\right\}\right]$$



The loss of load probability structure defines the demand curve elements.



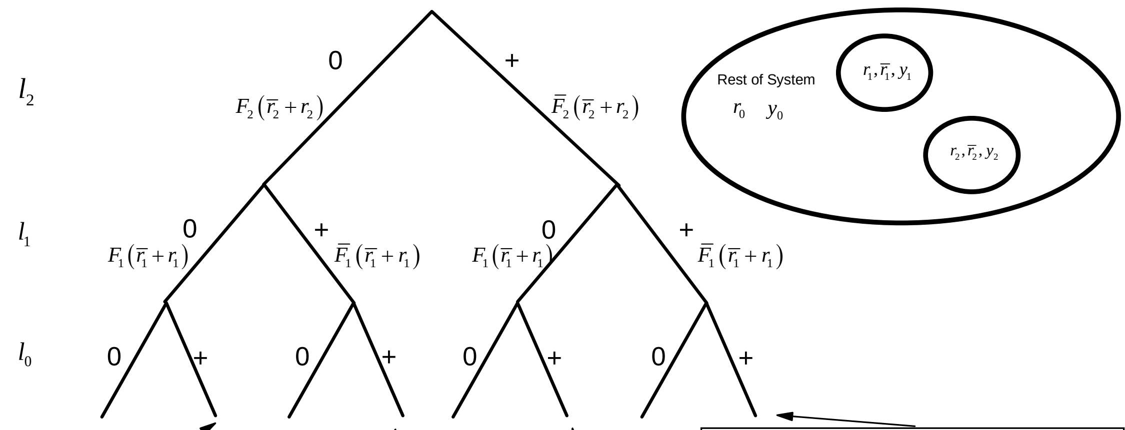
The tree structure identifies the loss probability dependencies and the paths where incremental capacity affects the losses.

- **Outages and Demand Changes.** The zonal convolutions of capacity outages and demand changes determine the (assumed independent) elementary zonal probability distributions of changes in net load.
- **Tree Structure.** The dependencies for losses and binding interface constraints defined by the probability tree structure determine the path probabilities for loss of load in each location as a function of the underlying independent elementary distributions.
- **Demand Curve.** The demand curve is determined by the value of lost load in each zone and the dependencies in the tree structure determining when reserves or interface capacity would be substitutable for losses.
  - **Value of Loss Load.** Assume embedded zones have higher incremental values of lost load.
  - **Substitution of Capacity.** Identify substitution possibilities on alternative paths for zonal losses and binding constraints. For example:
    - **Zonal Losses.** Apply only when interface constraint is binding.
    - **Reserve Substitution.** Higher level reserves substitute for lower level losses only when interface constraint is not binding.
    - **Interface Capacity.** Increased interface capacity for binding interface substitutes lower level losses for higher level losses.

The loss outcomes determine demand for rest of system operating reserve.

Demand Curve Elements: Rest of System

$$p_{r_0}=v_o\left[\int_{-\infty}^{\overline{r}_2+r_2}\int_{-\infty}^{\overline{r}_1+r_1}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2}f_i\left(x_i\right)dx_i+\overline{F}_1\left(\overline{r}_1+r_1\right)\int_{-\infty}^{\overline{r}_2+r_2}\int_{r_0+r_2-\overline{r}_1-x_2}^{\infty}\prod_{i=0,2}f_i\left(x_i\right)dx_i+\overline{F}_2\left(\overline{r}_2+r_2\right)\int_{-\infty}^{\overline{r}_1+r_1}\int_{r_0+r_1-x_1-\overline{r}_2}^{\infty}\prod_{i=0}^{1}f_i\left(x_i\right)dx_i+\overline{F}_0\left(r_0-\overline{r}_1-\overline{r}_2\right)\overline{F}_1\left(\overline{r}_1+r_1\right)\overline{F}_2\left(\overline{r}_2+r_2\right)\right]$$



$$\int_{-\infty}^{\overline{r}_2+r_2}\int_{-\infty}^{\overline{r}_1+r_1}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2}f_i\left(x_i\right)dx_i$$

$$\int_{-\infty}^{\overline{r}_2+r_2}\int_{\overline{r}_1+r_1}^{\infty}\int_{r_0+r_2-\overline{r}_1-x_2}^{\infty}\prod_{i=0}^{2}f_i\left(x_i\right)dx_i$$
$$=\overline{F}_1\left(\overline{r}_1+r_1\right)\int_{-\infty}^{\overline{r}_2+r_2}\int_{r_0+r_2-\overline{r}_1-x_2}^{\infty}\prod_{i=0,2}f_i\left(x_i\right)dx_i$$

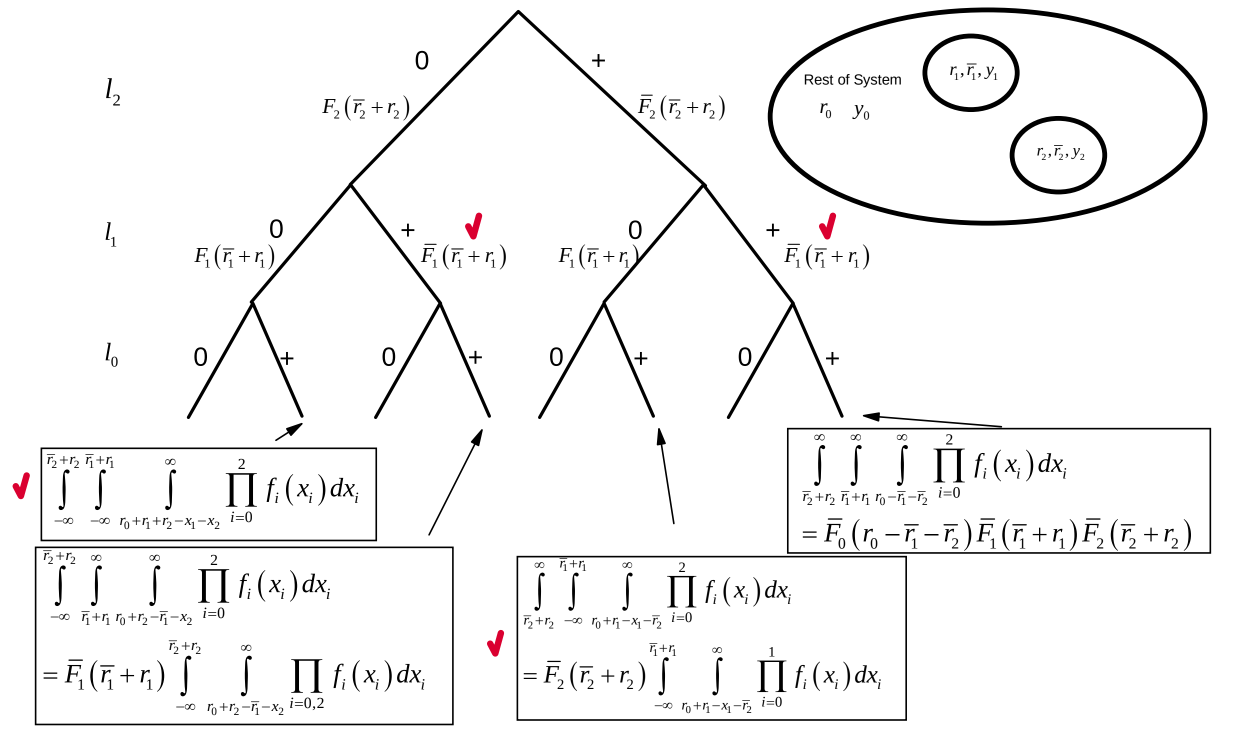
$$\int_{\overline{r}_2+r_2}^{\infty}\int_{-\infty}^{\overline{r}_1+r_1}\int_{r_0+r_1-x_1-\overline{r}_2}^{\infty}\prod_{i=0}^{2}f_i\left(x_i\right)dx_i$$
$$=\overline{F}_2\left(\overline{r}_2+r_2\right)\int_{-\infty}^{\overline{r}_1+r_1}\int_{r_0+r_1-x_1-\overline{r}_2}^{\infty}\prod_{i=0}^{1}f_i\left(x_i\right)dx_i$$

$$\int_{\overline{r}_2+r_2}^{\infty}\int_{\overline{r}_1+r_1}^{\infty}\int_{r_0-\overline{r}_1-\overline{r}_2}^{\infty}\prod_{i=0}^{2}f_i\left(x_i\right)dx_i$$
$$=\overline{F}_0\left(r_0-\overline{r}_1-\overline{r}_2\right)\overline{F}_1\left(\overline{r}_1+r_1\right)\overline{F}_2\left(\overline{r}_2+r_2\right)$$

The loss outcomes and dependencies determine the demand for zone 1 operating reserves.

Demand Curve Elements: Zone 1

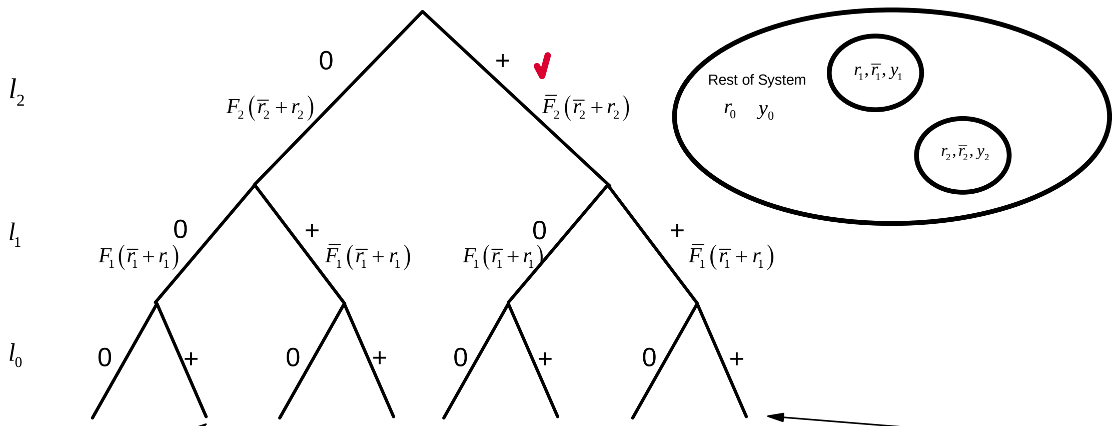
$$p_{r_r} = v_1 \overline{F}_1\left(\overline{r}_1 + r_1\right) + v_0 \left[\int_{-\infty}^{\overline{r}_2+r_2} \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i + \overline{F}_2\left(\overline{r}_2 + r_2\right) \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1-x_1-\overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i \right]$$



The loss outcomes and dependencies determine the demand for zone 2 operating reserves.

Demand Curve Elements: Zone 2

$$p_{r_2} = v_2\bar{F}_2\left(\bar{r}_2+r_2\right)+v_0\left[\int_{-\infty}^{\bar{r}_2+r_2}\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i\left(x_i\right)dx_i+\bar{F}_1\left(\bar{r}_1+r_1\right)\int_{-\infty}^{\bar{r}_2+r_2}\int_{r_0+r_2-\bar{r}_1-x_2}^{\infty}\prod_{i=0,2} f_i\left(x_i\right)dx_i\right]$$



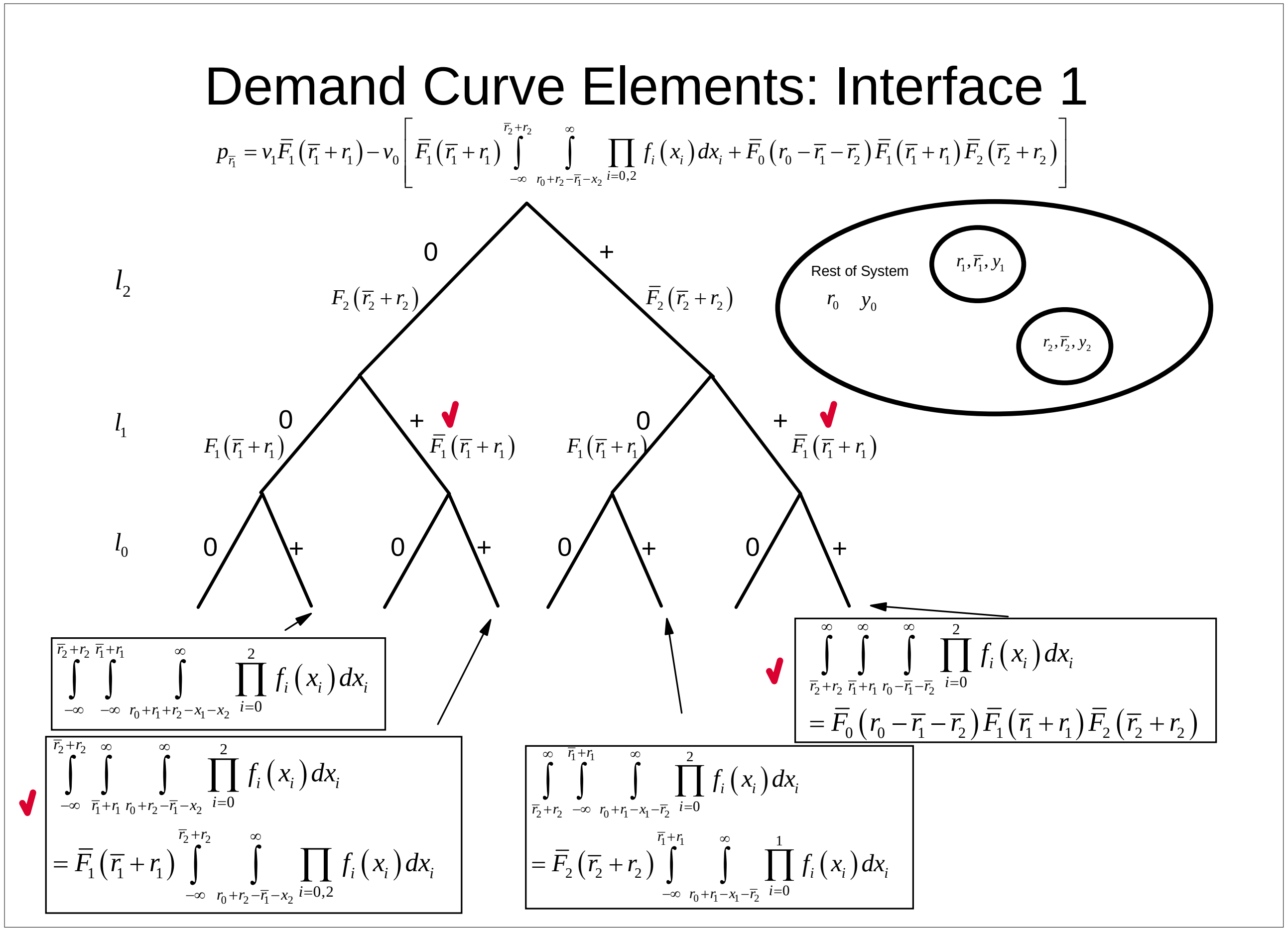
$$\int_{-\infty}^{\bar{r}_2+r_2}\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i\left(x_i\right)dx_i$$

$$\int_{-\infty}^{\bar{r}_2+r_2}\int_{\bar{r}_1+r_1}^{\infty}\int_{r_0+r_2-\bar{r}_1-x_2}^{\infty}\prod_{i=0}^{2} f_i\left(x_i\right)dx_i = \bar{F}_1\left(\bar{r}_1+r_1\right)\int_{-\infty}^{\bar{r}_2+r_2}\int_{r_0+r_2-\bar{r}_1-x_2}^{\infty}\prod_{i=0,2} f_i\left(x_i\right)dx_i$$

$$\int_{\bar{r}_2+r_2}^{\infty}\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1-x_1-\bar{r}_2}^{\infty}\prod_{i=0}^{2} f_i\left(x_i\right)dx_i = \bar{F}_2\left(\bar{r}_2+r_2\right)\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1-x_1-\bar{r}_2}^{\infty}\prod_{i=0}^{1} f_i\left(x_i\right)dx_i$$

$$\int_{\bar{r}_2+r_2}^{\infty}\int_{\bar{r}_1+r_1}^{\infty}\int_{r_0-\bar{r}_1-\bar{r}_2}^{\infty}\prod_{i=0}^{2} f_i\left(x_i\right)dx_i = \bar{F}_0\left(r_0-\bar{r}_1-\bar{r}_2\right)\bar{F}_1\left(\bar{r}_1+r_1\right)\bar{F}_2\left(\bar{r}_2+r_2\right)$$

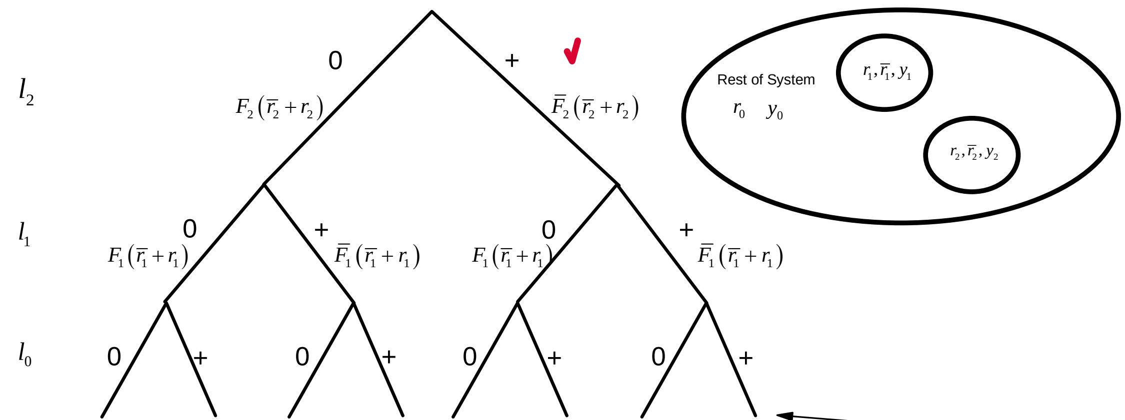
The loss outcomes and dependencies determine the demand for zone 1 interface capacity.



The loss outcomes and dependencies determine the demand for zone 2 interface capacity.

Demand Curve Elements: Interface 2

$$p_{\overline{r}_2} = v_2 \overline{F}_2\left(\overline{r}_2 + r_2\right) - v_0 \left[\overline{F}_2\left(\overline{r}_2 + r_2\right) \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i + \overline{F}_0\left(r_0 - \overline{r}_1 - \overline{r}_2\right) \overline{F}_1\left(\overline{r}_1 + r_1\right) \overline{F}_2\left(\overline{r}_2 + r_2\right) \right]$$



$$\int_{-\infty}^{\overline{r}_2 + r_2} \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 + r_2 - x_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i$$

✓

$$\int_{-\infty}^{\overline{r}_2 + r_2} \int_{\overline{r}_1 + r_1}^{\infty} \int_{r_0 + r_2 - \overline{r}_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_1\left(\overline{r}_1 + r_1\right) \int_{-\infty}^{\overline{r}_2 + r_2} \int_{r_0 + r_2 - \overline{r}_1 - x_2}^{\infty} \prod_{i=0,2} f_i\left(x_i\right) dx_i$$

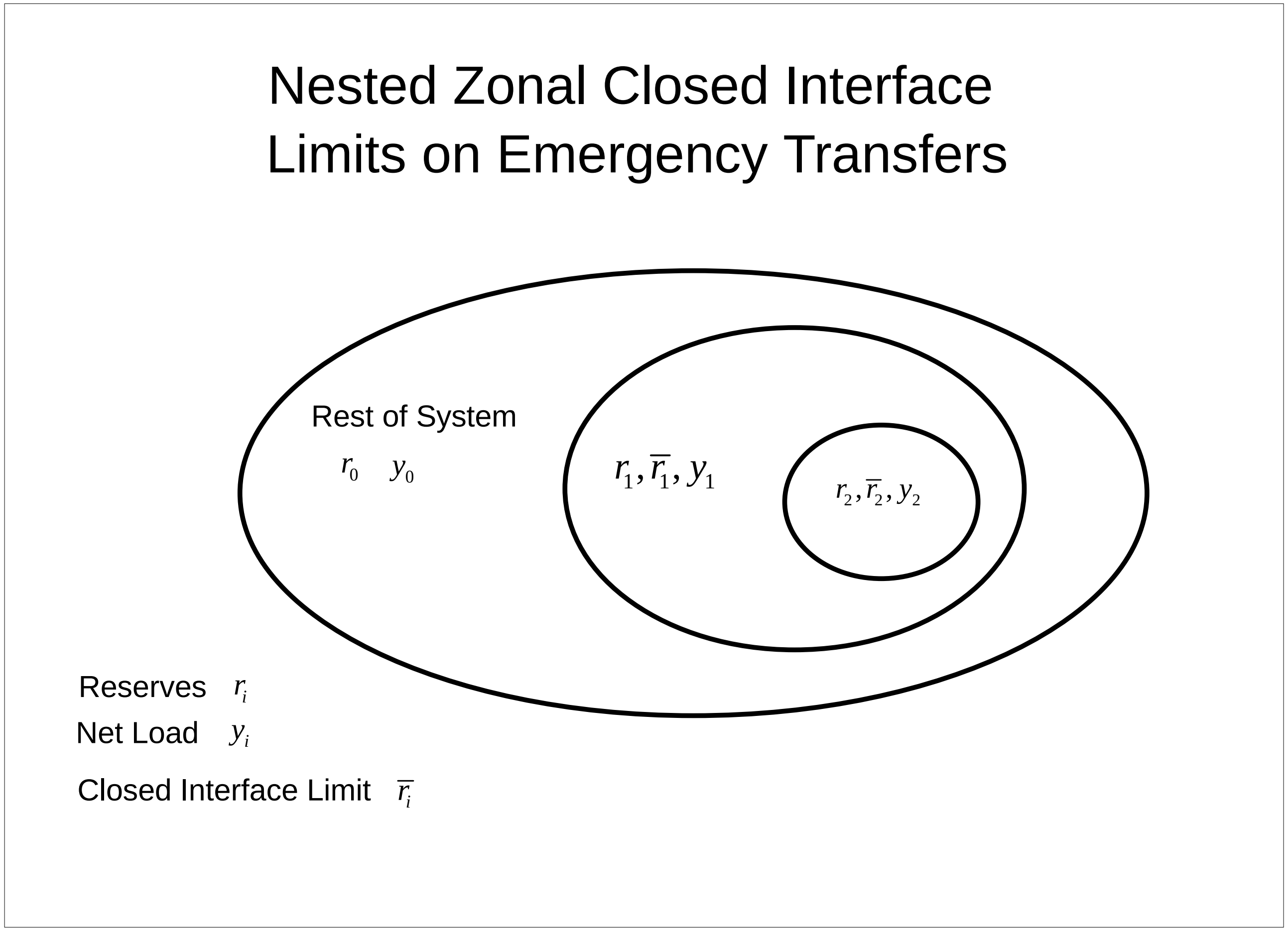
✓

$$\int_{\overline{r}_2 + r_2}^{\infty} \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_2\left(\overline{r}_2 + r_2\right) \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i$$

✓

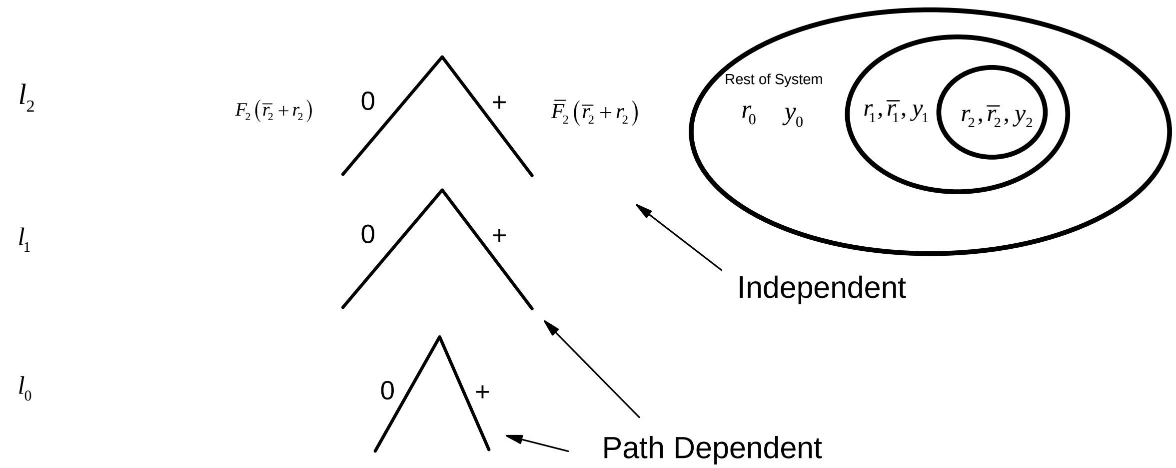
$$\int_{\overline{r}_2 + r_2}^{\infty} \int_{\overline{r}_1 + r_1}^{\infty} \int_{r_0 - \overline{r}_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_0\left(r_0 - \overline{r}_1 - \overline{r}_2\right) \overline{F}_1\left(\overline{r}_1 + r_1\right) \overline{F}_2\left(\overline{r}_2 + r_2\right)$$

Nested constrained zones define an alternative extension of the case for a single constrained zone.



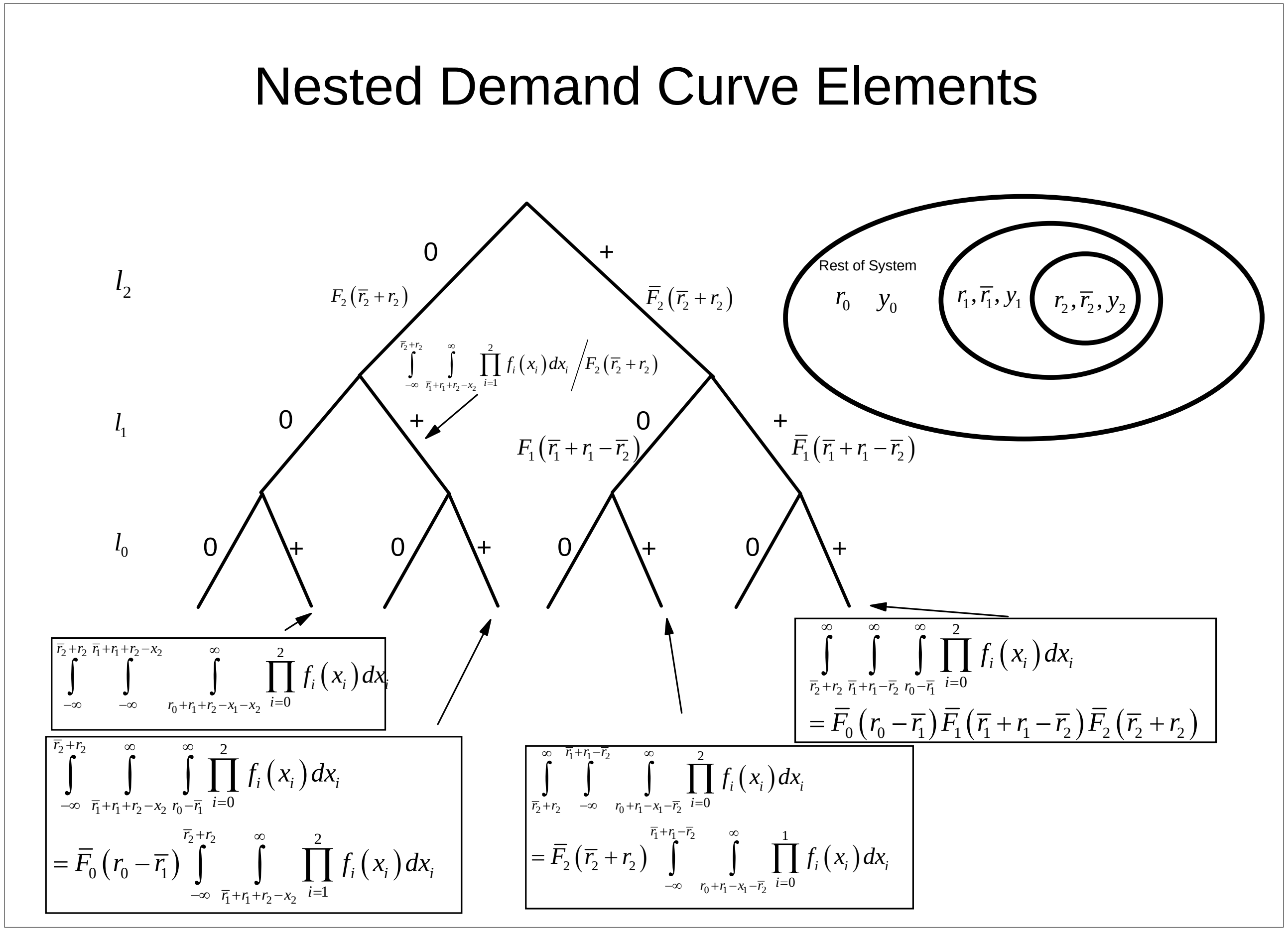
The probability tree for the nested zones captures the dependencies of loss of load.

Nested Loss of Load Probability Structure



$$y_i \sim f_i,\ F_i(y_i) = \int_{-\infty}^{y_i} f_i(x_i)\,dx_i$$

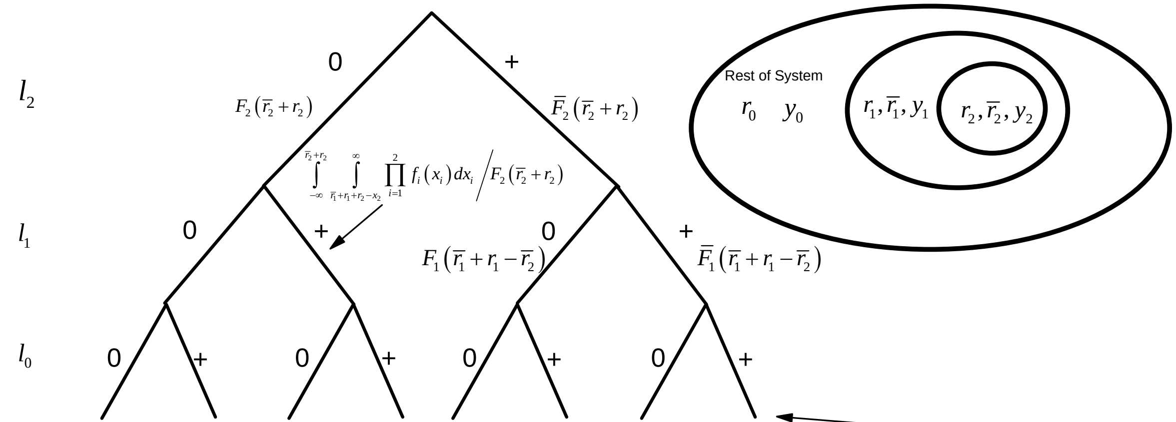
The nested loss of load probability structure defines the demand curve elements.



The nested loss outcomes and dependencies determine the demand for rest of system operating reserves.

Nested Demand Curve Elements: Rest of System

$$p_{r_0} = v_0\left[\int_{-\infty}^{\bar r_2+r_2}\int_{-\infty}^{\bar r_1+r_1+r_2-x_2}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i + \bar F_0(r_0-\bar r_1)\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_2(\bar r_2+r_2)\int_{-\infty}^{\bar r_1+r_1-\bar r_2}\int_{r_0+r_1-x_1-\bar r_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i + \bar F_0(r_0-\bar r_1)\bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right]$$



$$\int_{-\infty}^{\bar r_2+r_2}\int_{-\infty}^{\bar r_1+r_1+r_2-x_2}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i$$

$$\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\int_{r_0-\bar r_1}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_0(r_0-\bar r_1)\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i$$

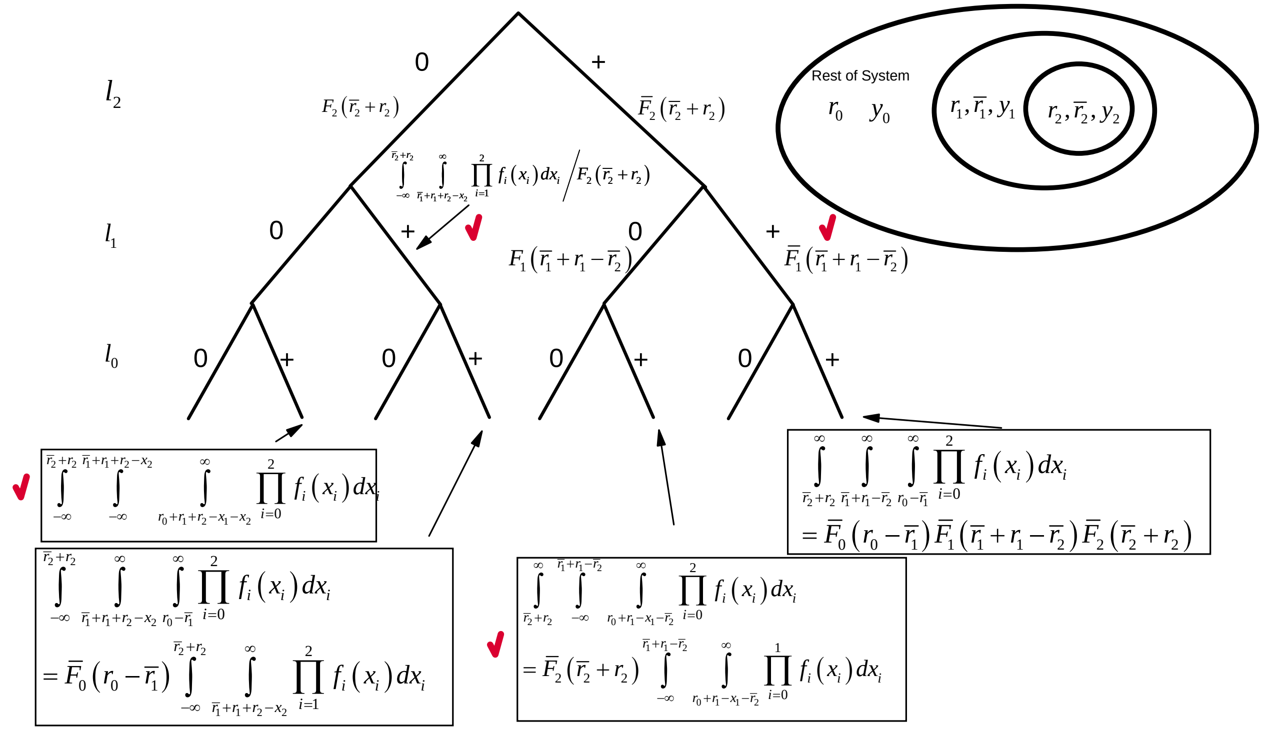
$$\int_{\bar r_2+r_2}^{\infty}\int_{-\infty}^{\bar r_1+r_1-\bar r_2}\int_{r_0+r_1-x_1-\bar r_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_2(\bar r_2+r_2)\int_{-\infty}^{\bar r_1+r_1-\bar r_2}\int_{r_0+r_1-x_1-\bar r_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i$$

$$\int_{\bar r_2+r_2}^{\infty}\int_{\bar r_1+r_1-\bar r_2}^{\infty}\int_{r_0-\bar r_1}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_0(r_0-\bar r_1)\bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)$$

The nested loss outcomes and dependencies determine the demand for zone 1 operating reserves.

Nested Demand Curve Elements: Zone 1

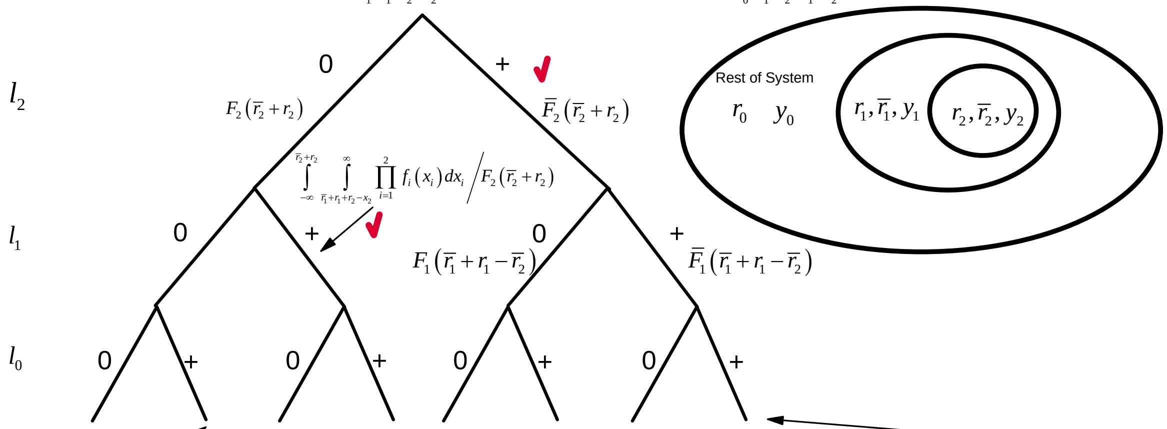
$$p_{r_1} = v_1\left[\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right] + v_0\left[\int_{-\infty}^{\bar r_2+r_2}\int_{-\infty}^{\bar r_1+r_1+r_2-x_2}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i + \bar F_2(\bar r_2+r_2)\int_{-\infty}^{\bar r_1+r_1-\bar r_2}\int_{r_0+r_1-x_1-\bar r_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i\right]$$



The nested loss outcomes and dependencies determine the demand for zone 2 operating reserves.

Nested Demand Curve Elements: Zone 2

$$p_{r_2} = v_2 \overline{F}_2\left(\overline{r}_2 + r_2\right) + v_1 \int_{-\infty}^{\overline{r}_2+r_2} \int_{\overline{r}_1+r_1+r_2-x_2}^{\infty} \prod_{i=1}^{2} f_i\left(x_i\right) dx_i + v_0 \int_{-\infty}^{\overline{r}_2+r_2} \int_{-\infty}^{\overline{r}_1+r_1+r_2-x_2} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i$$



$$\int_{-\infty}^{\overline{r}_2+r_2} \int_{-\infty}^{\overline{r}_1+r_1+r_2-x_2} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i$$

$$\int_{-\infty}^{\overline{r}_2+r_2} \int_{\overline{r}_1+r_1+r_2-x_2}^{\infty} \int_{r_0-\overline{r}_1}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_0\left(r_0 - \overline{r}_1\right) \int_{-\infty}^{\overline{r}_2+r_2} \int_{\overline{r}_1+r_1+r_2-x_2}^{\infty} \prod_{i=1}^{2} f_i\left(x_i\right) dx_i$$

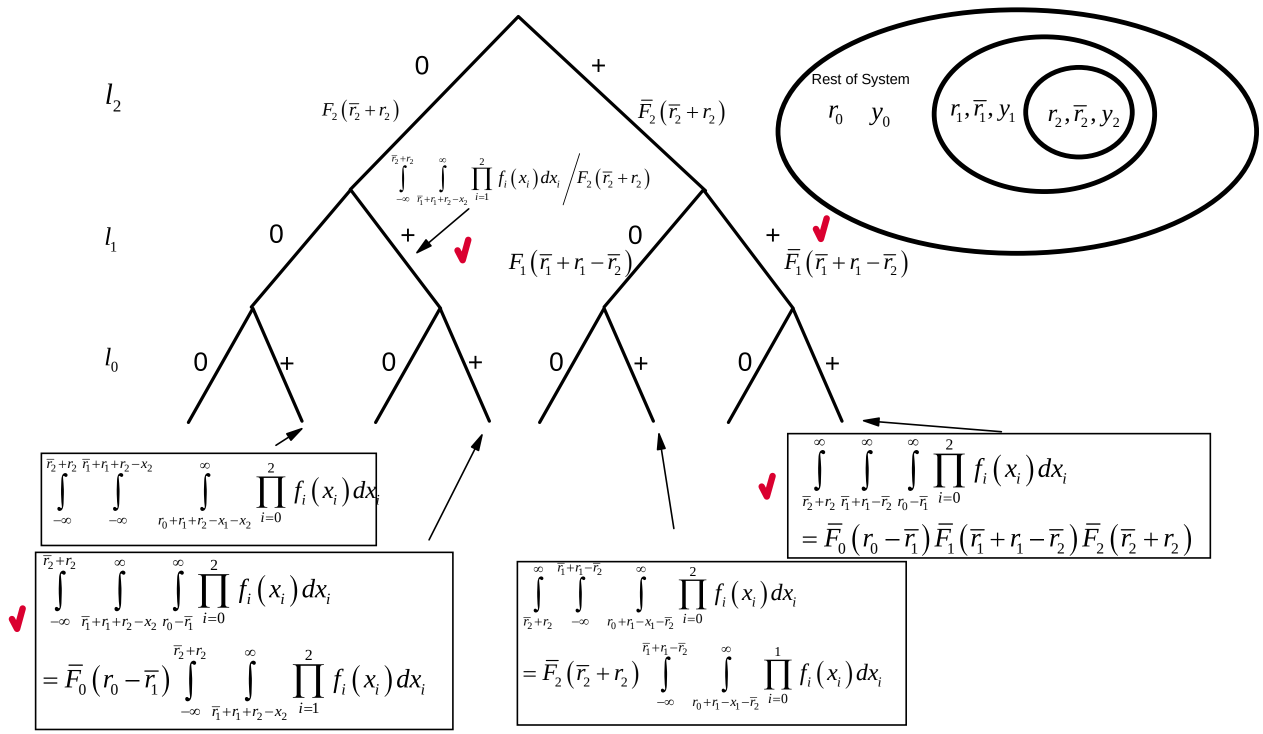
$$\int_{\overline{r}_2+r_2}^{\infty} \int_{-\infty}^{\overline{r}_1+r_1-\overline{r}_2} \int_{r_0+r_1-x_1-\overline{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_2\left(\overline{r}_2 + r_2\right) \int_{-\infty}^{\overline{r}_1+r_1-\overline{r}_2} \int_{r_0+r_1-x_1-\overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i$$

$$\int_{\overline{r}_2+r_2}^{\infty} \int_{\overline{r}_1+r_1-\overline{r}_2}^{\infty} \int_{r_0-\overline{r}_1}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_0\left(r_0 - \overline{r}_1\right) \overline{F}_1\left(\overline{r}_1 + r_1 - \overline{r}_2\right) \overline{F}_2\left(\overline{r}_2 + r_2\right)$$

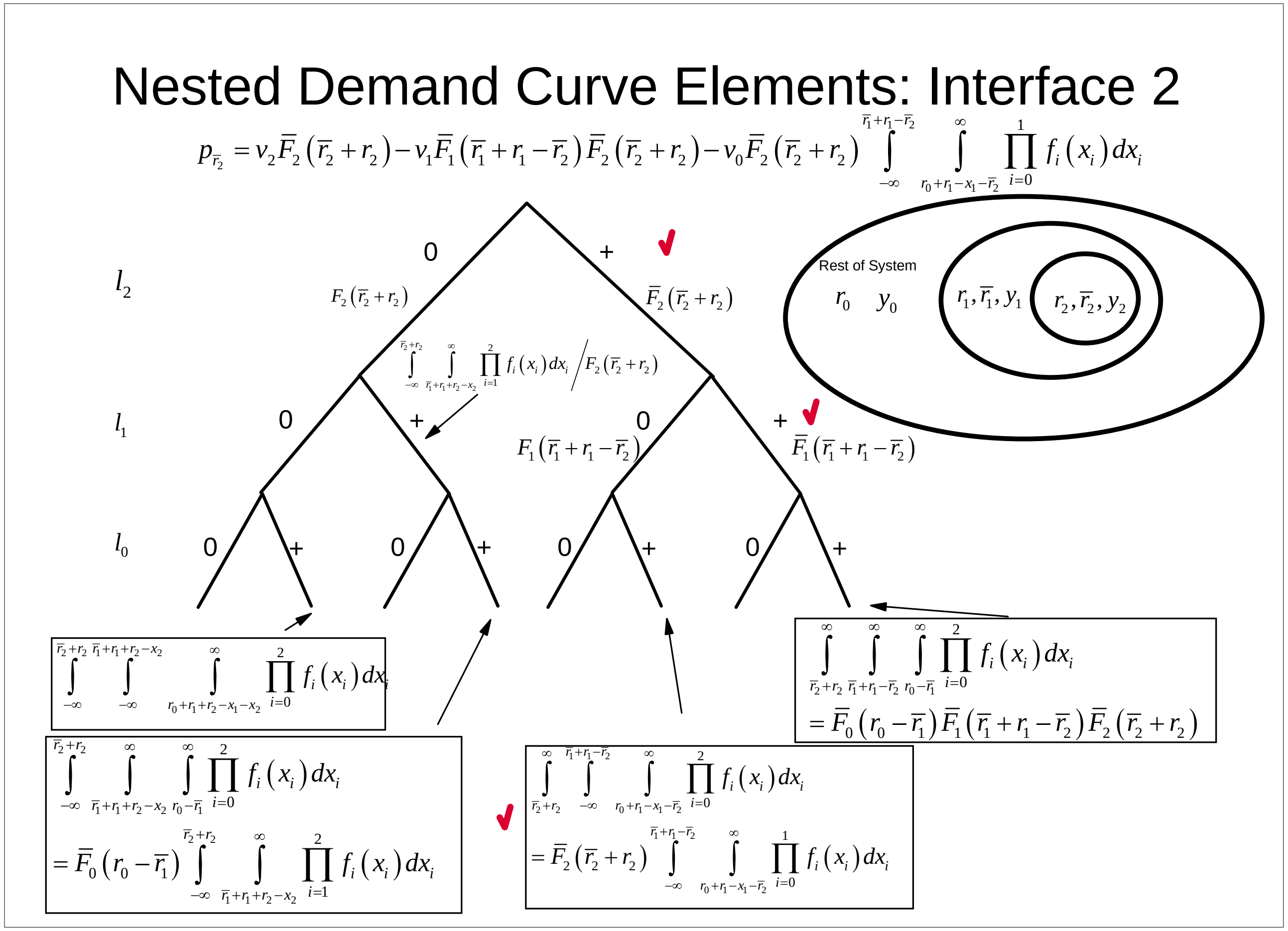
The nested loss outcomes and dependencies determine the demand for interface 1 capacity.

Nested Demand Curve Elements: Interface 1

$$p_{\bar r_1} = v_1\left[\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right] - v_0\left[\bar F_0(r_0-\bar r_1)\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_0(r_0-\bar r_1)\bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right]$$

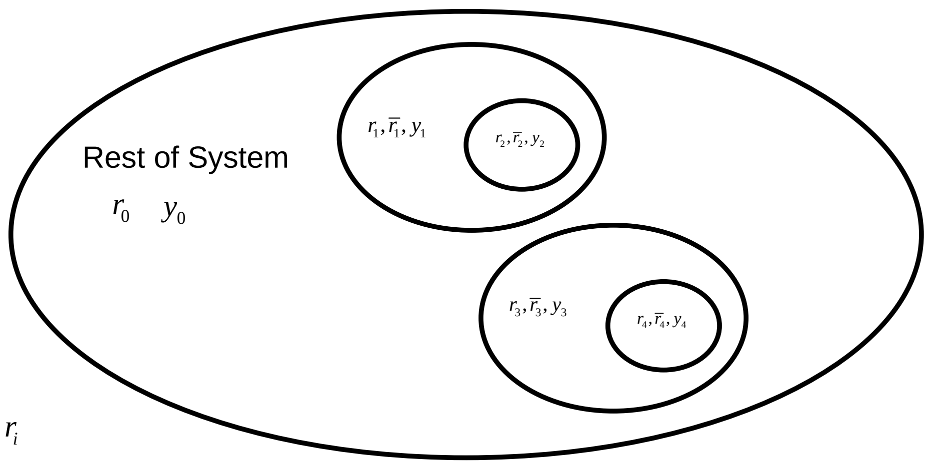


The nested loss outcomes and dependencies determine the demand for interface 2 capacity.



Mixed constrained zones define a more general extension of a constrained zonal structure.

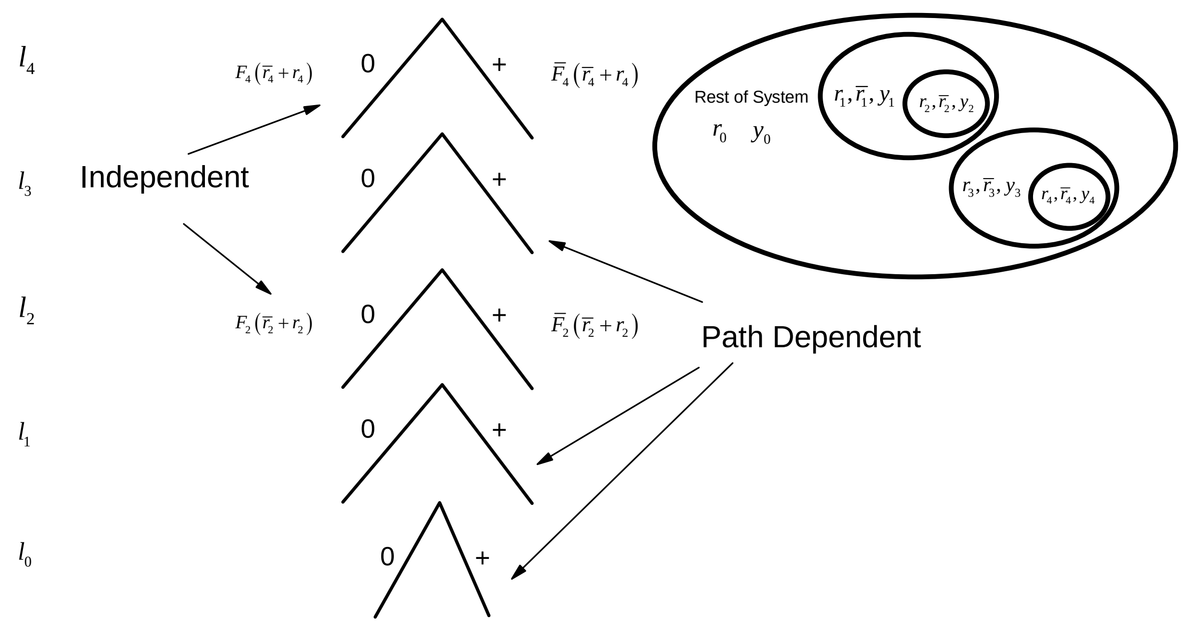
Mixed Zonal Closed Interface
Limits on Emergency Transfers



Reserves r_i
Net Load y_i
Closed Interface Limit $\bar{r}_i$

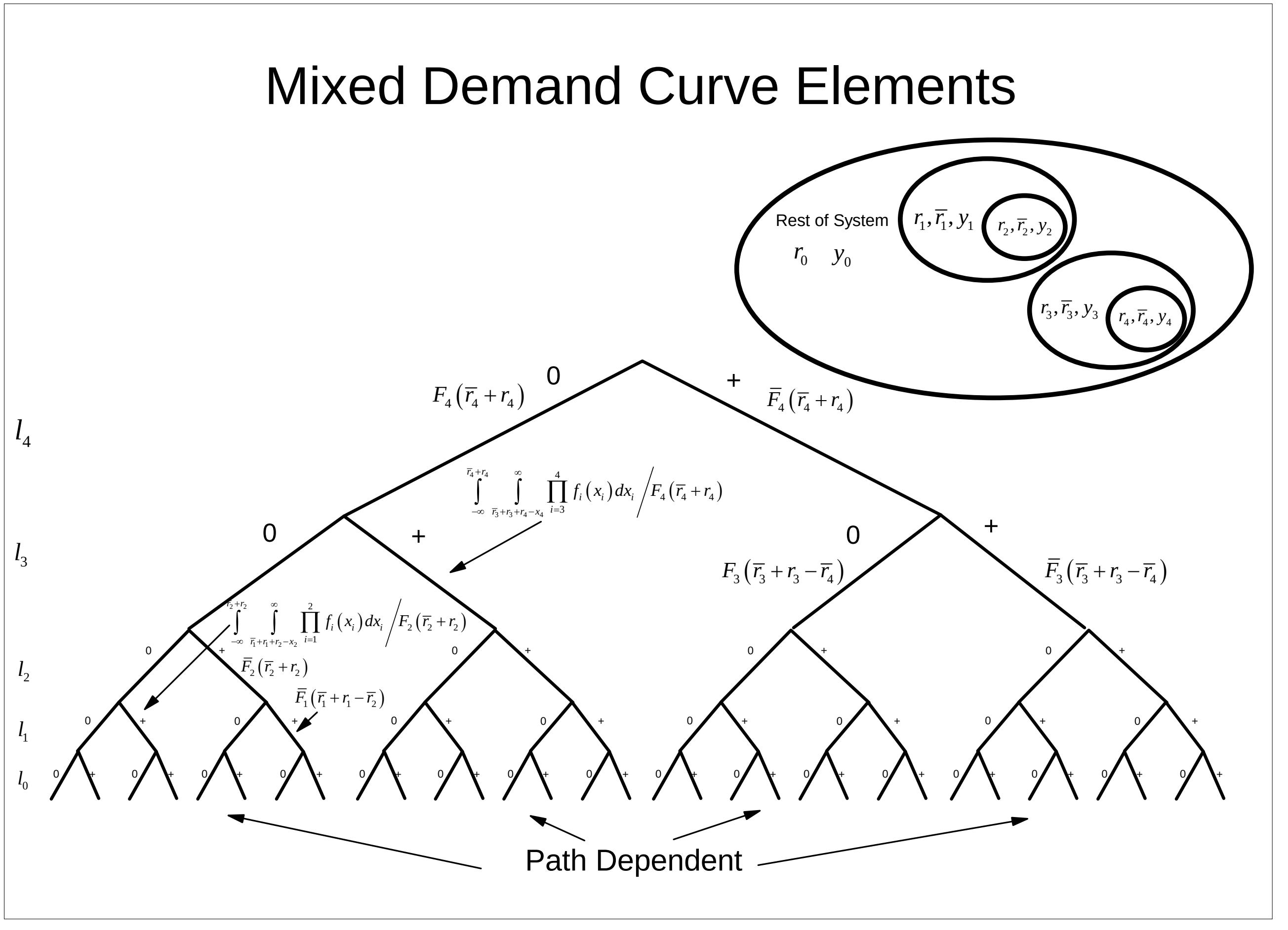
The mixed probability tree for the nested zones captures the dependencies of loss of load.

Mixed Loss of Load Probability Structure



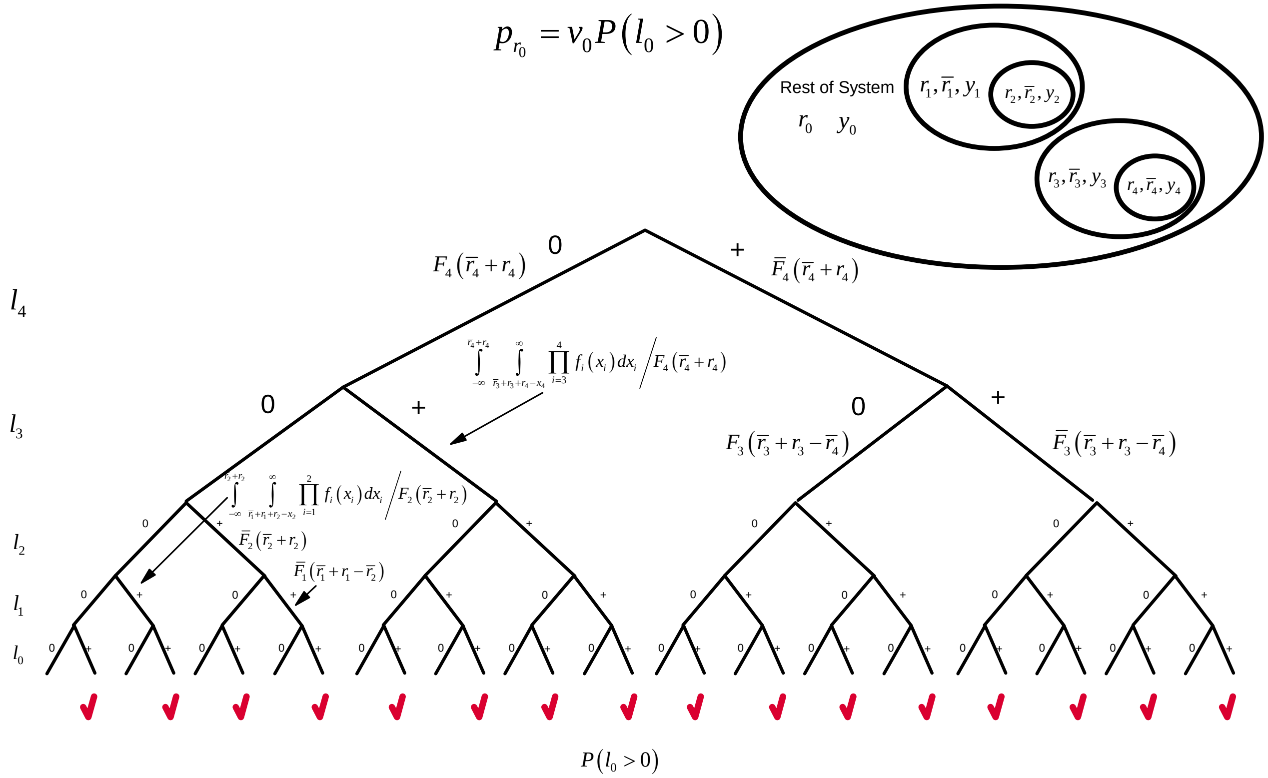
$$y_i \sim f_i,\ F_i(y_i) = \int_{-\infty}^{y_i} f_i(x_i)\,dx_i$$

The mixed loss of load probability structure defines the demand curve elements.



The mixed loss outcomes and dependencies determine the demand for rest of system operating reserves.

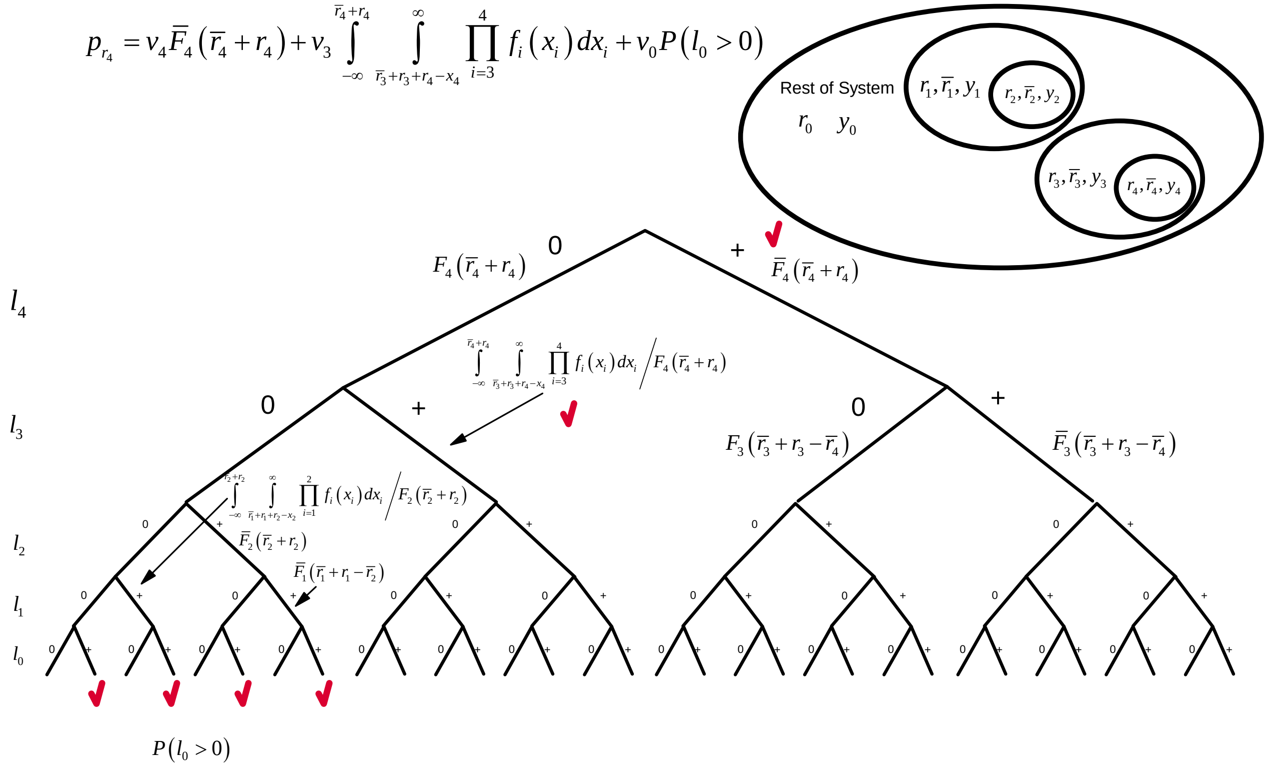
Mixed Demand Curve Elements: Rest of System



The mixed loss outcomes and dependencies determine the demand for zone 4 operating reserves.

Mixed Demand Curve Elements: Zone 4

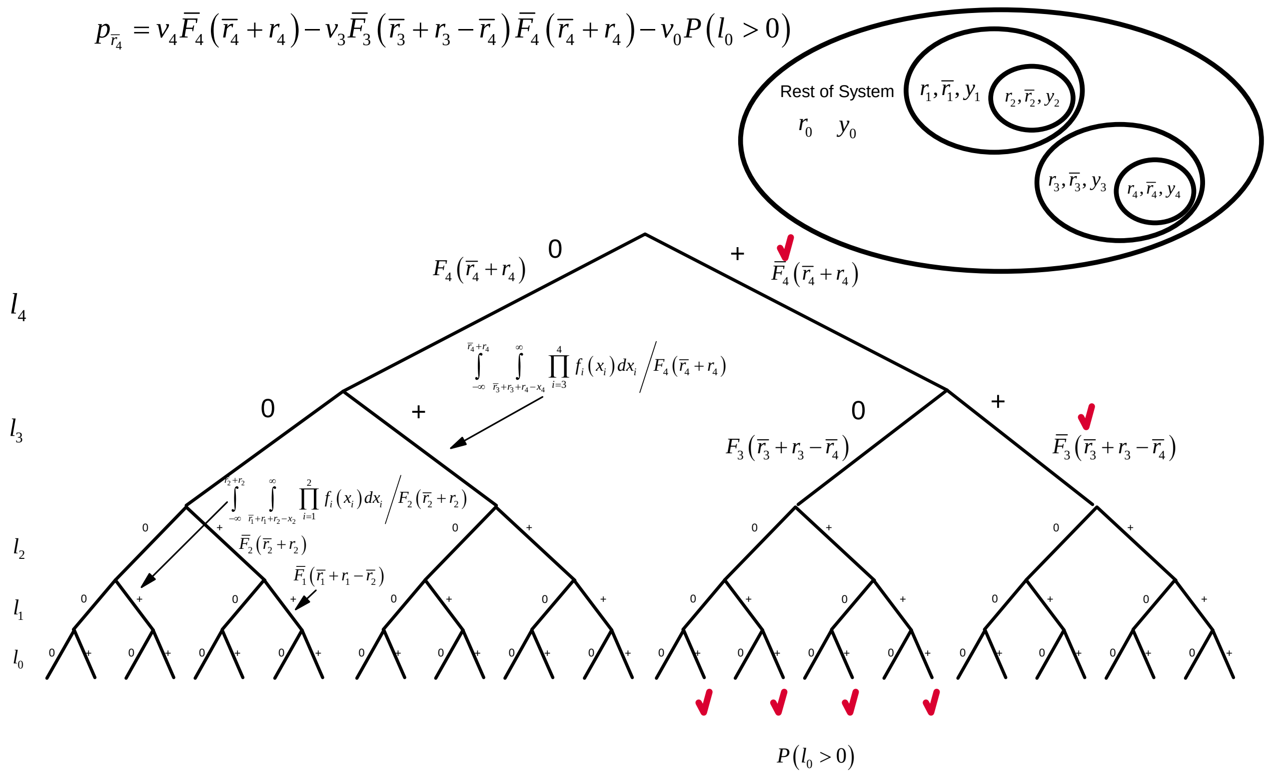
$$p_{r_4} = v_4 \bar{F}_4\left(\bar{r}_4 + r_4\right) + v_3 \int_{-\infty}^{\bar{r}_4 + r_4} \int_{\bar{r}_3 + r_3 + r_4 - x_4}^{\infty} \prod_{i=3}^{4} f_i\left(x_i\right) dx_i + v_0 P\left(l_0 > 0\right)$$



The mixed loss outcomes and dependencies determine the demand for interface 4 capacity.

Mixed Demand Curve Elements: Interface 4

$$p_{\bar{r}_4} = v_4 \bar{F}_4\left(\bar{r}_4 + r_4\right) - v_3 \bar{F}_3\left(\bar{r}_3 + r_3 - \bar{r}_4\right)\bar{F}_4\left(\bar{r}_4 + r_4\right) - v_0 P\left(l_0 > 0\right)$$



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