SCARCITY PRICING AND
LOCATIONAL OPERATING RESERVE DEMAND CURVES

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The US experience illustrates successful market design and remaining challenges for both theory and implementation.

- **Design Principle: Integrate Market Design and System Operations**

  Provide good short-run operating incentives.
  Support forward markets and long-run investments.

- **Design Framework: Bid-Based, Security Constrained Economic Dispatch**

  Locational Marginal Prices (LMP) with granularity to match system operations.
  Financial Transmission Rights (FTRs).

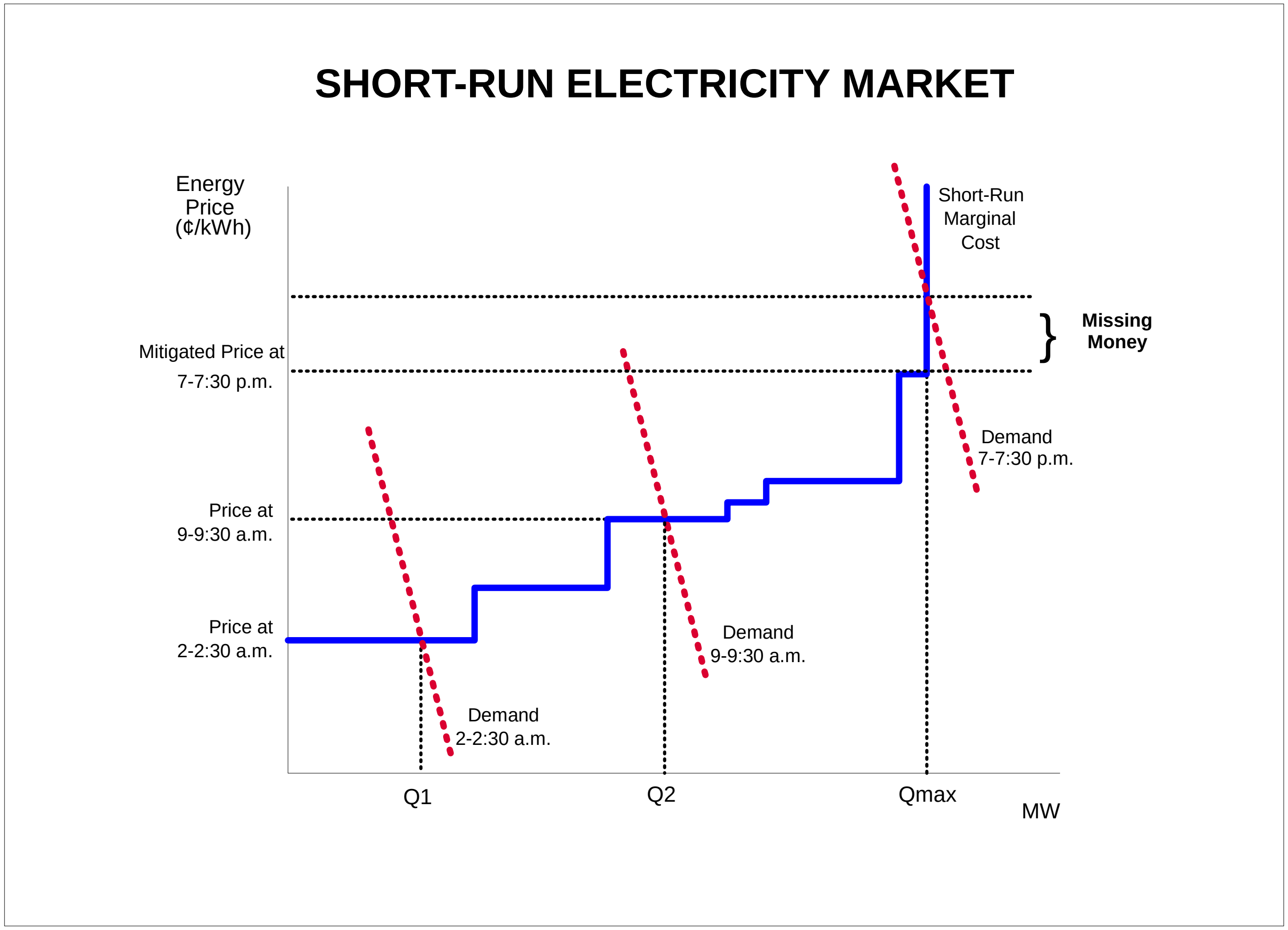
- **Design Implementation: Pricing Evolution**

  Better scarcity pricing to support resource adequacy.
  Unit commitment and lumpy decisions with coordination, bid guarantees and uplift payments.

- **Design Challenge: Infrastructure Investment**

  Hybrid models to accommodate both market-based and regulated investments.
  Applying beneficiary-pays principle to support integration with rest of the market design.

Early market designs presumed a significant demand response. Absent this demand participation most markets implemented inadequate pricing rules equating prices to marginal costs even when capacity is constrained. This produces a "missing money" problem.



Scarcity pricing presents an important challenge for Regional Transmission Organizations (RTOs) and electricity market design. Simple in principle, but more complicated in practice, inadequate scarcity pricing is implicated in several problems associated with electricity markets.

- **Investment Incentives.** Inadequate scarcity pricing contributes to the "missing money" needed to support new generation investment. The policy response has been to create capacity markets. Better scarcity pricing would reduce the challenges of operating good capacity markets.

- **Demand Response.** Higher prices during critical periods would facilitate demand response and distributed generation when it is most needed. The practice of socializing payments for capacity investments compromises the incentives for demand response and distributed generation.

- **Renewable Energy.** Intermittent energy sources such as solar and wind present complications in providing a level playing field in pricing. Better scarcity pricing would reduce the size and importance of capacity payments and improve incentives for renewable energy.

- **Transmission Pricing.** Scarcity pricing interacts with transmission congestion. Better scarcity pricing would provide better signals for transmission investment.

Smarter scarcity pricing would mitigate or substantially remove the problems in all these areas. While long-recognized, the need for smarter prices for a smarter grid promotes interest in better theory and practice of scarcity pricing.[1]

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1 FERC, Order 719, October 17, 2008.

The theory and practice of scarcity pricing intersect important elements of electricity systems and economic dispatch.

- **Reliability.** By definition, scarcity conditions arise when the system is constrained and dispatch is modified to respect reliability constraints.
- **Dispatch.** Simultaneous optimization of energy and reserves means that scarcity in either affects prices for both.
- **Resource Adequacy.** The standards for resource adequacy and operating security are not fully integrated or compatible.

A critical connection is the treatment of operating reserves and construction of operating reserve demand curves. The basic idea of applying operating reserve demand curves is well tested and found in operation in important RTOs.

- **NYISO.** See NYISO Ancillary Service Manual, Volume 3.11, Draft, April 14, 2008, pp, 6-19-6-22.
- **ISONE.** FERC Electric Tariff No. 3, Market Rule I, Section III.2.7, February 6, 2006.
- **MISO.** FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009. [2]

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[2] "For each cleared Operating Reserve level less than the Market-Wide Operating Reserve Requirement, the Market-Wide Operating Reserve Demand Curve price shall be equal to the product of (i) the Value of Lost Load ("VOLL") and (ii) the estimated conditional probability of a loss of load given that a single forced Resource outage of 100 MW or greater will occur at the cleared Market-Wide Operating Reserve level for which the price is being determined. … The VOLL shall be equal to $3,500 per MWh." MISO, FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009, Sheet 2226.

The underlying models of operating reserve demand curves differ across RTOs. One need is for a framework that develops operating reserve demand curves from first principles to provide a benchmark for the comparison of different implementations.

- **Operating Reserve Demand Curve Components.** The inputs to the operating reserve demand curve construction can differ and a more general model would help specify the result.
- **Locational Differences and Interactions.** The design of locational operating reserve demand curves presents added complications in accounting for transmission constraints.
- **Economic Dispatch.** The derivation of the locational operating demand curves has implications for the integration with economic dispatch models for simultaneous optimization of energy and reserves.

A series of approximations to a probabilistic unit commitment and economic dispatch models provides a framework for incorporating scarcity pricing and operating reserve demand curves. The resulting model is a workable extension of existing unit commitment and economic dispatch formulations.

Begin with an expected value formulation of a single period, lossless economic dispatch that might appeal in principle. Given benefit (*B*) and cost (*C*) functions, demand (*d*), generation (*g*), plant capacity (*Cap*), reserves (*r*), commitment decisions (*u*), transmission constraints (*H*), and state probabilities (*p*):

$$\underset{y^i,d^i,g^i,r,u\in(0,1)}{Max}\; p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N}p_i\left(B^i\left(d^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)$$

$$\begin{aligned}
&s.t.\\
&\quad y^i=d^i-g^i, \quad i=0,2,\cdots,N,\\
&\quad \iota^t y^i=0, \quad i=0,1,2,\cdots,N,\\
&\quad H^i y^i\le b^i, \quad i=0,1,2,\cdots,N,\\
&\quad g^0+r\le u\bullet Cap^0,\\
&\quad g^i\le g^0+r, \quad i=1,2,\cdots,N,\\
&\quad r\le r_{\max},\\
&\quad g^i\le u\bullet Cap^i, \quad i=0,2,\cdots,N.
\end{aligned}$$

Suppose there are *K* possible contingencies. The interesting cases have $K \gg 10^3$. The number of possible system states is $N=2^K$, or more than the stars in the Milky Way. Some approximation will be in order.[3]

[3] Shams N. Siddiqi and Martin L. Baughman, "Reliability Differentiated Pricing of Spinning Reserve," IEEE Transactions on Power Systems, Vol. 10, No. 3, August 1995, pp.1211-1218. José M. Arroyo and Francisco D. Galiana, "Energy and Reserve Pricing in Security and Network-Constrained Electricity Markets," IEEE Transactions On Power Systems, Vol. 20, No. 2, May 2005, pp. 634-643. François Bouffard, Francisco D. Galiana, and Antonio J. Conejo,

Introduce random changes in load ε^i and possible lost load l^i in at least some conditions.

$$\underset{y^i,d^i,g^i,r,u\in(0,1)}{Max}\; p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N}p_i\left(B^i\left(d^o+\varepsilon^i,-l^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)$$

$$\begin{aligned}
&s.t.\\
&\quad y^0=d^0-g^0,\\
&\quad y^i=d^0+\varepsilon^i-g^i-l^i,\quad i=1,2,\cdots,N,\\
&\quad \iota^t y^i=0,\quad i=0,1,2,\cdots,N,\\
&\quad H^i y^i\le b^i,\quad i=0,1,2,\cdots,N,\\
&\quad g^0+r\le u\bullet Cap^0,\\
&\quad g^i\le g^0+r,\quad i=1,2,\cdots,N,\\
&\quad r\le r_{\max},\\
&\quad g^i\le u\bullet Cap^i,\quad i=0,2,\cdots,N.
\end{aligned}$$

Simplify the benefit and cost functions:

$$B^i\left(d^o+\varepsilon^i,-l^i,d^0\right)\approx B^0\left(d^0\right)+k_d^i-v^t l^i \quad , \qquad C^i\left(g^i,g^0,r,u\right)\approx C^0\left(g^0,r,u\right)+k_g^i.$$

This produces a separable approximate objective function:

$$p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N}p_i\left(B^i\left(d^o,-l^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)=B^0\left(d^0\right)-C^0\left(g^0,r,u\right)+\sum_{i=1}^{N}p_i\left(k_d^i-k_g^i\right)-v^t\sum_{i=1}^{N}p_i l^i\ .$$

“Market-Clearing With Stochastic Security—Part I: Formulation,” IEEE Transactions On Power Systems, Vol. 20, No. 4, November 2005, pp. 1818-1826; “Part II: Case Studies,” pp. 1827-1835.

The revised formulation highlights the pre-contingency objective function and the role of the value of the expected undeserved energy.

$$\underset{y^i, d^i, g^i, r, u \in (0,1)}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - v^t \sum_{i=1}^{N} p_i l^i$$

$$s.t.$$

$$\begin{aligned}
&y^0 = d^0 - g^0, \\
&y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N, \\
&\iota^t y^i = 0, \quad i = 0, 1, 2, \cdots, N, \\
&H^i y^i \le b^i, \quad i = 0, 1, 2, \cdots, N, \\
&g^i \le g^0 + r, \quad i = 1, 2, \cdots, N, \\
&r \le r_{\max}, \\
&g^i \le u \bullet Cap^i, \quad i = 0, 2, \cdots, N.
\end{aligned}$$

There are still too many system states.

Define the optimal value of expected unserved energy (VEUE) as the result of all the possible optimal post-contingency responses given the pre-contingency commitment and scheduling decisions.

$$
\begin{aligned}
& VEUE\left(d^0, g^0, r, u\right) = \underset{y^i, g^i, l^i}{Min}\, v^t \sum_{i=1}^{N} p_i l^i \\
& s.t. \\
& \quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N, \\
& \quad \iota^t y^i = 0, \quad i = 1, 2, \cdots, N, \\
& \quad H^i y^i \le b^i, \quad i = 1, 2, \cdots, N, \\
& \quad g^i \le g^0 + r, \quad i = 1, 2, \cdots, N, \\
& \quad g^i \le u \bullet Cap^i, \quad i = 1, 2, \cdots, N.
\end{aligned}
$$

This second stage problem subsumes all the redispatch and curtailment decisions over the operating period after the commitment and scheduling decisions.

The expected value formulation reduces to a much more manageable scale with the introduction of the implicit VEUE function.

$$\underset{y^0, d^0, g^0, r, u \in (0,1)}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - VEUE\left(d^0, g^0, r, u\right)$$

$$s.t.$$

$$y^0 = d^0 - g^0,$$

$$\iota^t y^0 = 0,$$

$$H^0 y^0 \le b^0,$$

$$g^0 + r \le u \bullet Cap^0,$$

$$r \le r_{\max},$$

$$g^0 \le u \bullet Cap^0.$$

The optimal value of expected unserved energy defines the demand for operating reserves. This formulation of the problem follows the outline of existing operating models except for the exclusion of contingency constraints.

The deterministic approach to security constrained economic dispatch includes lower bounds on the required reserve to ensure that for a set of monitored contingencies (e.g., an n-1 standard) there is sufficient operating reserve and transmission capacity to maintain the system for an emergency period.

Suppose that the maximum generation outage contingency quantity is r_{Min}. Let the K_M monitored transmission contingency constraints be $H^i y^0 \le \tilde{b}^i$. Then we obtain a standard form of security constrained unit commitment and economic dispatch with the addition of the value of expected unserved energy.

$$\underset{y^0,d^0,g^0,r,u\in(0,1)}{Max} B^0\left(d^0\right)-C^0\left(g^0,r,u\right)-VEUE\left(d^0,g^0,r-r_{\min},u\right)$$

$$\begin{aligned}
&s.t.\\
&\quad y^0 = d^0 - g^0,\\
&\quad \iota^t y^0 = 0,\\
&\quad H^0 y^0 \le b^0,\\
&\quad H^i y^0 \le \tilde{b}^i, \quad i = 1,2,\cdots,K_M,\\
&\quad g^0 + r \le u \bullet Cap^0\\
&\quad r \le r_{\max},\\
&\quad r \ge r_{\min},\\
&\quad g^0 \le u \bullet Cap^0.
\end{aligned}$$

A difficulty with defining a locational operating reserve demand curve is the complexity of the interactions among locations plus interactions with the transmission grid. A similar problem appears in the evaluation of planned transmission and generation investment.

- **Expected Values.** The basic formulation of the real-time economic dispatch problem is built on a particular configuration of the transmission grid and the usual application of Kirchoff's laws. The operating reserve and long-term planning problem share a focus on the expected values of outcomes across different conditions. The expected value in principle applies probabilities across many configurations and the expected value need not follow the individual dictates of Kirchoff's laws.

- **Zonal Model.** The expected value formulation rationalizes approximation in a zonal model. The zonal application across a wide range of conditions is a regular feature of RTO transmission planning and resource adequacy calculations.

  - **Zones with Closed Interfaces.** Areas with limited transmission are defined and treated as having a close interface with a capacity limit for emergency transfers from the rest of the system.

  - **Capacity Emergency Transfer Limit (CETL).** Conservative transmission standards (e.g., 1 day in 25 years) apply to simulations that determine the transfer limit.[4]

- **Interface Limits.** Although the exact CETL calculations might not be appropriate for short-term reserve management, the analogy could be applied to determine closed interface limits.

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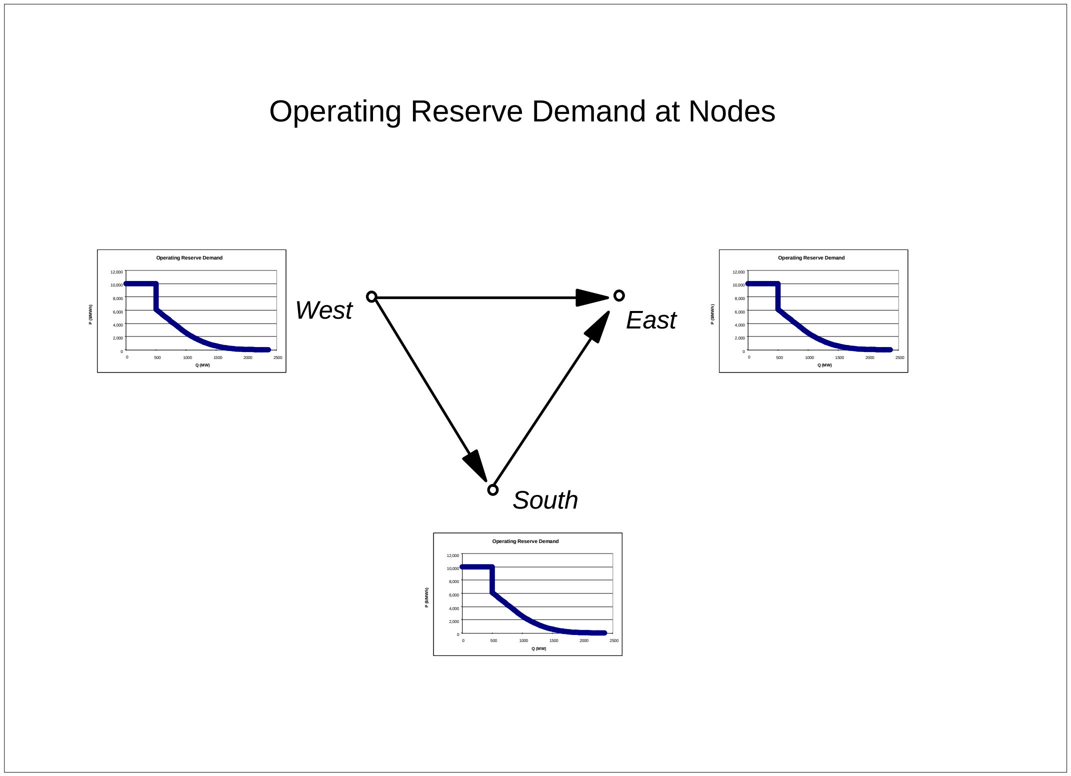
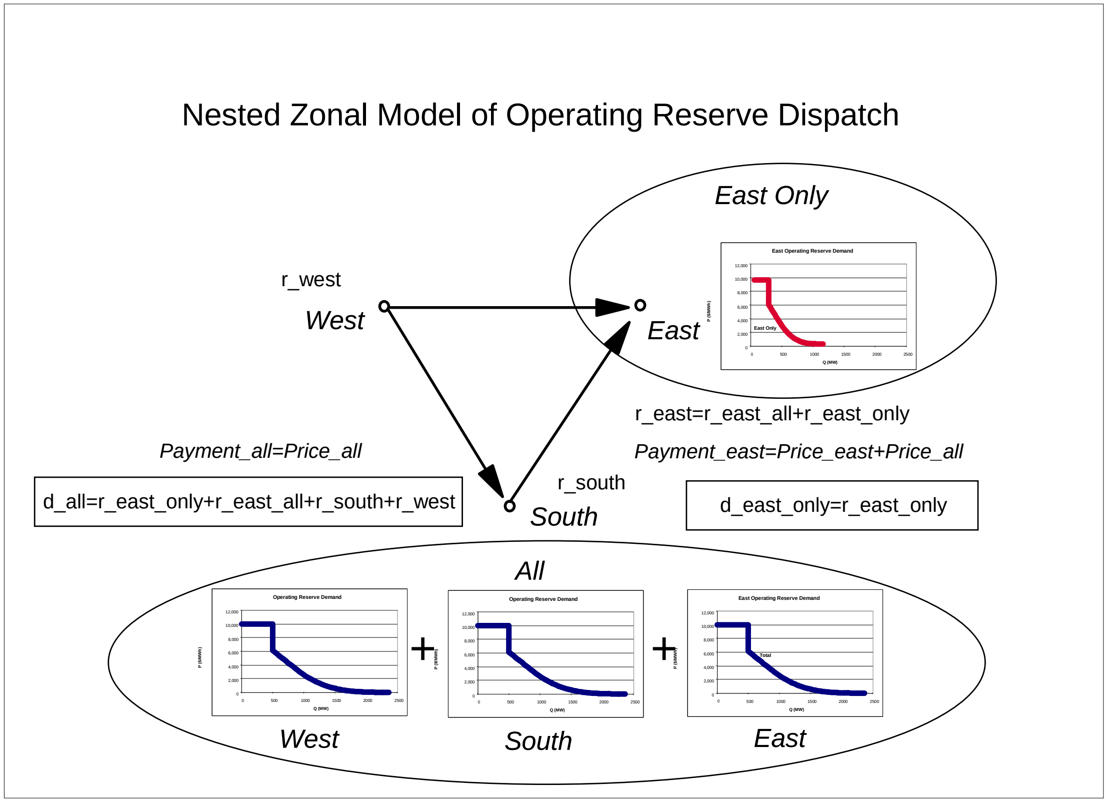
4 PJM , *2008 PJM Reserve Requirement Study*, October 8, 2008, Appendix H.

Suppose that the *LOLP* distribution at each node could be calculated.[5] **This would give rise to an operating reserve demand curve at each node.**



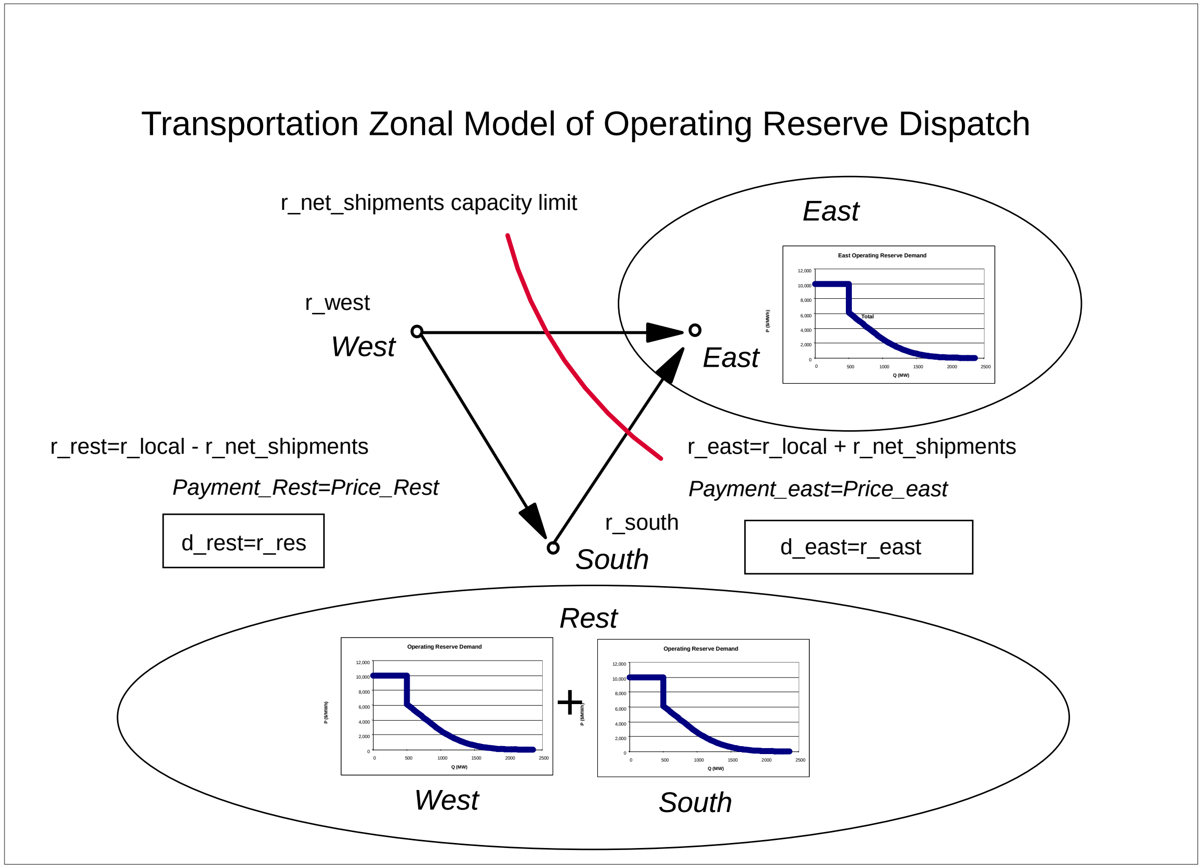
[5] Eugene G. Preston, W. Mack Grady, Martin L. Baughman, "A New Planning Model for Assessing the Effects of Transmission Capacity Constraints on the Reliability of Generation Supply for Large Nonequivalenced Electric Networks," IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997, pp. 1367-1373. J. Choi, R. Billinton, and M. Futuhi-Firuzabed, "Development of a Nodal Effective Load Model Considering Transmission System Element Unavailabilities," IEE Proceedings - Generation, Transmission and Distribution, Vol. 152, No. 1, January 2005, pp. 79-89.

The next piece is a model of simultaneous dispatch of operating reserves and energy. One approach for the operating reserve piece is a cascading zonal model (e.g., NYISO reserve pricing).



The result is that the input operating reserve price functions are additive premiums that give rise to an implicit operating reserve demand curves with higher prices.

An alternative approach would be to overlay a transportation model with interface transfer limits on operating reserve "shipments." The resulting prices are on the demand curves, but the model requires estimation of the (dynamic) transfer capacities. This is similar to the PJM installed capacity deliverability model, but specified an hour ahead rather than years ahead.



The expected value formulation with zonal approximation of the value of expected unserved energy provides a model for integrating zonal locational operating reserve requirements.

$$\underset{y^0, d^0, g^0, r, \bar{r}, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - ZVEUE\left(d^0, g^0, r - r_{\min}, \bar{r}, u\right)$$

$$\begin{aligned}
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad \iota^t y^0 = 0, \\
&\quad H^0 y^0 \le b^0, \\
&\quad H^i y^0 \le \tilde{b}^i, \quad i = 1, 2, \cdots, K_M, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad A^0 y^0 + \bar{r} \le \bar{r}_{int}, \\
&\quad r \le r_{\max}, \\
&\quad r \ge r_{\min}, \\
&\quad g^0 \le u \bullet Cap^0.
\end{aligned}$$

The operating reserves trade off against energy dispatch ($g^0 + r \le u \bullet Cap^0$), allocation of zonal interface capacity $\bar{r}$ trades off with power flow dispatch across the interface ($A^0 y^0 + \bar{r} \le \bar{r}_{int}$), and there are limits on use of individual reserves ($r \le r_{\max}$).

With sufficient regularity assumptions and a given unit commitment ($\tilde{u}$), if ($\tilde{d}^0, \tilde{g}^0, \tilde{r}, \tilde{\bar{r}}$) is a solution of the optimal dispatch, then it is also a solution of the approximation problem using "demand curves" to characterize the zonal value of expected unserved energy.

$$\underset{y^0, d^0, g^0, r, \bar{r}}{Max}\; B^0\left(d^0\right) - C^0\left(g^0, r, \tilde{u}\right) - \nabla ZVEUE\left(\tilde{d}^0, \tilde{g}^0, \tilde{r} - r_{\min}, \tilde{\bar{r}}, \tilde{u}\right)^t \left(d^0, g^0, r - r_{\min}, \bar{r}\right)$$

$$s.t.$$

$$y^0 = d^0 - g^0,$$

$$\iota^t y^0 = 0,$$

$$H^0 y^0 \leq b^0,$$

$$H^i y^0 \leq \tilde{b}^i, \quad i = 1, 2, \cdots, K_M,$$

$$g^0 + r \leq \tilde{u} \bullet Cap^0,$$

$$A^0 y^0 + \bar{r} \leq \bar{r}_{int},$$

$$r \leq r_{\max},$$

$$r \geq r_{\min},$$

$$g^0 \leq \tilde{u} \bullet Cap^0.$$

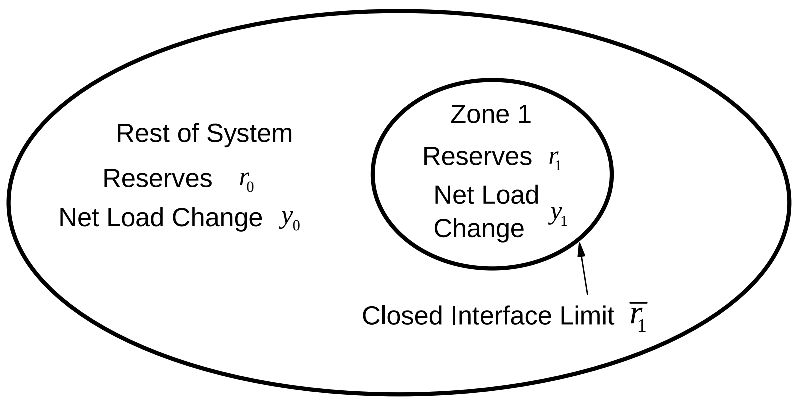
Hence if we could characterize the gradient of $ZVEUE$, this would open the way to an iterative solution of the dispatch problem with a demand curve for operating reserves (e.g, PIES method with diagonal demand). The gradient also allows estimation of $ZVEUE$ needed for full unit commitment problem.

Different variants of operating reserve demand curves can be and have been integrated with energy dispatch. A challenge for any locational operating reserve demand curve is to define a framework for deriving the form of the demand curve.

- **Generalize Loss of Load Probability (LOLP) and expected unserved energy from the aggregate system.** The simple model of loss of load from random changes in demand and generation provides a starting point but does not address locational interactions.
- **Integrate reservation of interface capacity.** A zonal model of interface capacity would include tradeoffs between normal energy dispatch and reservation of interface capacity to allow transfer of operating reserves.
- **Derive interaction between reserves in different locations.** Under some conditions, reserves in one location can support outages in another location.

The task is to define a locational operating reserve model that approximates and prices the dispatch decisions made by operators. To illustrate, consider the simplest case with one constrained zone and the rest of the system. The reserves are defined separately and there is a known transfer limit for the closed interface between the constrained zone and the rest of the system.

Zonal Interface Limit on Emergency Transfers



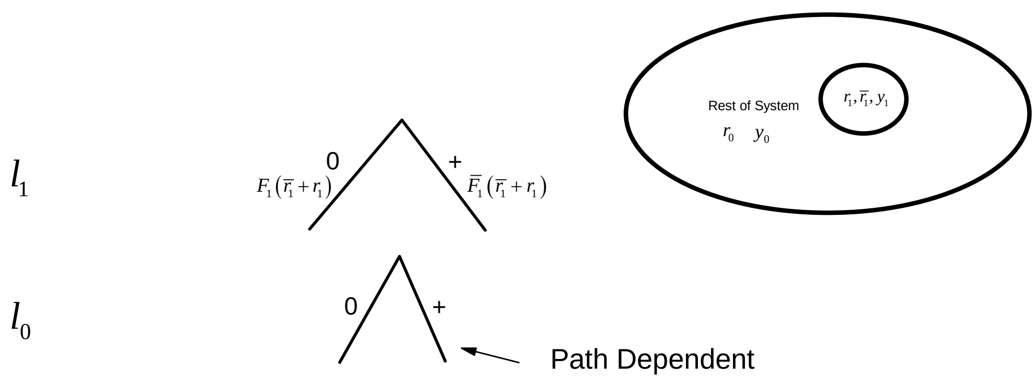
$$y_0 \sim f_0, y_1 \sim f_1, F_o(y_0) = \int_{-\infty}^{y_0} f_0(x_0)\,dx_0, F_1(y_1) = \int_{-\infty}^{y_1} f_1(x_1)\,dx_1$$

The zonal value of expected unserved energy (ZVEUE) would be an added component of the objective function in economic dispatch. The basic problem determines the configuration of lost load. The derivatives of ZVEUE define the demand curves for operating reserves.

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \middle| y_0+y_1-l_0-l_1\leq r_0+r_1,\, y_1-l_1\leq \overline{r}_1+r_1\right\}\right]$$

Loss of Load Probability Structure

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \middle| y_0+y_1-l_0-l_1\leq r_0+r_1,\, y_1-l_1\leq \overline{r}_1+r_1\right\}\right]$$



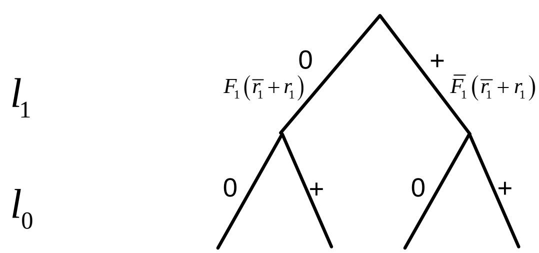
$$y_i \sim f_i,\; F_i\left(y_i\right)=\int_{-\infty}^{y_i} f_i\left(x_i\right)dx_i \qquad VOLL_0=v_0\leq VOLL_1=v_1.$$

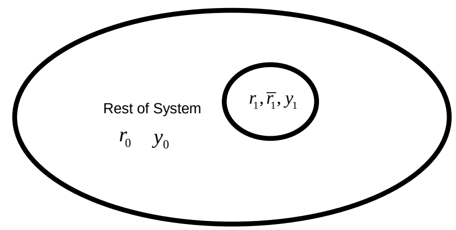
The full ZVEUE is difficult to characterize and calculate. However, inspection of the possible configurations of outages reveals the marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves

Loss of Load Probabilities

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \middle| y_0+y_1-l_0-l_1\leq r_0+r_1,\, y_1-l_1\leq \overline{r}_1+r_1\right\}\right]$$

Reserve Incremental Values



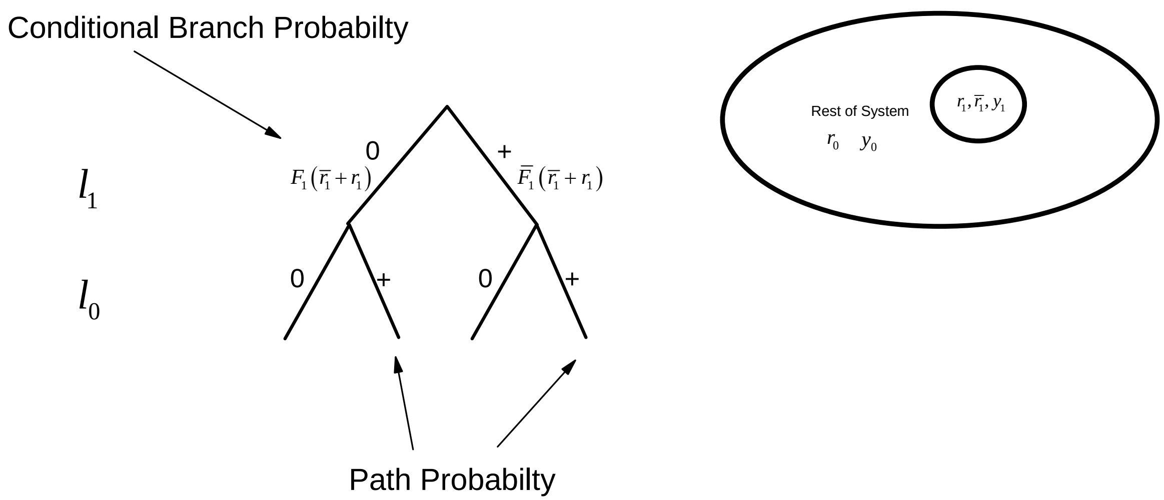


r_1	0	v_0	v_1	v_1
$\overline{r}_1$	0	0	v_1	v_1-v_0
r_0	0	v_0	0	v_0

The full ZVEUE is difficult to characterize and calculate. However, inspection of the possible configurations of outages reveals the probabilities for the possible marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves.

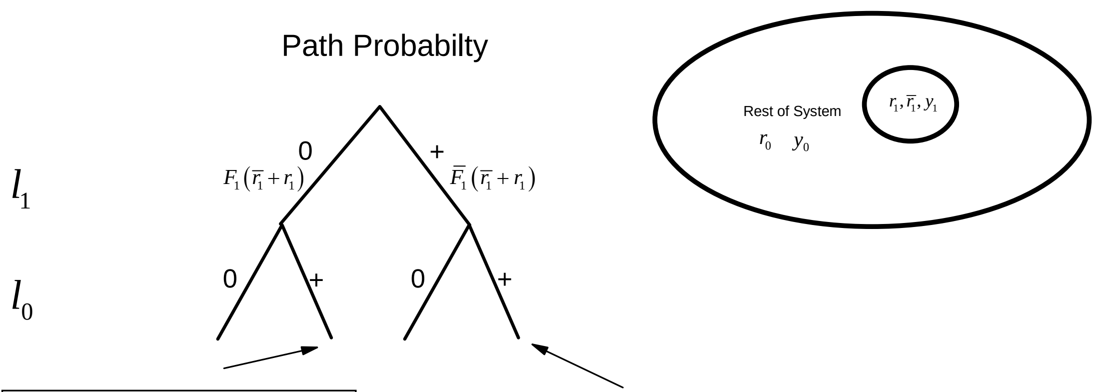
Loss of Load Probabilities

$$ZVEUE\left(r_0,\bar{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \,\middle|\, y_0+y_1-l_0-l_1\leq r_0+r_1,\ y_1-l_1\leq \bar{r}_1+r_1\right\}\right]$$



Assuming locational independence of outages, it is straightforward to calculate the probabilities on each path. The loss of load probabilities times the locational VOLL yields the operating reserve demand as a function of all the locational reserves and interface capacities.

Demand Curve Elements



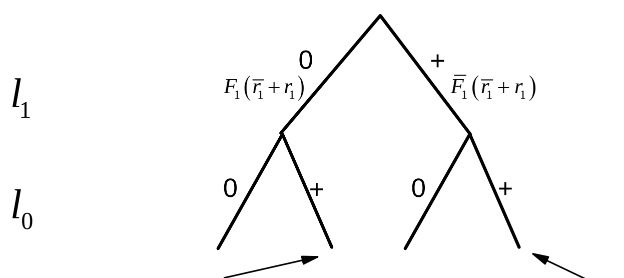
$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i = \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0(r_0 + r_1 - x_1)\, f_1(x_1)\, dx_1$$

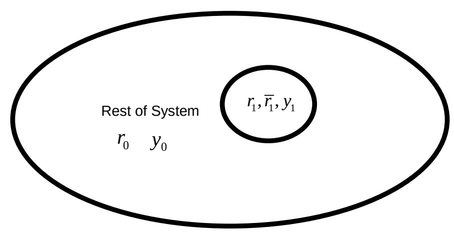
$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i(x_i)\, dx_i = \bar{F}_1(\bar{r}_1 + r_1)\, \bar{F}_0(r_0 - \bar{r}_1)$$

Assuming locational independence of outages, it is straightforward to calculate the probabilities on each path. The loss of load probabilities times the locational VOLL yields the operating reserve demand as a function of all the locational reserves and interface capacities.

Demand Curve Elements: Rest of System

$$p_{r_0} = v_o \left[\int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1 + \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_0\left(r_0 - \bar{r}_1\right) \right]$$





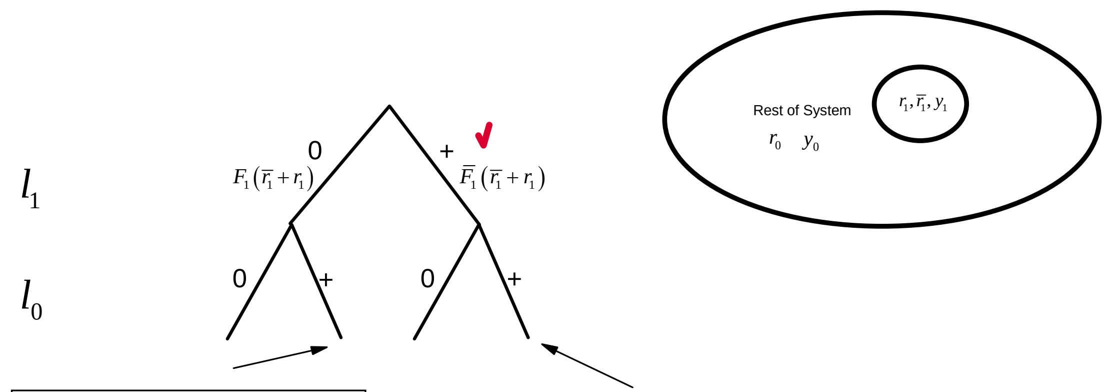
$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1$$

$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_0\left(r_0 - \bar{r}_1\right)$$

A similar inspection of the possible paths in the trees identifies the probability that an increment of operating reserve would change the unserved energy. The possible configurations of outages reveals the marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves.

Demand Curve Elements: Zone 1

$$p_{r_1} = v_1 \bar{F}_1\left(\bar{r}_1 + r_1\right) + v_o \left[\int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1 \right]$$



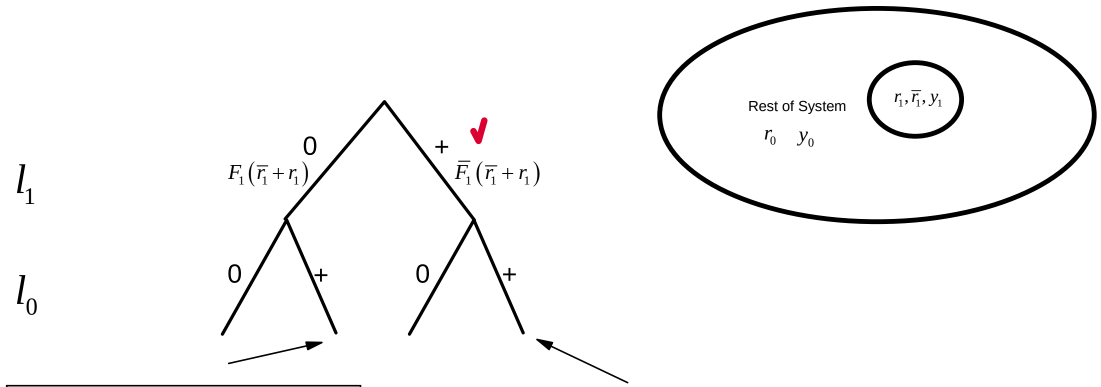
$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1$$

$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_0\left(r_0 - \bar{r}_1\right)$$

A similar calculation provides the demand for interface capacity as a function of the level of locational operating reserves and interface capacity.

Demand Curve Elements: Interface

$$p_{\bar{r}_1} = v_1 \bar{F}_1\left(\bar{r}_1 + r_1\right) - v_0\left[\bar{F}_1\left(\bar{r}_1 + r_1\right)\bar{F}_0\left(r_0 - \bar{r}_1\right)\right]$$



$$\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1-x_1}^{\infty}\prod_{i=0}^{1} f_i\left(x_i\right)dx_i = \int_{-\infty}^{\bar{r}_1+r_1}\bar{F}_0\left(r_0+r_1-x_1\right)f_1\left(x_1\right)dx_1$$

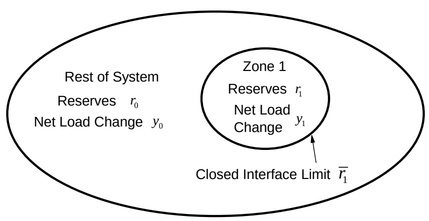
$$\int_{\bar{r}_1+r_1}^{\infty}\int_{r_0-\bar{r}_1}^{\infty}\prod_{i=0}^{1} f_i\left(x_i\right)dx_i = \bar{F}_1\left(\bar{r}_1+r_1\right)\bar{F}_0\left(r_0-\bar{r}_1\right)$$

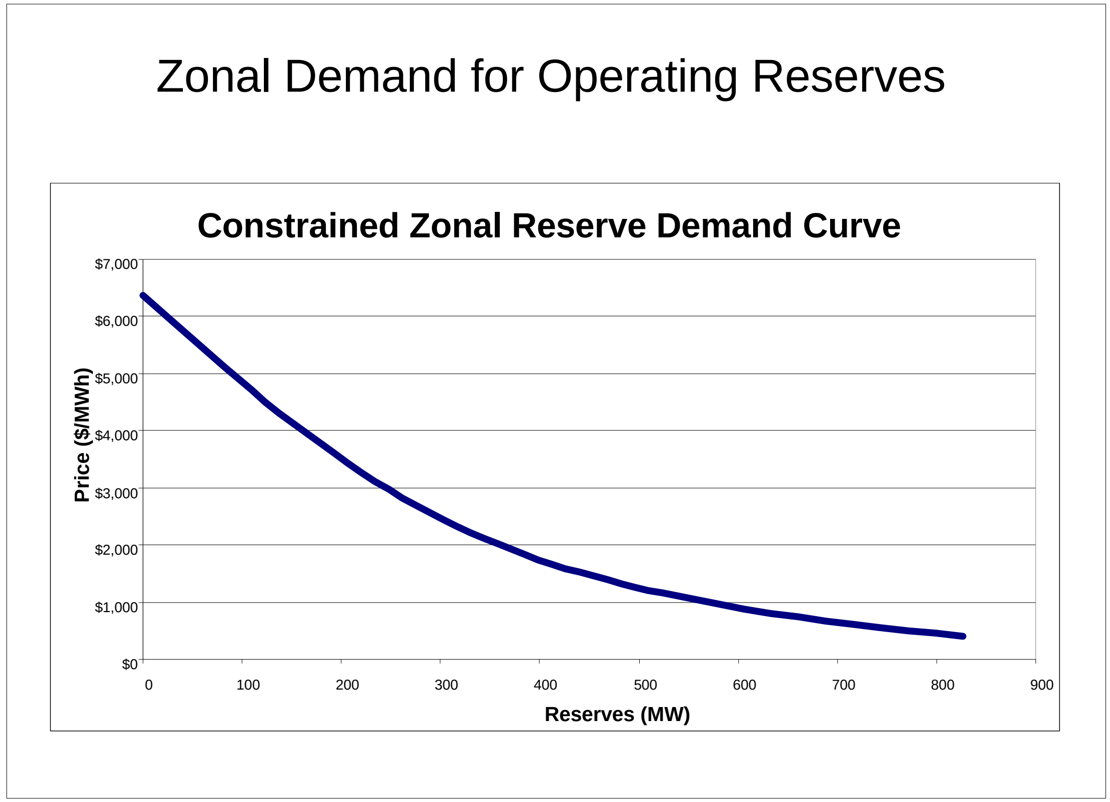
An illustrative demand curve for the constrained zone.

	ROS	Zone 1
Expected Total (MW)	107.10	45.90
Std Dev (MW)	488.99	209.57
VOLL ($/MWh)	7000	10000

	ROS	Zone 1	Interface
Benchmark (MW)	160.65	45.90	68.85

Zonal Interface Limit on Emergency Transfers





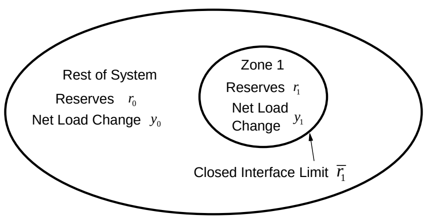
$$p_{r_1} = v_1\left(1 - F_1\left(\bar{r}_1 + r_1\right)\right) + v_0 \int_{-\infty}^{\bar{r}_1 + r_1} \left[1 - F_0\left(r_0 + r_1 - x_1\right)\right] f_1\left(x_1\right) dx_1$$

An illustrative demand curve for the rest of the system.

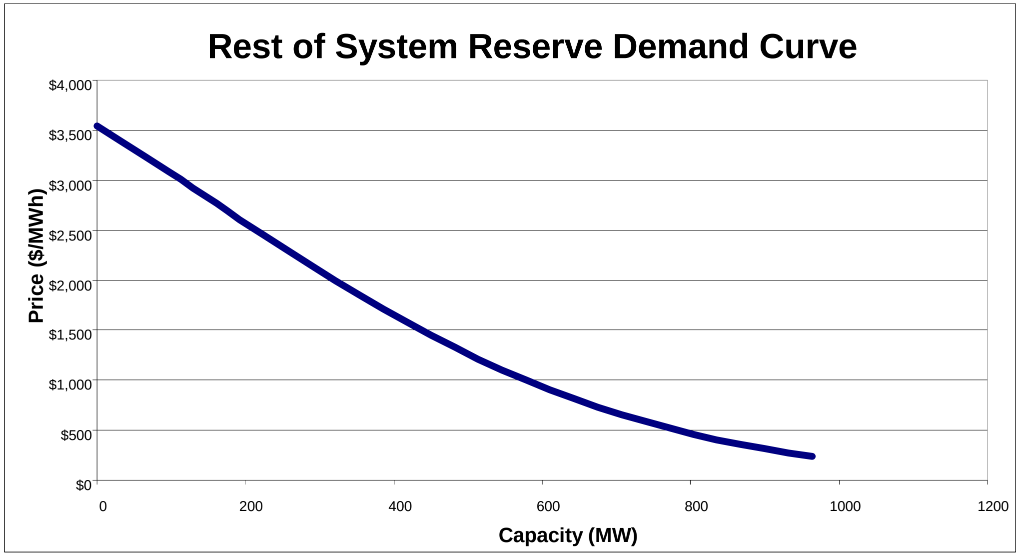
	ROS	Zone 1
Expected Total (MW)	107.10	45.90
Std Dev (MW)	488.99	209.57
VOLL ($/MWh)	7000	10000

	ROS	Zone 1	Interface
Benchmark (MW)	160.65	45.90	68.85

Zonal Interface Limit on Emergency Transfers



Demand for Operating Reserves



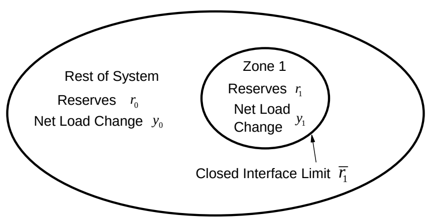
$$p_{r_0} = v_0 \int_{r_0 - \overline{r}_1}^{\infty} \left[1 - F_1 \left(r_0 + r_1 - x_0 \right) \right] f_0 \left(x_0 \right) dx_0$$

An illustrative demand curve for the interface capacity.

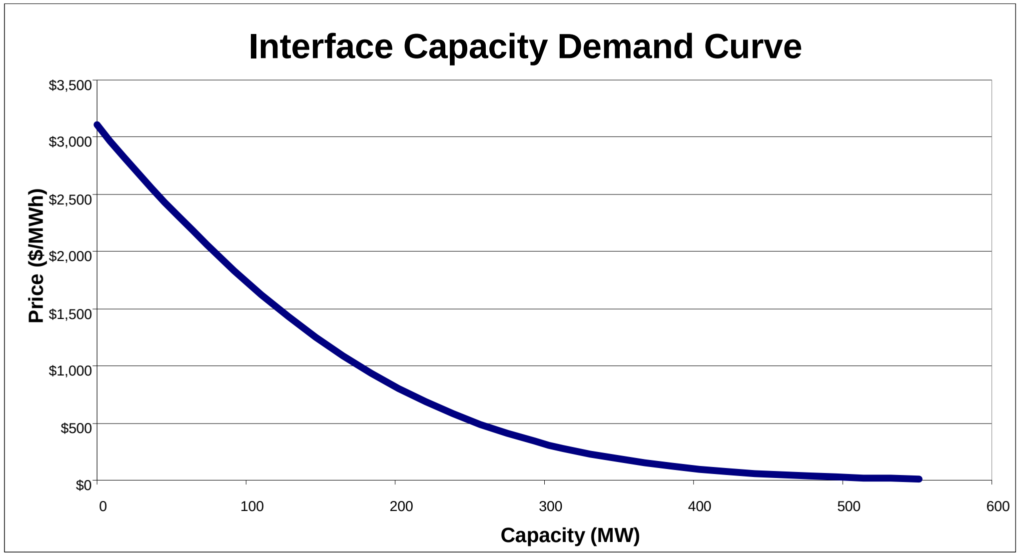
	ROS	Zone 1
Expected Total (MW)	107.10	45.90
Std Dev (MW)	488.99	209.57
VOLL ($/MWh)	7000	10000

	ROS	Zone 1	Interface
Benchmark (MW)	160.65	45.90	68.85

Zonal Interface Limit on Emergency Transfers



Zonal Demand for Transfer Limit

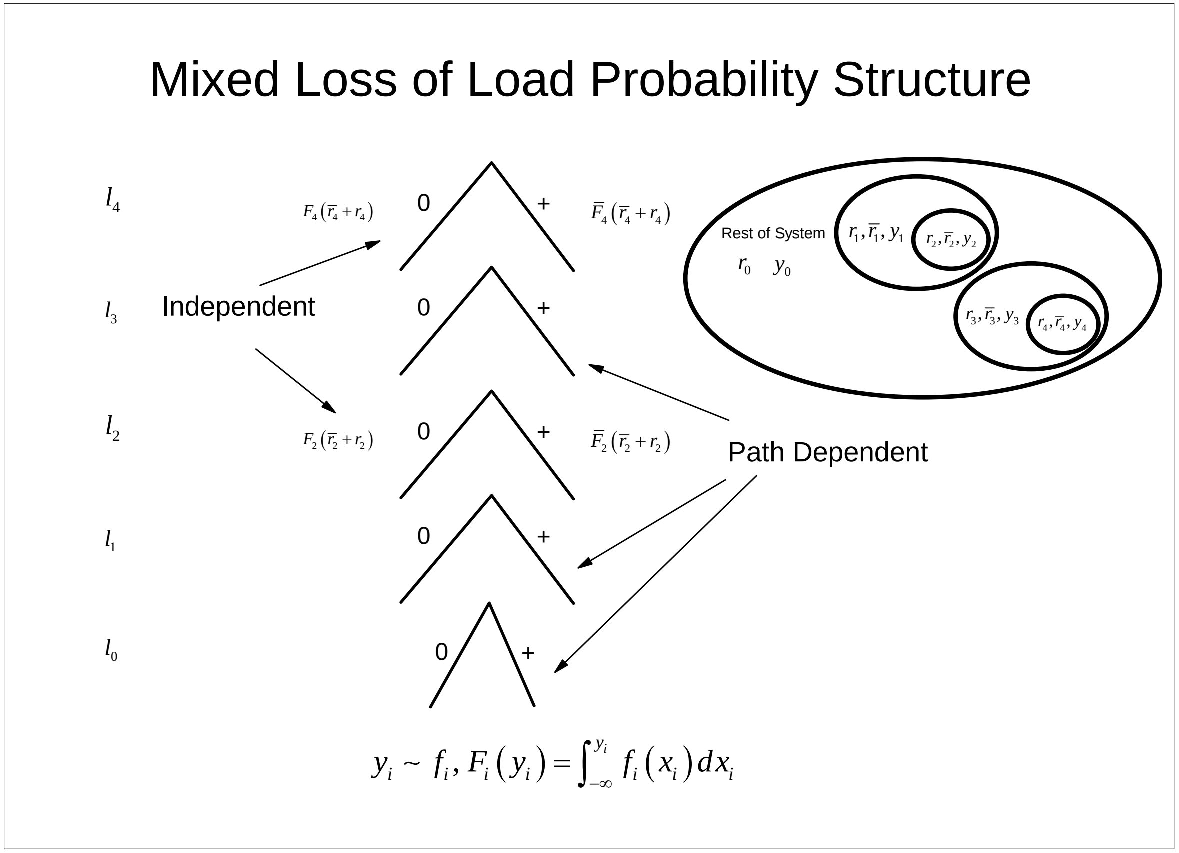


$$p_{\bar{r}_1} = v_1\left(1 - F_1\left(\bar{r}_1 + r_1\right)\right) - v_0\left(1 - F_0\left(r_0 - \bar{r}_1\right)\right)\left(1 - F_1\left(\bar{r}_1 + r_1\right)\right)$$

The tree structure identifies the loss probability dependencies and the paths where incremental capacity affects the losses.

- **Outages and Demand Changes.** The zonal convolutions of capacity outages and demand changes determine the (assumed independent) elementary zonal probability distributions of changes in net load.
- **Tree Structure.** The dependencies for losses and binding interface constraints defined by the probability tree structure determine the path probabilities for loss of load in each location as a function of the underlying independent elementary distributions.
- **Demand Curve.** The demand curve is determined by the value of lost load in each zone and the dependencies in the tree structure determining when reserves or interface capacity would be substitutable for losses.
    - **Value of Loss Load.** Assume embedded zones have higher incremental values of lost load.
    - **Substitution of Capacity.** Identify substitution possibilities on alternative paths for zonal losses and binding constraints. For example:
        - **Zonal Losses.** Apply only when interface constraint is binding.
        - **Reserve Substitution.** Higher level reserves substitute for lower level losses only when interface constraint is not binding.
        - **Interface Capacity.** Increased interface capacity for binding interface substitutes lower level losses for higher level losses.

The zonal model generalizes to multiple nested zones with a straightforward algorithm for calculating the implied operating reserve demand curve.[6]



6 For details, see W. Hogan, "Illustrating Two Models Of Locational Operating Reserve Demand Curves," October 2009. (available at www.whogan.com)

The probability trees provide a workable means for beginning with the locational probability distributions of load and outages and calculating the resulting demand curves. The appendix outlines the extensions to multiple nested and parallel zones.

The implied demand curves illustrate critical properties.

- **Interaction.** The demand curves are interdependent, but the dependence is not in the form of the nested or cascading model often assumed.
- **Maximum Value.** The value of loss load in the zone is an upper bound for the reserve price in the zone.
- **Convergence.** As the interface capacity increases, the implied demand curves in the constrained zone and for the rest of the system converge to the same prices.
- **Interface Demand.** In addition to the demand for operating reserves, there is an implied demand curve for the interface transfer limit.
- **No Thresholds.** The implied demand curve scarcity prices are positive at all levels. At higher reserves the prices are lower, but there is no threshold where the scarcity price falls to zero.

Improved pricing through an explicit operating reserve demand curve raises a number of issues.

Demand Response: Better pricing implemented through the operating reserve demand curve would provide an important signal and incentive for flexible demand participation in spot markets.

Price Spikes: A higher price would be part of the solution. Furthermore, the contribution to the “missing money” from better pricing would involve many more hours and smaller price increases.

Practical Implementation: The NYISO, ISONE and MISO implementations dispose of any argument that it would be impractical to implement an operating reserve demand curve. The only issues are the level of the appropriate price and the preferred model of locational reserves.

Operating Procedures: Implementing an operating reserve demand curve does not require changing the practices of system operators. Reserve and energy prices would be determined simultaneously treating decisions by the operators as being consistent with the adopted operating reserve demand curve.

Multiple Reserves: The demand curve would include different kinds of operating reserves, from spinning reserves to standby reserves.

Reliability: Market operating incentives would be better aligned with reliability requirements.

Market Power: Better pricing would remove ambiguity from analyses of high prices and distinguish (inefficient) economic withholding through high offers from (efficient) scarcity pricing derived from the operating reserve demand curve.

Hedging: The Basic Generation Service auction in New Jersey provides a prominent example that would yield an easy means for hedging small customers with better pricing.

Increased Costs: The higher average energy costs from use of an operating reserve demand curve do not automatically translate into higher costs for customers. In the aggregate, there is an argument that costs would be lower.

The expected value formulation with zonal approximation of the value of expected unserved energy provides a model for integrating zonal locational operating reserve requirements.

$$\underset{y^0, d^0, g^0, r, \bar{r}, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - ZVEUE\left(d^0, g^0, r - r_{\min}, \bar{r}, u\right)$$

$$
\begin{aligned}
& s.t. \\
& \quad y^0 = d^0 - g^0, \\
& \quad \iota^t y^0 = 0, \\
& \quad H^0 y^0 \le b^0, \\
& \quad H^i y^0 \le \tilde{b}^i, \quad i = 1, 2, \cdots, K_M, \\
& \quad g^0 + r \le u \bullet Cap^0, \\
& \quad A^0 y^0 + \bar{r} \le \bar{r}_{int}, \\
& \quad r \le r_{\max}, \\
& \quad r \ge r_{\min}, \\
& \quad g^0 \le u \bullet Cap^0.
\end{aligned}
$$

The operating reserves trade off against energy dispatch ($g^0 + r \le u \bullet Cap^0$), allocation of zonal interface capacity $\bar{r}$ trades off with power flow dispatch across the interface ($A^0 y^0 + \bar{r} \le \bar{r}_{int}$), and there are limits on use of individual reserves ($r \le r_{\max}$).

Compared to a perfect model, there are many simplifying assumptions needed to specify and operating reserve demand curve. The sketch of the operating reserve demand curve(s) in a network could be extended.

- **Empirical Estimation.** Use existing LOLP models or LOLP extensions with networks to estimate approximate LOLP distributions at nodes.
- **Value of Lost Load.** There are different estimates of lost load. For demand curve estimation the relevant value is the marginal of the average VOLL across the group that would first be curtailed in the event of an outage greater than the available reserves.
- **Multiple Periods.** Incorporate multiple periods of commitment and response time. Handled through the usual supply limits on ramping.
- **Operating Rules.** Incorporate up and down ramp rates, deratings, emergency procedures, etc.
- **Pricing incidence.** Charging participants for impact on operating reserve costs, with any balance included in uplift.[7]
- **Extended LMP.** Dispatch-based pricing that resolves inconsistencies by minimizing the total value of the price discrepancies in uplift difference between market clearing and economic dispatch.
- **…**

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[7] Brendan Kirby and Eric Hirst, “Allocating the Cost of Contingency Reserves,” *The Electricity Journal*, December 2003, 99. 39-47.

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